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Categorising different ways secondary school mathematics teachers use written materials for classroom work.

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CATEGORISING DIFFERENT WAYS SECONDARY SCHOOL MATHEMATICS TEACHERS USE WRITTEN MATERIALS FOR CLASSROOM WORK

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To Daniel, whose happy laughs are contagious.
To Alice, who came straight from Wonderland.
With all my love

ABSTRACT

The objectives of this work were to categorise ways in which experienced secondary mathematics teachers have been using written materials for classroom work and to investigate links between these practices and students' achievement of each teacher's short term aims.

Eight experienced English school teachers had their lessons observed with different groups of students: mixed-ability and groups set by levels of attainment (high, medium and low attainers). Six of these teachers were 'textbook users' and two were 'non-textbook users'. Each teacher eventually used some sort of written materials. A methodology based on qualitative analysis of data obtained mainly from classroom observation was built for this research.

From this study a categorisation of the teachers based on their use of written materials, was obtained, which took into account (1) the choice of materials made and (2) the strategies of use of the chosen materials. Among other findings, the investigation showed that most teachers in the sample changed the ways they used written materials when working with different attainment level groups. Examples of different strategies used by the teachers with different groups were produced by this study, indicating that teachers make their own decisions on how to use written materials.

For the investigation of the links between teacher's practices and student's achievement, teachers' decisions towards use of written materials were classified. Tests were developed based on each teacher's short term aims, with emphasis on tasks related to written material used during the lessons. All observed groups were tested. These tests were used only as indicators of the effects associated with the adopted strategies in using written materials, whether commercially produced or not. The results of this study also indicated that the ways written materials are used seem to be at least as important as the choice of materials itself.

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ACKNOWLEDGEMENTS

I would like to thank my supervisor, Professor Kath Hart for her dedication beyond the limits of a supervisor's obligation. Thank you Prof. Hart, for your patience, your suggestions, for the serious way you do and teach research and, most of all, for your friendship.

I would like also to acknowledge the financial support offered by the Federal University of Rio de Janeiro and by CAPES - Ministery of Education of Brazil for this research.

To the teachers in the sample, my most sincere admiration and gratitude. I know it was not easy having me snooping around. Good teachers you are, you taught me the complexity of the work you do. I also want to acknowledge the research students and lecturers from the School of Education, King's College London and from the Shell Centre for Mathematics Education, The University of Nottingham for many useful contributions to the present research. My special thanks to my co-supervisor, Professor David Johnson, for his suggestions and to Professor Margaret Brown, my examiner for the 'upgrading', for her recommendations at that time. My recognition to Chiz Dubé for her administrative support.

My love to Lilian Nasser, the best colleague someone could have when first arriving in London, to Dora Santos, who shared the 'hut' with me, and to my brave friend Marcia Pinto and her four lovely kids for many happy moments. 'Obrigado' to my colleagues from the 'Instituto de Matemática', Federal University of Rio de Janeiro for their encouragement. In special, to Maria Laura Leite Lopes, for introducing me to Mathematics Education. My recognition to Lucia Tinoco and Claudia Segadas, who were always very supportive.

My love and gratitude to Vera, or Veronica Getirana Gomes Ferreira, or the 'fellow Mathematics Educator' in this thesis. Her support during the last six months I spent in England can only be compared to her support in taking me to her place, in Recife, when this work was finally concluded. Thanks to Isabel, for helping me with the 'driver seat'. Thanks to my family and friends, for being so supportive during the time I was never there. Special thanks to Raffaella, Giuseppe and Rita, who seasoned the Nottingham years with Italian flavours and to Lúcio and Bia, for many nice walks by the seashore, back in Rio. To Luiz Carlos, who (in between projects) took care of my English; to Cesar, who took care of the kids; to my mother and my brother Alexandre, who took care of everything else, my love and gratitude. My only complaint goes to my father: Dad, you could have stayed a bit longer!

Thank you, Alice and Daniel, for being the best kids and the best friends a mother could ask for. This work is dedicated to you. I'm sorry for all the time I could not spend with you. I'm sorry to have had to ask you to become so responsible so early in your lives. Well, I suppose I should say I'm sorry to have moved you around like nomads but, looking back, it was great fun, was it not?

CHAPTER 1

INTRODUCTION AND BASIC DEFINITIONS

"When I use a word," Humpty Dumpty said in rather a scornful tone, "it means just what I choose it to mean - neither more nor less".

Lewis Carrol - 'Through the Looking-Glass'.

The use of printed materials for classroom work (especially those classified as 'textbooks') has been heavily criticised, even though the findings of surveys involving large numbers of schools suggest that teachers use these materials on a regular basis and that a great amount of classroom work is based on them.

The first objective of the present work can be described as obtaining a picture of how secondary mathematics teachers have been using written materials - either published or their own produced ones - when they give these materials to their students for mathematical school work.

It was assumed that different teachers would use written materials in different ways, so similarities and differences were expected. Based on this assumption, criteria should be generated to categorise the different ways teachers have been using these materials. It was also hoped that this work could establish whether there was any relationship between these categories of usage and students' achievement of teachers' aims. In fact, this can be considered as the second objective of this research.

In the end of this chapter, some basic definitions are introduced. In chapter two, the theoretical framework for the present study is established. A brief discussion of Piaget's theory of development is followed by summaries of the educational ideas of Ausubel, Vygotsy and Gagné. The constructivist theory of knowledge acquisition is also presented.

It is argued in chapter three that few research results could be found on use of textbooks. A review of this literature is presented. Research on teaching that could be relevant to the present work is also discussed in this chapter. On the other hand, literature on advice for teachers on the use of written materials is less rare. Chapter four contains a review of such materials.

From the discussion of the literature, it seemed that additional research was needed on the actual ways teachers use written materials for mathematical classroom work. It was also important to investigate the links between the ways materials are used and students' associated results. In chapter five, these problems are formulated as the two research questions which guided this work. Also, the associated design for data collection is built. Issues such as the samples of teachers and students, and the sources of data to be used are

discussed. A brief description of a pilot study, intended to test the instruments for the main data collection, closes the chapter.

In chapter six the methodology for the analysis of the data related to the first research question is established, with the aim of categorising the ways teachers use written materials for classroom work. In chapter seven, the methodology for the analysis of links between identified strategies in use of written materials and the results of the tests applied to measure the student's achievement of the teacher's short term aims are discussed.

In chapter eight, the data obtained from classroom observation of each teacher in the sample are analysed, searching for similarities and differences in the ways teachers use written materials when different groups of students are considered. This chapter is concluded by the main result of the first aspect of this study: a categorisation of the teachers in the sample as written material users.

The main objective of the analysis developed in chapter nine is to associate the ways materials are used when working with different groups of students, and the results of the corresponding tests. A discussion on the effects of different observed strategies is presented therein.

Finally, in chapter ten the results of this study are discussed in the light of the theoretical framework and of the review of literature. In chapter eleven, the conclusions of the present work are summarised, its implications and limitations are discussed, and some possibilities for future research are suggested.

This introduction cannot end before some definitions are presented, in order to clarify the meaning of the basic terms necessary for this research. For instance, what is a textbook? Is it any kind of written material published for classroom work or just those that are published in book format? So, for the purposes of this work:

written materials: are any sort of materials produced on paper to be used by the students for classroom work (or sometimes as homework). The term **'own produced written materials'** is used to designate unpublished materials developed either by the teacher or by the school.

printed materials: written materials that have been published, usually as small books, booklets or series of worksheets. Some of these materials are complemented by teacher's guides, with suggestions on how they should be used, but they do not necessarily provide either a complete source of instruction or suggestions for the progression of the content.

textbooks: printed materials that form a complete source for instruction. Usually a textbook is designed to provide a complete basis for classroom work, including explanations, examples and exercises and sometimes suggestions for assessment. It is also common for a textbook to be complemented by an answer book and/or a teacher's guide, offering suggestions on how the textbook should be used. They can be presented as a series of books, booklets or worksheets. For the purposes of this research, what characterises printed matter as a textbook is that it carries in itself a suggestion for the progression of the content.

During this work, other terms may require definition. Appendix 1.a. is a glossary of the main terms and their corresponding definitions. Finally, notice that even though the spelling of words adopted in this present report was the one used in England, whenever authors in the literature were quoted, their original spelling was kept.

CHAPTER 2

THEORETICAL FRAMEWORK

> It is important that researchers make explicit, to themselves as well as others, the theory or theories of teaching and learning and the conceptualizations of the nature of mathematics with which they are approaching the study ... Without explicit attention to them, the significance of a study may be obscured, making it easy for readers to dismiss the research as inconsequential, albeit interesting. (Thompson, 1992, p. 130)

This research is about texts in school mathematics. To be more specific, it is about the ways secondary mathematics teachers have been using printed written materials, such as textbooks, or have been using written materials produced by themselves instead. Issues such as the progression of the mathematical content, the pre-requisites for learning a specific topic, the acquisition of symbolic language and the differentiation among students will be part of the core of this work. On the other hand, mathematics teachers' decisions will be interpreted and analysed. In the present chapter, teaching and learning theories that have considered such issues are reviewed. They will provide theoretical support for research questions as well as theoretical support for data collection and analysis.

The works of Ausubel and Vygotsky are considered, and among the behaviourists, the work of Gagné, because his research took into consideration several ideas from the developmental theory of Jean Piaget. Piaget's work can also be considered as part of the common root which links Ausubel's and Vygotsky's views.

In this chapter, the former theories will be briefly presented and commented on, emphasising those aspects which are particularly relevant to the present work. In section 2.5 Constructivism is discussed. Constructivist ideas have been some of the most popular during the last few years among Mathematics Educators. Unfortunately, most of the material written under Constructivist influence does not take much notice of such issues as textbooks and written materials, as it is going to be discussed in chapter 4 (Advice Given to Teachers on How to Use Printed Materials). Nevertheless, the Constructivist theory has influenced teachers during the last few years and a study based on school observation would have to take in consideration its main ideas in order to obtain a better understanding of teachers' beliefs and attitudes.

To conclude the chapter, a brief summary is presented, linking the main ideas discussed in it with the present study's objectives, research questions and methodology.

2.1. Piaget's Theory of Development

> *In reality development is the essential process and each element of learning occurs as a function of total development rather than being an element which explains development.*
> *(Piaget, 1964, p. 176)*

Jean Piaget's theories have been considered fundamental for the understanding of the development of cognitive structures and functions during childhood. Although Piaget himself made clear that 'the problem of development in general and the problem of learning' are different (Piaget, 1964, pg. 176), educators who follow his ideas are aware that the developmental stage of a child is the very basis for deciding what, when and how to teach a specific topic. Probably due to the mathematical or scientific character of most of the Piagetian tasks, this influence shows very clearly among mathematics teachers and mathematics educators.

In this section, the stages of development proposed by Piaget are briefly described, with special emphasis on his ideas on an individual's construction of knowledge and cognitive structures.

Development and Learning

Piaget explained the difference between development and learning saying that 'the development of the knowledge is a spontaneous process ...[that] we must "re-situate" in its general biological and psychological context' and 'learning presents the opposite case. In general learning is provoked by situations ... opposed to spontaneous. In addition, it is a limited process, limited to a single problem, or to a single structure' (Piaget, 1964, p. 176)

To Piaget, the 'operational structures' are the basis of knowledge. Operational structures can be defined as a series of logical rules that allow the individual to act on his knowledge, modifying it. Operational structures also include the capacity 'to systematise or organise their process into coherent systems... All these structures interact and are co-ordinated into an efficient system... as a result of the organisation tendency [of an individual]' (Ginsburg and Opper, 1969, p. 137). Piaget (1961) provided some examples of operational structures such as: the capacity of using the number structure, the capacity to isolate an element from a complex structure, the understanding of the causes of a physical phenomenon and the ability of grouping objects to construct a classification. Piaget stated that 'the central problem of development is to understand the formation, elaboration, organisation and functioning of these structures' (Piaget, 1964, p. 177).

As a child grows older, he/she will be maturing and amplifying such structures. To Piaget, this process is a natural consequence of the general principle of adaptation: 'All organisms are born with a tendency to adapt to the environment. The ways in which

adaptation occurs differ from species to species, from individual to individual... from stage to stage within any one individual' (Ginsburg and Opper, 1969, p. 18). In order to progress in the process of adaptation to the surrounding world, a child will use mainly two mechanisms: assimilation and accommodation. Assimilation occurs when an organism treats or modifies a new experience in such a way as to become part of the existing structures. In other words, 'assimilation occurs when an organism uses something in its environment for some activity which is already part of its repertoire...accommodation, on the other hand, means the addition of new activities to an organism's repertoire or the modification of old activities in response to the impact of environmental events' (Berlyne, 1957, p. 38).

According to Piaget, by using these mechanisms, the child will slowly and continuously pass through stages of development. Piaget identified four basic stages of intellectual development, each underlined by different patterns of behaviour:

(a) **sensori-motor stage** (from 0 to 2 years) - 'it is the preverbal period or, speaking more generally, a period of direct action without representation' (Sinclair, 1968, p. 2). Sinclair (1968) described the changes in perception during this stage: 'the world around the subject becomes more and more stable and organized' (p. 2). Berlyne (1957, p. 39) explained further: 'the concept of an *object* is bound with objective notions of *space* and *causality* which the child ... has to build up gradually through interaction with the world' [his italics].

(b) **pre-operational stage** (from 2 to 7 years) - This is the period when the child starts developing the symbolic function. Some authors, as Berlyne (1957) sub-divided this period in two:

(b.1) the pre-conceptual thought period (from 2 to 4 years) - when the child will 're-learn on a conceptual level some lessons he has already mastered in the sensori-motor level' (Berlyne, 1957, p. 43). It is characterised by imitation, symbolic play and verbal achievements.

(b.2) the period of intuitive thought (from 4 to 7 years) - when 'the child ... is ... still dominated by its perceptions' (Berlyne, 1957, p. 44). This period is characterised by the acquisition of the ability to draw, mental images and verbal evocation, as well as the inability to take into account several aspects of the situation at once or in turns.

(c) **concrete operations stage** (from 7 to 13 years) - The child becomes able to demonstrate 'reasoning processes that would satisfy logicians' (Berlyne, 1957, p. 45). At this stage the child will become able to operate in the presence of the object or even in the presence of a representation of the object. 'The child can think in a logically coherent manner about objects that exist [in their previous experience]' (Sinclair, 1968, p. 5)

(d) **formal operation stage** (from about 13 years of age) - The child becomes able to 'bear on hypothesis ... and on behaviour and properties of objects that cannot be directly observed' (Sinclair, 1968, p. 8). The child will become capable of reasoning about propositions or ideas.

According to Piaget, the ages mentioned above are approximate. It is also important to notice that Piaget and Inhelder (1969) described the transitional period from one stage to the next as a product of a complex system of interactions. They identified some factors that contribute to such progression: (1)organic growth, (2)acquired experience in actions performed (not only physical experiences but also logical-mathematical ones), (3)social interaction and transmission, and (4)equilibration and self-regulation.

Finally, Piaget has also defined learning, as a process of accommodation-assimilation provoked by situations. He stated also that learning 'can only occur if there is active assimilation' (Piaget, 1964, p. 185).

Criticisms of Piaget's Theory

Piaget's theories generated a great amount of research projects, some of which confirmed some of Piaget's ideas while others refuted them. Berlyne (1957) presented a general view on the main points of criticism on Piaget's theory:

> Doubts have been expressed about the validity of the method of interrogation used for these studies and about the generality of the findings. Repetitions with other populations have not always produced the results that Piaget's work would lead one to expect' (p. 38)

He also pointed to the fact that data on the sensori-motor period 'came mainly from observation of Piaget's own children' and that 'he still does not pay much attention to questions of sampling' (Berlyne, 1957, p. 48) . Another criticism is based on the fact that

> except for some means and mean deviations in his reports of perceptual experiments, he provides few statistics. There are generally no measures of variance, which one suspects must be considerable, no test of significance, just a categorical statement... with a few specific illustrations (Berlyne, 1957, p. 48)

Several research projects have challenged Piaget's views. As an example, Gelman (1972) introduced a whole new set of experiments on number invariance for young children that challenges Piaget's conclusions. Gelman justified the 'importance of developing a number invariance test that differs from Piaget's' (p. 76) based on the fact that 'some investigations have suggested that failure on the number conservation task may derive from extra-logical difficulties' (p. 76). Such difficulties could be related with misunderstanding of the words used, or that the children could be led by the wrong clues while watching the experiment.

On the other hand, there have been several research projects involving large samples of school age children in western countries which either confirmed Piaget's main ideas (with some differences in the age range for each stage) or concluded that the same child could perform in different Piagetian stages depending on the task proposed - as an example of the

first kind of research see Inhelder (1954) - also reported in Berlyne (1957) - who studied a large sample of adolescents in Geneva and as an example of the second kind see the C. S. M. S. research (Hart (ed.), 1981, p. 20), that adapted a series of Piagetian tasks and was based on a large sample of children aged from 11 to 16 years in England.

Piaget's theory is the basis for the other educational theories discussed in this chapter. These theories show clearer relations with the issue 'printed materials in mathematics lessons', and will be better understood in the light of Piaget's ideas.

2.2. Ausubel's Learning Theory.

> *If I had to reduce all of educational psychology to just one principle, I would say this: the most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly (Ausubel, 1968, p. vi)*

This statement in the preface of '*Education Psychology: A Cognitive View*' by David Ausubel is the starting point to understand his theory of education. Ausubel considered that the human brain stores information in a well organised cognitive structure, where new information is stored linked with older related elements, forming a framework of conceptual hierarchies in which a more general concept will be the starting point of a series of branches leading to minor or more specific elements related with it. Ausubel called such structures 'subsumers' and used the term to refer to the existing relevant concepts of an individual's general cognitive structure. Figure 2.2.1, from Novak (1977, p. 75) shows the former ideas in a schematic form.

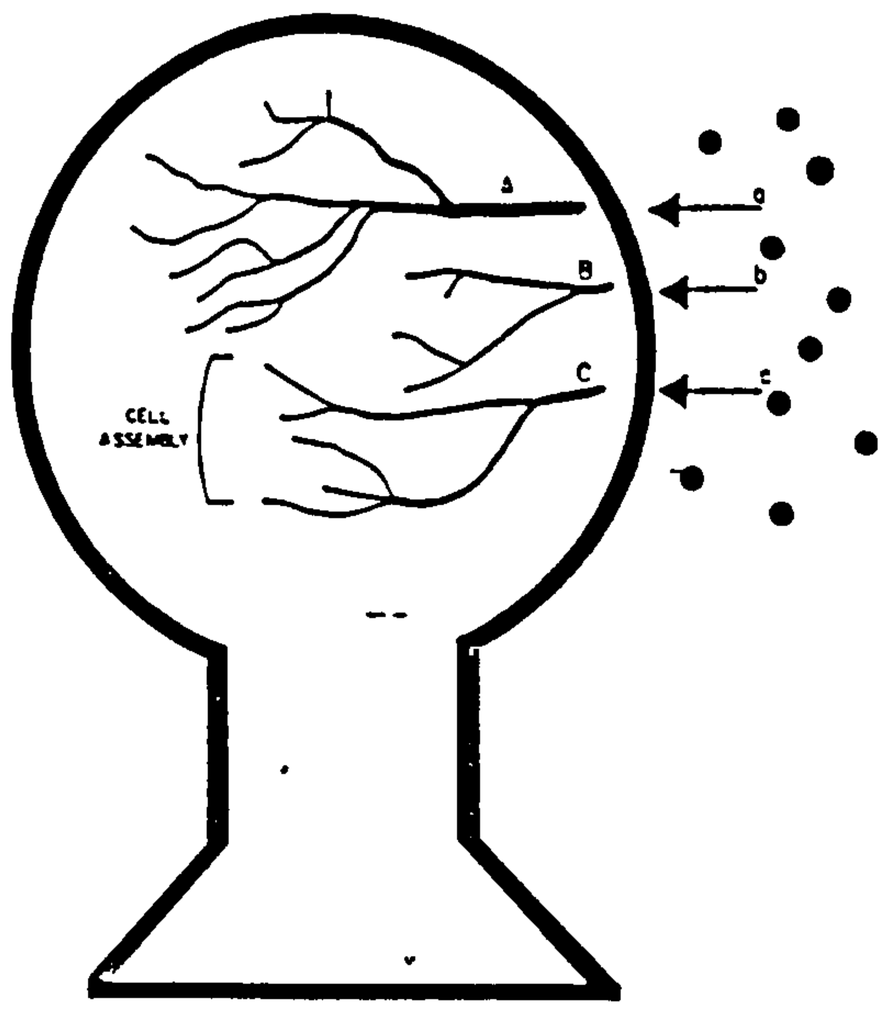


figure 2.2.1: In meaningful learning, new information a, b, and c are linked with existing relevant structures (subsumers) A, B, and C, respectively. Subsumer A is more highly differentiated than subsumers B or C. Biologically, subsumers may be considered complex cell sets.
(from Novak, 1977, p. 75)

So, by 'ascertain what a learner already knows' Ausubel was emphasising the importance of instructors determining what are the relevant concepts in an area of study as well as the branches related with it in the student's cognitive structure, and by 'teaching him accordingly', the importance of providing the students with activities leading to knowledge that could be linked to the recognised subsumers. As a consequence for teaching, teachers

should be aware of the importance of diagnosing the state of knowledge of the child, and matching the instruction to it.

Ausubel called 'meaningful learning' the process of linking new information to existing subsumers (modifying it during the assimilation process), as opposed to 'rote learning', when the information gathered does not have specific association with existing concepts in the cognitive structure. Novak (1977) studied Ausubel's theory in depth. In his book *'A Theory of Education'* he emphasised another relevant aspect:

> Since subsuming concepts can be substantially more differentiated in one individual than in another, the same new material can be learned very meaningfully by one person and almost by rote by another. (p. 26)

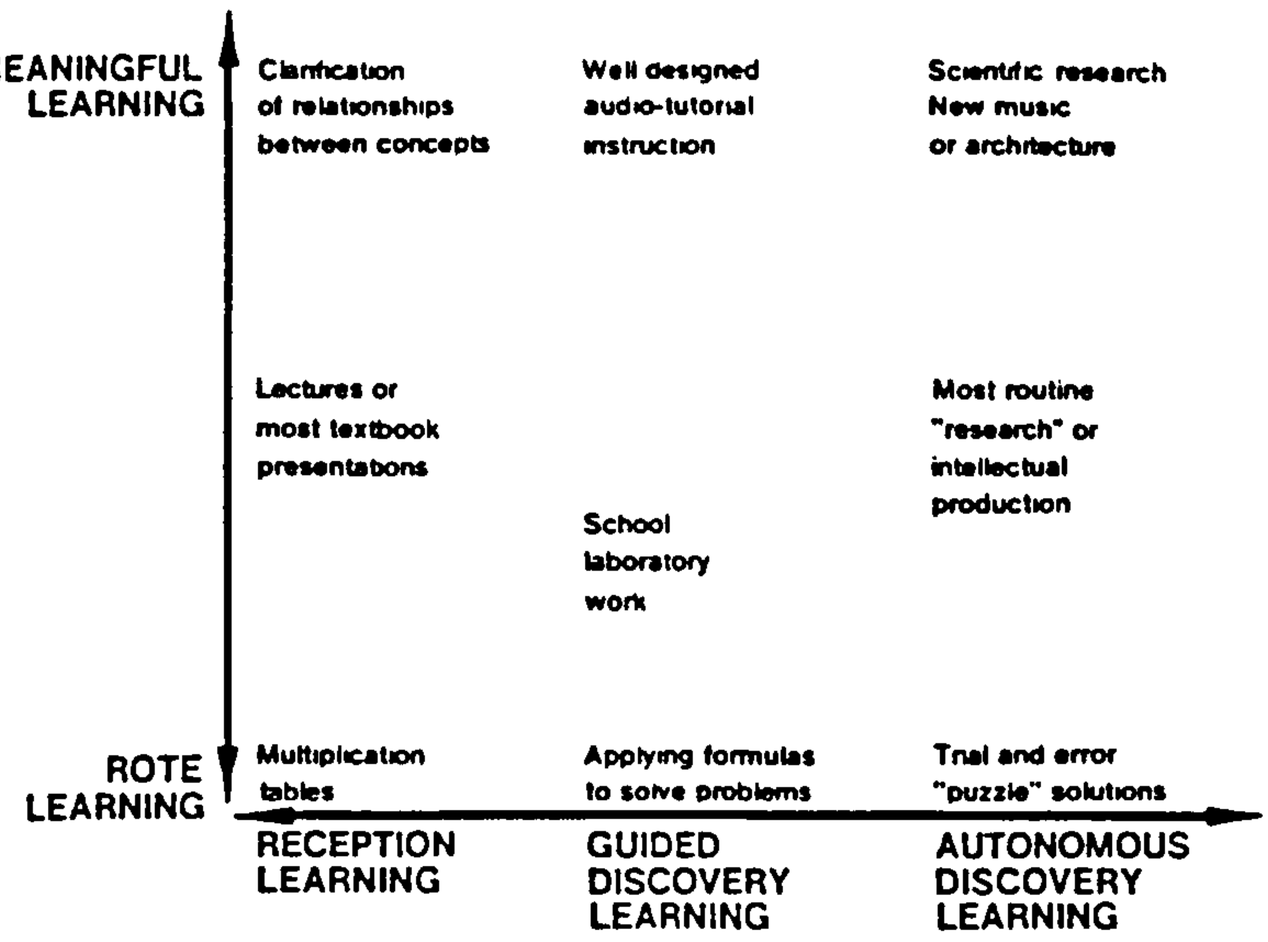


figure 2.2.2: Reception learning and discovery learning are on a continuum distinct from that rote learning and meaningful learning. Typical forms of learning are shown to illustrate where different representative activities would fit into the matrix (from Gowin and Novak, 1984, p. 8)

In fact, Novak (1977) considered a 'rote/meaningful continuum' rather than a dichotomy. He said:

> The real issue is not whether new information will be learned meaningfully or by absolute rote. The problem centres on the extent of meaningfulness in new learning...In some cases, whether some new information is learned by rote or meaningfully may be primarily a function of the individual's learning set, rather than a function of the learning material (p. 80-81).

Explaining the concept of 'rote learning' a bit further, Novak(1977) said that by school age 'complete rote learning' should not occur anymore, as the child already has enough subsumers to make at least some 'weak' links. He explained that 'rote learning occurs when

no conscious effort is made to associate new knowledge with a framework of concepts ... already in the cognitive structure' (p. 81).

This statement makes clear that, in Novak's point of view, Ausubel did not give priority to any teaching methodology. In fact, he and his followers distinguished between two continua: the 'rote/meaningful' one, and another considering the activities provided for the student. Figure 2.2.2, from Gowin and Novak (1984, p. 8) illustrates the different possible relationships between the rote/meaningful continuum and the reception/discovery learning continuum.

'The tremendous efficiency of meaningful learning as an information - processing and -storing mechanism' (Ausubel, 1968, p. 58), did not completely invalidate rote learning in Ausubel's point of view, as he admitted that there are some kinds of information (such as telephone numbers) that are inherently meaningless.

Another relevant feature of this theory is the importance given to affective aspects influencing cognitive learning. Ausubel reinforced that a positive attitude towards learning a specific subject is one of the most important factors in promoting learning. It is important to register here that Ausubel postulated that attitudes towards learning derive from internal sources and that some form of information from emotional signals is stored in the brain, although neither Ausubel nor his followers provided research evidence for this postulate. Figure 2.2.3, extracted from Novak (1977, p. 27) shows rote, meaningful and affective learning in schematic form.

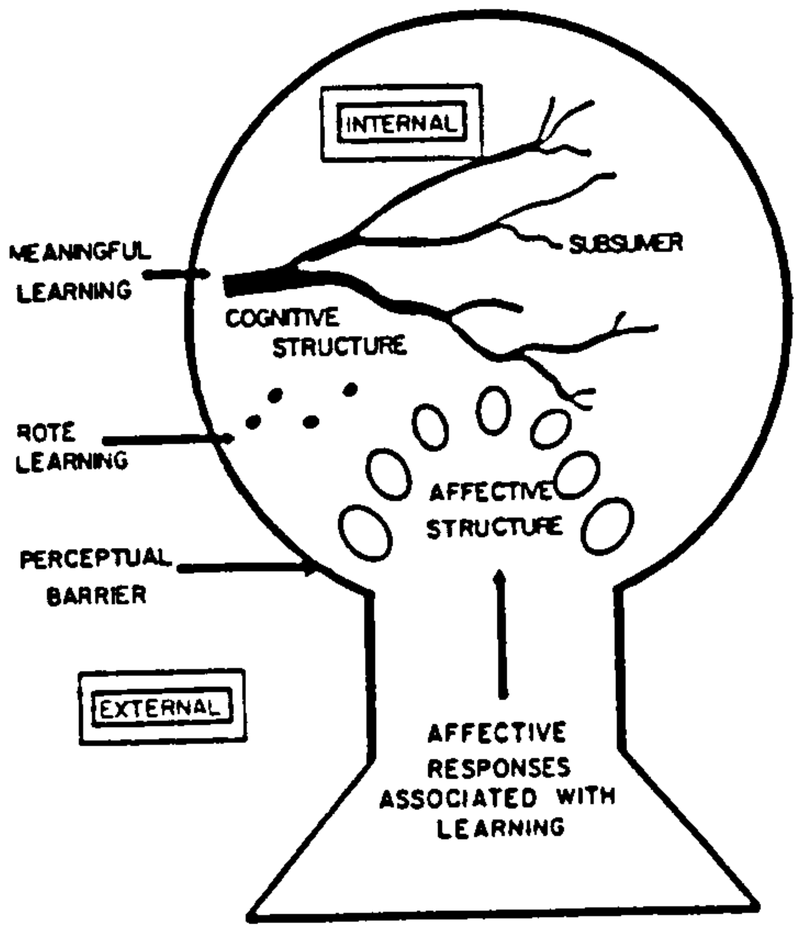


figure 2.2.3: Schema of cognitive learning (rote and meaningful) derived from external sources through perceptual barriers, and affective learning derived from internal sources. Affective structure cannot be defined theoretically at this time, but it is postulated that some form of information from internal (emotional) signals is stored in the brain (from Novak, 1977, p. 27)

In order to complete the picture on how learning occurs, Ausubel described the process starting from young children, under school age. He said that for those children the principal process by which concepts are acquired is 'concept formation'. Novak (1977) defined concept formation as a 'type of discovery learning involving hypothesis generation and testing as well as generalisation from specific instances (p. 77). Ausubel considered that process as the starting point, and it is linked with the acquisition of vocabulary. As the children go to school, the process becomes more complex, as Novak (1977) described it:

> ...most children have an adequate framework of concepts to allow meaningful reception learning to proceed and although concept formation may occasionally occur, most new concepts are acquired through concept assimilation, progressive differentiation and integrative reconciliation. (p. 78).

'Concept assimilation' is another way to say 'meaningful learning'. It is the process in which new information is added to a subsumer, modifying it. 'Progressive differentiation' is defined as being the process of organising subsumers themselves, in such a way that ' greater inclusiveness and greater specificity of regularities in objects or events are discerned and more propositional linkages with other related concepts are recognised' (Gowin and Novak, 1984, p. 97). 'Integrative reconciliation' occurs when 'two or more concepts are recognised as relatable in new propositional meanings and/or when conflicting meanings of concepts are resolved'. (Gowin and Novak, 1984, p. 97-98).

Ausubel's theory provided answers to a series of questions educators have been posing. The fitness of one single material for different students and the capacity of learning meaningfully from different types of materials and styles of teaching are some of the issues addressed. Nevertheless, some of his suggestions on how to improve teaching to solve these problems have been heavily criticised by researchers who sought to provide evidence for these suggestions. In order to facilitate the learning process, Ausubel suggested the use of 'Advance(Organizers', which should provide a bridge linking new knowledge with prior concepts. Figure 2.2.4, from Novak (1977, p. 79) illustrates the process. The Advance(Organisers should be 'the more general and inclusive ideas of a discipline' (Ausubel, 1968, p. 148) in order to provide an anchorage to related, more specific ideas in the same discipline. Ausubel (1968) suggested that *'the principal function of the organizers is to bridge the gap between what the learner already knows and what he needs to know before he can successfully learn the task at hand'* (p. 148, Ausubel's Italics).

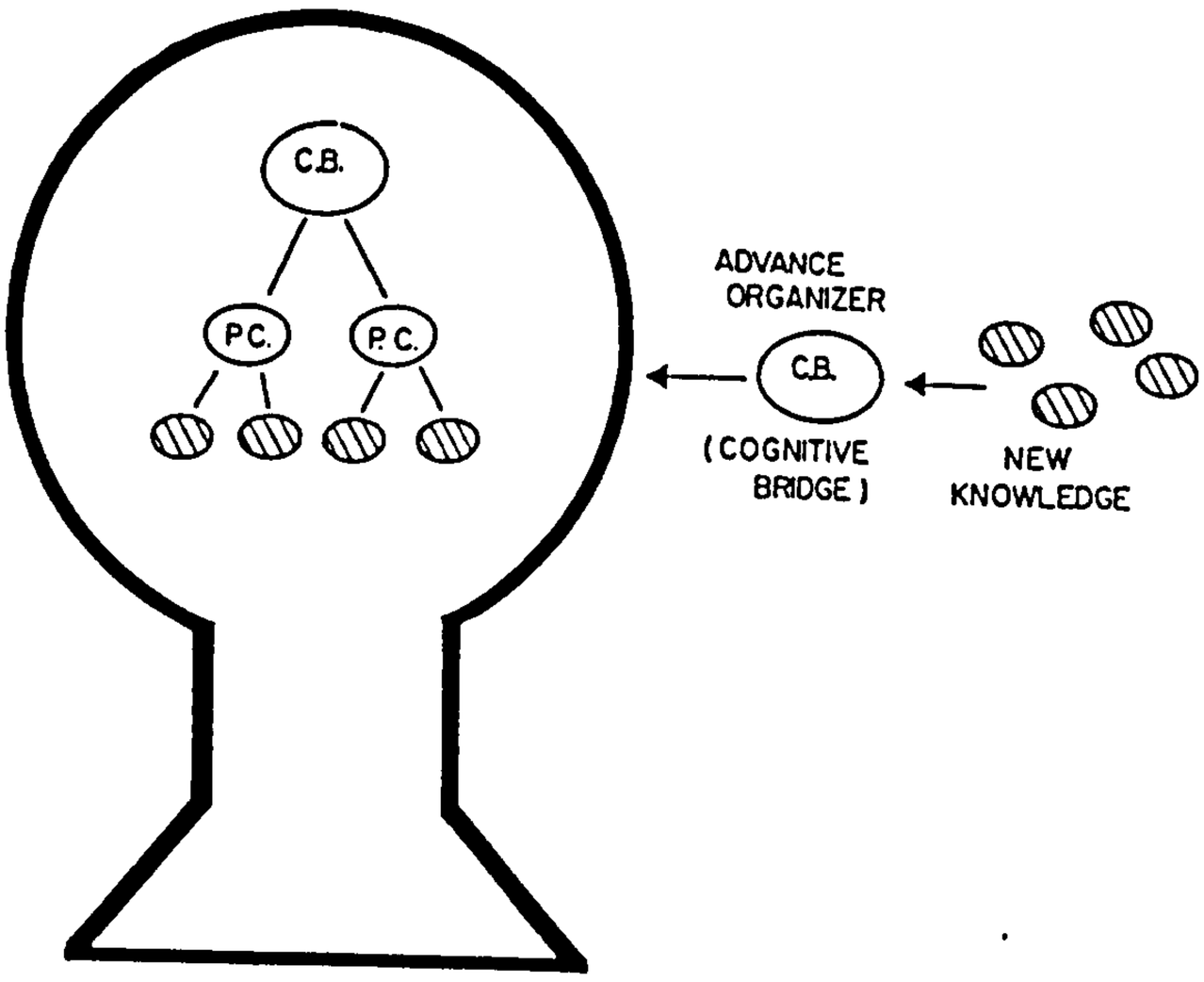


figure 2.2.4: Cognitive bridge (C. B.) serves to facilitate linkage of new information with prior concepts in cognitive structure; or to link previously learned concepts, thus facilitating integrative reconciliation (from Novak, 1977, p. 79)

Novak (1977) discussed such ideas in the light of several pieces of research carried out to implement Advance Organisers and with a wide range of results, from success to failure. He concluded that:

> Advance Organizers probably function only to the extent that some relevant subsumers exists, and association between existing subsumers and new information is perceived by the learner (which requires a little more than just a meaningful learning set)... Our research group has come to view advance organizers as functional only to the extent that they facilitate explicit cognitive bridging (p. 78).

Ausubel's idea of using 'more general and inclusive ideas' as anchorage was criticised based on pieces of research that pointed in the opposite direction: the more general ideas need to be built upon a series of previous related pieces of knowledge.

During the early sixties, Ausubel had a dispute with Bruner and other educators who defended discovery learning and problem solving as the best possible ways to promote learning. Ausubel defended a different point of view and a series of papers were published by both sides. In one of these papers (Ausubel, 1963) he stated his personal point of view as far as different methodologies and types of materials are concerned.

He said that it is not reasonable to embrace the views 'widely accepted ... that, by definition, all problem-solving and laboratory experience is ... meaningful and all expository verbal learning consists of rotely memorized glib verbalisms' (p. 84). Instead, he defended a view 'that both expository and problem solving techniques can be either rote or meaningful... In both instances meaningful learning takes place if the learning task can be related in non-arbitrary, substantive fashion to what the learner already knows, and if the learner adopts a corresponding learning set to do so' (p. 84).

Ausubel considered the proposition 'every man must discover for himself every bit of knowledge that he really wishes to possess' as a 'repudiation of the very concept of culture. For perhaps the most unique attribute of human culture ...[is] the fact that accumulated discoveries of millennia can be transmitted to each succeeding generation' (p. 85).

Ausubel made clear that he did not deny the importance of learning by discovery as one possible teaching methodology, but he defended another point of view: different students and different age groups will have different needs. He reinforced the importance of discovery when 'pupils are in the concrete stage of cognitive development' and even for 'older students... in learning subject-matter content on an intuitive basis' because 'even generally more mature students still tend to function on a relatively concrete level when confronted with a new ... area, in which they are as yet totally unsophisticated' (p. 87).

In this paper, Ausubel most of all defended an equilibrium between the acquisition of problem solving ability and the acquisition of subject matter by the students, otherwise, 'despite their adeptness at problem solving they would be unable to solve simple problems involving the applications of such content' (Ausubel, 1963, p. 94). Ausubel showed his point of view related with progression of the contents and critical thinking when criticising an overdose of discovery learning. He said:

> One principal difficulty with this approach, apart from the fact that it fails to promote the orderly, sequential growth of knowledge, is that critical thinking ability can only be enhanced within the context of a specific discipline (p. 93)

Ausubel accepted Piaget's ideas, although Novak (1977) assumed that 'Ausubel and I are closer to the thinking of Lev Vygotsky... who places stress on the role of language in mediating higher levels of cognitive functioning' (p. 120). Nevertheless, he concluded that 'no operational conflict exists between the ideas of Piaget and Ausubel... and Piaget's developmental periods have a descriptive validity even though they may not be the most useful functional characterisation of a cognitive learning process' (Novak, 1977, p. 122).

Stressing the differences between both theories, Novak (1977) said 'We see in Piaget's views a kind of psychological preformationism' (p. 124) and quoted Toumin (1972) to conclude that Piaget claims 'common destination of rational development in human individuals and communities alike' (p. 425). Compared with Piaget's ideas, Ausubel's ideas of meaningful learning seem to be more 'idiosyncratic, and the development of cognitive structures ... will depend upon the past sequences of ...experiences and on the kind of heritage in with he is embedded. Conceptual ontogeny of the individual proceeds not in a definitive pathway, but rather in a pattern determined by experience' (Novak, 1977, p. 124).

To Ausubel, critical thinking and reflectiveness can only be enhanced as the individual progresses within the content of a discipline. In this sense, reflectiveness becomes a consequence of knowledge. The consequences that are of interest to this research are those related to which material should be used by teachers in classroom situations. These materials

should match the child's state of knowledge as well as being organised in a progression that would facilitate a link between new acquisitions and previous related knowledge.

2.3. Vygotsky's Zones of Proximal Development

> *Learning is not development, however, properly organized learning results in development (Vygotsky, 1978, p. 90)*

Lev Vygotsky published in Russia during the thirties, but his books were first translated to English during the seventies, which means that it took a long time before his work became well known and accepted internationally.

Vygotsky reinforced the use of language, 'first acquired as a means of communication between the child and the people in his environment' (Vygotsky, 1986, p. 81) as subsequently becoming the organiser of the child's thought, 'upon conversion to internal speech' (Vygotsky, 1978, p. 37).

Vygotsky considered the social aspects of learning as being of major relevance when it comes to explain how knowledge is acquired. He said that '*human learning presupposes a specific social nature and a process by which children grow into the intellectual life of those around them*' [Vygotsky's italics] (1978, p. 88). In considering such social need to learn, Vygotsky emphasised the importance of asking and answering questions, as well as the process of imitating. He also valued positively the instructional role of an adult.

Although Vygotsky did not deny that pre-school (or non-school) learning is fundamentally different from school learning as far as systematisation is concerned, he stressed that 'systematicness' is not 'the only issue, there is also the fact that school learning introduces something fundamentally new into the child's development' (1978, p. 85). In order to explain this new issue, he introduced a new concept in theory of education - the zone of proximal development.

Vygotsky believed in development, but he did not use the word in the same sense as Piaget: Piaget saw development as organically determined and well categorised in stages while Vygotsky saw development as a dynamic process, strongly related to the acquisition of concepts, which can be spontaneous (based on a child's everyday experience) or 'scientific' (based on systematic lessons). Such a vision of development implied that he considered more than one level of development as fundamental in understanding a child's learning processes. He said that 'we cannot limit ourselves merely to determining developmental levels if we wish to discover the actual relations of the developmental process and learning capabilities. We must determine at least two developmental levels' (1978, p. 85). The first of these levels is called **actual developmental level** and can be defined as the 'level of development of a child's mental functions that has been stabilised as a result of certain already *completed* [his italics] developmental cycles' 1978, p. 85). The second one is called **potential developmental level** and is characterised by the functions that are in the process of maturation, but can be used when the child performs with the assistance of others. Vygotsky emphasised the importance of this second level saying that 'what children can do with the

assistance of the others might be in some sense even more indicative of their mental development than what they can do alone' (1978, p. 85)

The **zone of proximal development** can be defined as 'the distance between the actual developmental level as determined by independent problem solving and the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers' (Vygotsky, 1978, p. 86). Vygotsky also stated that the zones of proximal development permit one to 'delineate the child's immediate future and his dynamic developmental state' (1978, p. 87) and the 'zones of proximal development today will be the actual developmental level tomorrow' (1978, p. 86).

As a consequence for teaching, Vygotsky strongly criticised the orientation of learning towards the actual level of development. His experimental work was focused on process rather than in performance, and was based upon three main techniques as described in '*Thought and Language*' (Vygotsky, 1986):

(1) Introduction of obstacles or difficulties into the task, in order to disrupt conventional methods adopted by children.

(2) Provision of 'external aids' (a series of all kind of materials) or alternative ways to problem solving.

(3) Setting tasks that exceeded the child's knowledge and abilities, in order to determine his/her zones of proximal development.

The brief description of Vygotsky's main ideas above shows that it is not only in the different use of the word 'development' that his work differed from Piaget. While Piaget concentrated his theory on personal and internal acquisitions, Vygotsky focused his attention on social interactions and considered the unique and personal experiences of a child as fundamental in determining his/her cognitive development, which is close to Ausubel's views. Several research projects have been emphasising the importance of group interactions as an effective methodology. For example, Denvir and Brown (1986) commented on their conclusions for a research on Understanding of Number Concepts in Low Attaining 7-9 Years Old, which applied both individual and group teaching, that 'the advantages of individual teaching were not so clear cut as it seemed at first' and 'the children taught in a group seemed considerably more relaxed, were able to learn by watching other children and more eager to use materials, often responding to questions with actions rather than words' (p. 163).

Commenting about Piaget's ideas, Vygotsky said: 'We would have to point out that it is reality that is missing in his theory' (1986, p. 51). Notice also that Vygotsky considered instruction, guidance and even imitation as fundamental tools a child can use to learn and to develop fully the cognitive structures that are already in the zone of proximal development. Vygotsky considered instruction and guidance as fundamental tools to be used by a teacher and considered imitation and culture transmission as effective ways of learning. This implies

that for him, as for Ausubel, discovering and constructing are not the only ways a child could learn but only two of many possible ways.

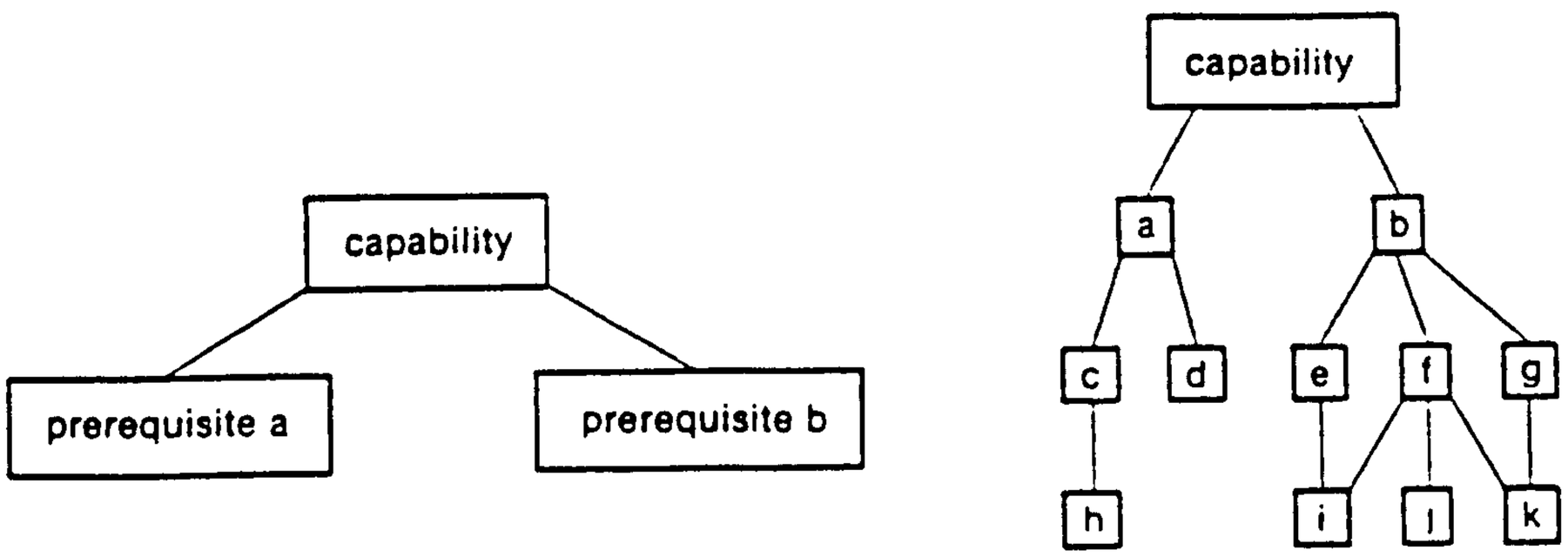
Like Ausubel's, Vygotsky's work is strongly related with school learning, in this sense it implies roles for the teacher and the teaching material used in classrooms. According to him, the teacher should provide suitable challenges for the students, as well as an environment where discussion can take place amongst students with effective help from the teacher, asking and answering questions. 'External aids', which include written materials are used to provide these challenges as well as a source of knowledge the children can refer to for help. Vygotsky's ideas could be used to evaluate teaching materials, as they should be suitable for the children in two different senses: not too easy, or no real learning would take place, and not too difficult, or the children would not even get started.

2.4. Gagné's Learning Theory

> *Learning is a change in human disposition or capability which can be retained and which is not simply ascribable to the process of growth. (Gagné, 1970, p. 3)*

Robert M. Gagné is considered an eclectic behaviourist. This designation came from the fact that, although he based his ideas on the classic behaviourist studies (for him, stimuli-response connections are the very root of the cognitive structure), he also accepted the fact that such a model was not adequate to explain 'higher levels' of learning. Gagné included part of Piaget's views in his work and from the group of Gestalt psychology, he borrowed the idea of "insight" in a restricted sense. Bigge (1982) commented that 'Gagné defines insight quite narrowly ... for him, an insight not only can be sudden, it must be.' (p. 141).

Although Gagné was aware of the importance of informal learning, his work is of special interest to this work because he mainly discussed formal learning. His work pointed in the direction of stabilising **conditions** influencing learning. He considered two different kinds of conditions: internal and external: ' These conditions, some of which are internal and some are external, make up the events of learning'.(Gagné, 1977, p. iv). He defined instruction as 'those events which are external, when deliberately planned and arranged' (Gagné, 1977, p. iv)



(a) (b)

figure 2.4.1 (a) and (b): Schematic forms for Gagné's learning hierarchies. (from Orton, 1992, p. 53)

Gagné's theory suggests that children learn in 'an ordered, additive, sequence of capabilities, each new capability being more complex and more advanced than the prerequisite capabilities on which it is built' (Orton, 1992, p. 52). Figure 2.4.1., from Orton (1992, p. 53) gives a schematic representation of this idea - which Gagné called **learning**

hierarchy (Gagné, 1977, p. 143). In order to learn a new capability, it is necessary to build from the pre-requisites (called a and b in figure 2.4.1. (a). The procedure is repeated by defining what prerequisites are required in order to attain pre-requisites a and b (figure 2.4.1. (b))

Orton (1992) said that 'as one might expect in education, things do not always work out perfectly. For example, one is likely to find there are pupils who possess the final capability but do not possess a or b or both. One also might find occasions when pupils can attain a or b without specific teaching in the process of learning the final capability' (p. 53). (see also Denvir and Brown, 1986). Gagné explained these differences by saying that a 'learning hierarchy ... describes an *on-the-average* efficient route to the attainment of an organised set of intellectual skills which represent "understanding" of a topic' [his italics] (Gagné, 1977, p. 151). Figure 2.4.2., (from Orton, 1992, p. 54, adapted from Gagné, 1970) shows an example of a learning hierarchy.

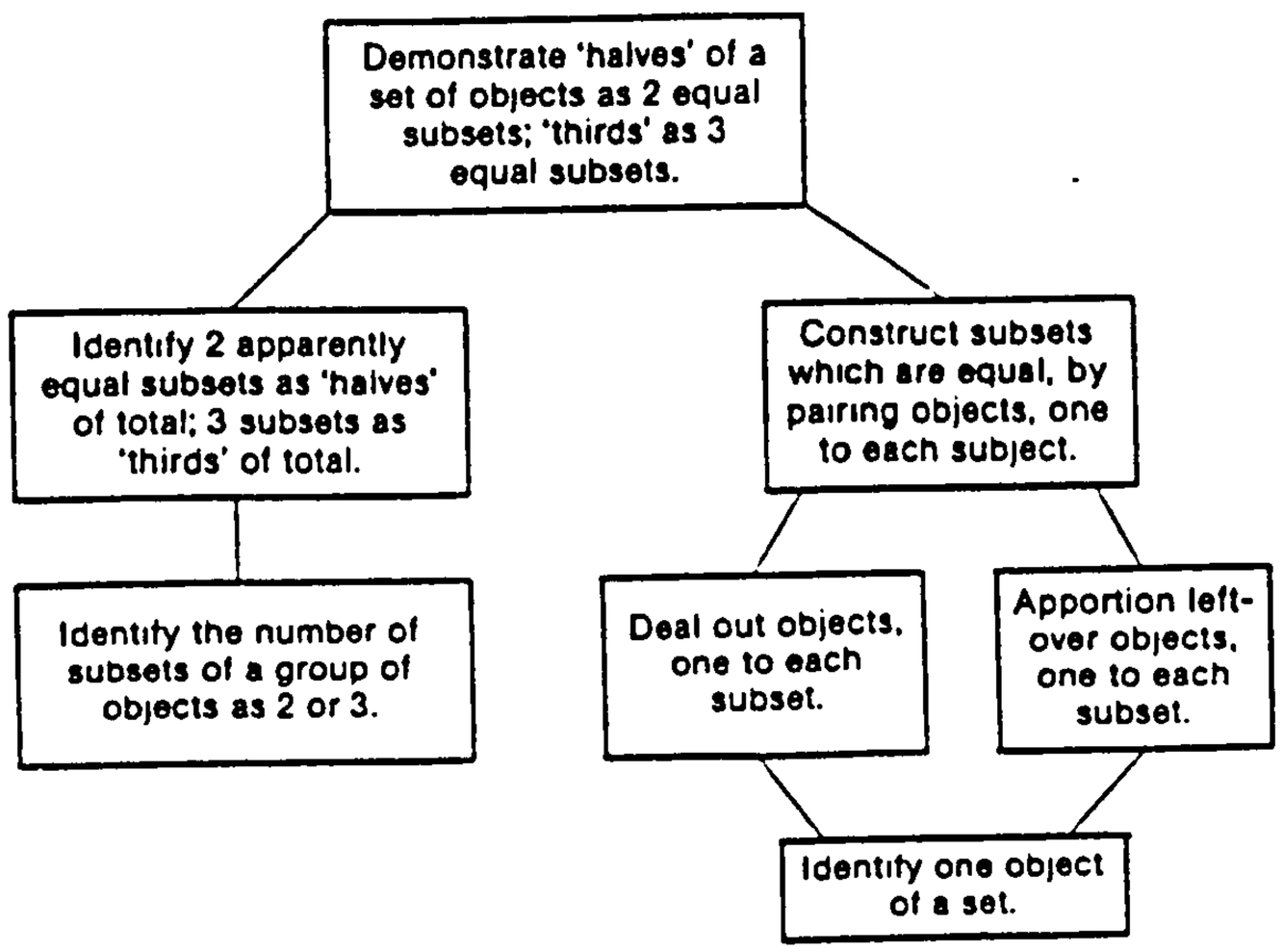


figure 2.4.2: A learning hierarchy for an early mathematical skill. (from Orton, 1992, p. 54, adapted from Gagné, 1970).

Gagné considered the existence of eight different basic forms of learning, hierarchically organised. They are: (1) signal learning; (2) stimulus-response learning; (3) chaining; (4) verbal associations; (5) discrimination learning; (6) concrete concept learning; (7) defined concepts and rule learning and (8) problem solving (Gagné, 1992, chapters 4, 5, 6 and 7). The first four levels involve predominantly early learning and are based on stimulus-response explanations. Orton (1992) provides a series of examples in mathematics that Gagné considered to be in these levels: learning number names, learning number symbols, ordering numbers, knowledge of the product of two numbers... (p. 56)

Gagné defined discrimination as a kind of capability which allows one individual to produce 'different responses to different members of a particular collection ... when several

distinctions are involved... [it is called] multiple discrimination' (Gagné, 1977, p. 105). About discrimination learning, Orton (1992) commented that 'the ability to discriminate is as important in mathematics as in any other subject... mathematics is full of subtle differences and similar symbolism' (p. 56).

Concept learning is defined by Gagné as the kind of 'learning which makes it possible for the individual to respond to things or events as a class' (Gagné, 1977, p. 111). Gagné studied separately the concrete concepts because

> the term "concept" has several meanings. Obviously, it [the theory] will not deal with them all at the same time and it would be a mistake to do so. First we should consider the most fundamental meaning of the term "concept". which is exhibited in individual behavior by responding to object-qualities such as those implied by the names red, double, circular and smooth or by common objects as cat, chair, tree and house (p. 111).

The seventh level is basically related to rules learning. Gagné considered the acquisition of defined concepts to be at the same level as learning rules because concepts of this kind are 'really rules for classifying objects and events' (Gagné, 1977, p. 127). "Defined concepts" are 'the kind of concept that are abstract in the sense that they involve relations. Some examples are physical concepts of mass and temperature... [and] mathematical concepts of square root and prime number' (Gagné, 1977, p. 111)

Gagné stated that 'these forms of learning build upon learnings that have preceded them. Although the learning of defined concepts and rules may well represent some frequent goals of a formal schooling process, it would be mistaken to believe that these goals can be reached by simply ignoring all other forms of learning or by pushing the latter into a trash can of unimportant events. The varieties of learning described here are possible only because they have been preceded by the acquisition of a set of simpler prerequisite capabilities' (Gagné, 1977, p. 128).

Problem solving is of special interest in this theory. Gagné stated that:

> one of the major reasons for learning rules is to use them in solving problems. The activity of problem solving is thus a natural extension of rule learning in which the most important part of the process takes place within the learner... Problem solving may be viewed as a process by which the learner discovers a combination of previous learned rules which can be applied to achieve a solution for a novel situation...one new learned entity is a "high order rule" which enables individuals to solve other problems of a similar type. (1977, pp. 155, 156)

Gagné included exploratory tasks as a kind of a problem solving situation and explained further why he considered problem solving as the highest level of learning.

Exploratory tasks are included in problem solving situations and Gagné explained the importance of problem solving, further saying that: '[the] nature and amount of guidance

provided by verbal instructions ... are minimal in problem solving situations... Problem solving as a method of learning requires that the learner *discover* the higher-order rule without specific help'. [his italics] (1977, p. 163). It is important to make clear that Gagné did not consider that it was possible for students to discover all the 'higher-order' rules. He stated that 'the evidence of experimental studies concerning the use of discovery in problem solving certainly does not demonstrate that higher-order rules must be learned by discovery' (1977, p. 164)

For each one of these learning levels, Gagné imposed a series of internal and external conditions in order to allow the learning situation to take place. Although these conditions are not going to be discussed here for all the levels, it's important to say that internal conditions for all levels (except the first) included previous acquisition of the prerequisites. The external conditions are defined as instructions when they are planned, Gagné presented those conditions in sections called '**educational implications**' in several chapters in his *Conditions of Learning* (1977). The matter for the higher levels is of special interest to this research because it includes suggestions on the use of didactic material.

Gagné said the acquisition of concepts is 'what makes instruction possible' (1977, p. 122). For him 'once the fundamental skill of reading has been acquired, concepts can often be introduced by means of instruction with accompanying pictures or diagrams' (p. 123). As to oral instruction, he stated that it can be used even earlier than written instructions, provided that the learner can understand all the concepts involved in the speech. For him, the acquisition of concepts liberated the students 'from the control of specific stimuli in their environment' (p. 123) and allowed them 'thereafter to learn by means of verbal instruction, presented orally or in printed form. They can also communicate their intentions, actions and thoughts to other people' (p. 123).

Coming to rules, Gagné emphasised that 'much school learning is a matter of learning rules, including rules that define. Typically, the learning of a topic or part of a course of study can be viewed as a hierarchy in which the most complex rules (representing the "target" objectives) require the learning of simpler rules as prerequisites ... learning hierarchies most often represent an ordered set of rules and concepts which the student needs to learn in order to achieve an understanding of the topic to be acquired' (Gagné, 1977, p. 152). He complemented this idea when talking about adult learning: 'much adult learning is accomplished by reading printed verbal statements in textbooks and other materials . Such verbal statements communicate the to-be-learned rule and often constitute the first step in its learning, particularly when the component concepts have previously been learned' (p. 151). So, it seems that for Gagné students should be moved during their school years to the adult position as learners, and should become able to learn from texts and other sources of self-instruction.

Finally, Gagné considered problem solving as the 'ultimate purpose of teaching' (Gagné, 1977, p. 177), stating that "much of the activity of problem solving is internally guided... Accordingly, the teachers' task is mainly one of finding and organising appropriate problem solving situations. Problems for students are most effective when they are (1) novel, in the sense of presenting unfamiliar situations, and (2) within the students' capabilities (that is, previously learned intellectual skills).' (p. 178).

As stated previously, Gagné accepted Piaget's views on development and discussed 'readiness to learn' several times, as in the quotation above. On the other hand, Gagné , like Ausubel, believed that a person needed not only to develop cognitive strategies but also to acquire a store of accessible knowledge in his/her memory: 'thus, it is not possible to concentrate education on the single aim of improving cognitive strategies, to the exclusion of other capabilities. Education programs must possess a balanced emphasis on all kinds of learning outcomes' (Gagné, 1977, p. 179). Novak (1977) compared the two theories as far as the higher levels are concerned and concluded that 'rule learning looks very much like Ausubel's subsumption and that new learning in problem solving reassembles Ausubel's... integrative reconciliation' (Novak, 1977, p. 125).

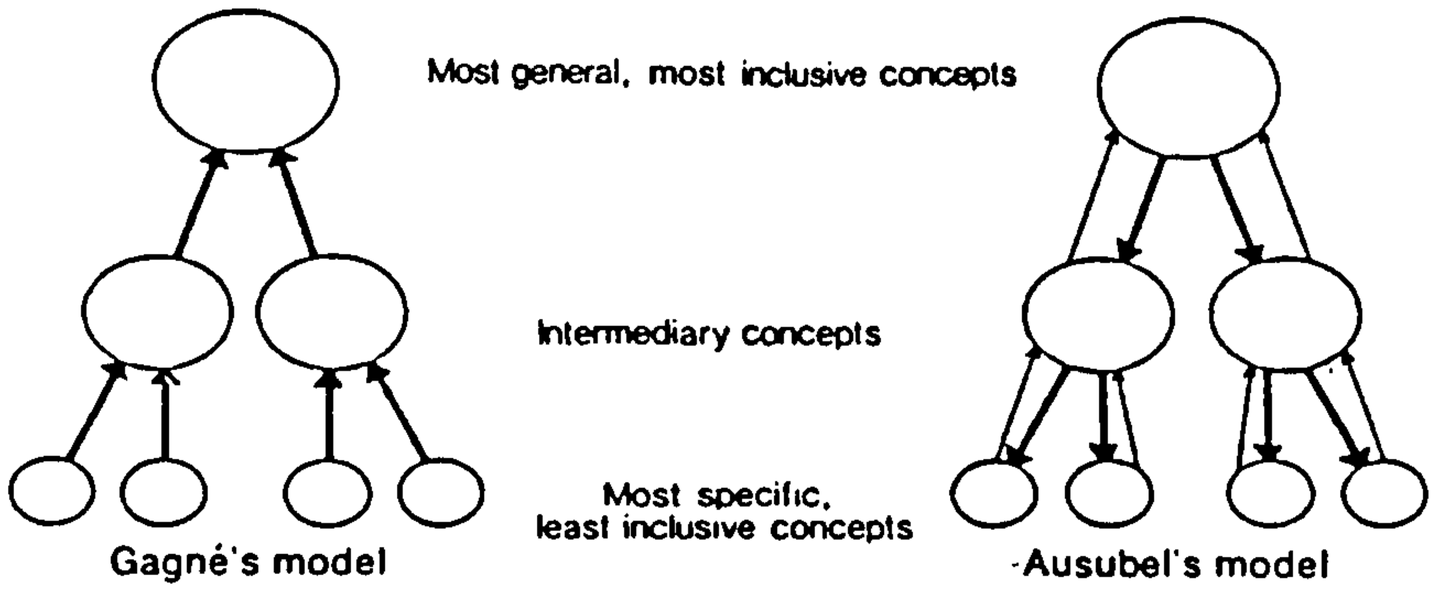


figure 2.4.3: Comparison of Gagné's and Ausubel's models of progressive concept development. (from Novak, 1977, p. 126)

Figure 2.4.3 (from Novak, 1977, p. 126) shows a comparison between Gagné and Ausubel's learning models. Gagné considered the arrows pointing upward, indicating the sequence in which learning must occur, while Ausubel considered the arrows pointing in the opposite direction. As Novak emphasises, 'the crucial factor is that, in whatever direction we proceed in teaching some knowledge hierarchy, our instruction must be planned so that relevant subsuming concepts exist or are developed... this problem becomes one of careful planning of instructional materials.' (p. 126).

Gagné has been criticised mainly for the behaviourist basis of his work. 'because his low esteem for the "insight" concept' (Bigge, 1982, p. 141) Gagné failed to give richer explanations for the acquisition of several capabilities that he considered as simple 'skills'. For example, 'he stated that a person cannot learn to read by developing insight. A Gestalt-field theorist would insist that not only can one learn to read through development of insight but that any genuine reading is an insightful process' (Bigge, 1982, p. 141). Summarising the criticism of Gagné's work, Bigge (1982) also said that Gagné did not provide adequate 'psychological bases for reflective teaching and learning, which is aimed towards students gaining significant understanding in an exploratory fashion' (p. 151) and concluded that many education theorists think that 'generalizations would not be products of mere cumulative process' (p. 151)

Novak (1977) also presented a criticism of Gagné's theory stating that 'the very foundation of all his learning model is an S-R association basic to all other learning' (pg. 126). In fact, radical behaviourists such as Skinner (1938) and Watson (1919) are quoted several times in the second edition of his *Conditions of Learning* (Gagné, 1970) and they seem to be quite influential in his work. Gagné believed a small child will start learning from signal and stimuli-response associations since these constitute the first levels of his hierarchy, which means that, in his point of view, all other learning processes are based on them. Novak (1977, p. 126) commented that somehow Gagné was contradicting his own statement as there 'can be little doubt that Watson's ideas that most forms of human learning could be accounted for by chains of conditioned responses is wildly incorrect' (Gagné, 1970, p. 13).

It's also important to say that 'much of the research carried out by Gagné and his colleagues has been concerned with whether the hypothesised prerequisites were necessary and sufficient [for the acquisition of the final capability]'. (Orton, 1992, p. 53). His learning hierarchies seem to be much better accepted among educators than Ausubel's advanced organisers, even among Ausubel's followers (see Novak, 1977, pp 125-127).

As far as this research is concerned, Gagné's ideas, if complemented by Ausubel's meaningful learning, provided a rich theoretical framework for classroom situations involving instruction and the use of written materials as well as other resources. In his point of view, written materials are sources of knowledge and can be used to help the learners to acquire new knowledge. They would provide guidance in relation to a progression of contents, which Gagné seems to defend in order to help the students to learn. These materials could also provide a good source of challenging problems and exploratory situations. Gagné also stated clearly that he did not defend the position adopted by radical constructivists (see next section) who say that transmission of knowledge does not occur.

2.5. Constructivist Theory of Knowledge Acquisition

> *Basically, to have "learned" means to have drawn conclusions from experience and to act accordingly.*
> *(von Glasersfeld, 1983, p. 55).*

Constructivism became one of the most popular theories among mathematics educators during the late eighties. Kilpatrick (1987), based on von Glasersfeld (1983), described it as 'an instrumentalist theory of cognition in which the mind is modelled as organising experience so as to deal with a real world that cannot itself be known' (p. 6).

According to von Glasersfeld (1990), the two basic principles that guide constructivism are:

1. Knowledge is not passively received either through the senses or by way of communication. Knowledge is actively built up by the cognizing subject.

2. (a). The function of cognition is adaptive, in the biological sense of the term, tending towards fit or viability.

(b). Cognition serves the subject's organisation of the experimental world, not the discovery of an objective ontological reality. (pp. 22-23).

The acceptance of these principles among mathematics educators has been discussed by several authors. Kilpatrick (1987) commented that

> As von Glasersfeld (1985) and Cobb (1986) have noted, the first of these principles is much more widely accepted than the second by people who think of themselves as constructivists. The first principle is one to which most cognitive scientists outside the behaviourist tradition would readily give assent, and almost no mathematics educator alive and writing today claims to believe otherwise. The second principle is the stumbling block for many people. It separates what von Glasersfeld (1985) calls *trivial* constructivism, what Cobb (1986) calls *empiricist oriented* constructivism, and what Davis and Mason (1986) call *simple* constructivism from the radical constructivism that is based on the acceptance of both principles' (his italics) (p. 7).

An attempt to unify these different views was made more recently by Ernest (1991), when defining 'social constructivism'. He proposed the acceptance of both principles of radical constructivism with the addition of two assumptions: realism ('there is an enduring physical world' (p. 23)) and social reality (' there is a language to be used as mediator in discussions' (p. 24)).

In the following paragraphs, a summary of the views defended by constructivists about the role of mathematics teachers in classroom situations is presented:

(1) Confrey (1990) presented an analysis of the 'usual' situation in mathematics classrooms (that she also called 'direct instruction' (p. 107)) followed by suggestions of changes:

Three key assumptions about mathematics instruction underlie direct instruction and are subject to challenge from a constructivist perspective:
1. Relatively short products are expected from students rather than process-oriented answers to questions; homework assignments and test items are accepted as providing adequate assessment to the success of instruction.
2. Teachers, for the most part, can simply execute their plans and routines, checking frequently to see if the student's responses are within desirable bounds, and only revising instruction when those bounds are exceeded...
3. The responsibility for determining if an adequate level of understanding has been reached lies primarily with the teacher (p. 107).

In a constructivist way of thinking, these assumptions are not acceptable, as von Glasersfeld (1983) said: 'mathematical knowledge cannot be reduced to a stock of retrievable "facts" but concerns the ability to compute new results.... and while reflection as such is not observable, its products *may* be inferred from observable responses' [his italics] (p. 58). Complementing this idea, Confrey (1990) suggested that the goal of instruction in mathematics could be stated in a constructivist way as :

> An instructor should promote and encourage the development for each individual within his/her class of a repertoire for powerful mathematical constructions for posing, constructing, exploring, solving and justifying mathematical problems and concepts and should seek to develop in students the capacity to reflect on and evaluate the quality of their constructions (p. 112).

She complemented this idea proposing three conditions that are necessary to the acceptance of this goal: '1. Teachers must build models of student's understanding of mathematics...; 2. Instruction is inherently interactive...; 3. ... the student must decide on the adequacy of his/her construction.' (p. 112).

(2) Steffe (1991) described one kind of interaction researcher-student called 'teaching experiment' in which 'the researcher acts as a teacher' (p. 177). These experiments are based on Piaget's clinical interviews and presuppose a one-to-one interaction (Because of that, Kilpatrick (1987) preferred to call them 'teaching interviews' (p. 17)). From these experiments, Steffe suggested ten principal goals for mathematics educators 'who opt for constructivism' (p. 191):

1. To learn to communicate mathematically with students.
2. To learn how to engage students in goal-directed mathematical activity.
3. To learn the mathematics of the students they teach.
4. To learn how to organize possible mathematical environments.
5. To learn the content of variable experiential fields - the mathematical experience of the students.
6. To learn the mathematics for the student they teach.

7. To learn how to foster reflection and abstraction in the context of goal-directed mathematical activities.
8. To learn how to encourage students to communicate mathematically among themselves.
9. To learn how to foster student motivation and how to sustain learning over a long period of time.
10. To learn to communicate pedagogically and mathematically with other mathematics educators (p. 191).

(3) Kilpatrick (1987) summarised radical constructivism's five consequences for educational practice as:

> (a) teaching (using procedures that aim at generating understanding) becomes sharply distinguished from training (using procedures that aim at repetitive behavior); (b) processes inferred as inside the student's head become more interesting than overt behavior; (c) linguistic communication becomes a process of guiding a student's learning, not a process for transferring knowledge; (d) student's deviations from teacher's expectations become means for understanding; and (e) teaching interviews become attempts not only to infer cognitive structures but also to modify them' (p. 12).

Kilpatrick discussed further these five points: About the differences between teaching and training he said: 'most people would probably argue that although the two concepts are different, training is a part of teaching when aimed at actions that display some intelligence' (p. 12). According to his point of view, there should be a clear distinction made between the two concepts, but not necessarily a dichotomy. When it comes to 'inside versus outside' (p. 13), he made clear that even radical constructivists have to make their inferences on what is happening in the pupil's mind from outside observable behaviour. He also suggested that several research works in Mathematics Education have been done emphasising the importance of unexpected errors (p. 16) and recognised that teaching interviews had been used before 'as a means of studying cognition' (p. 17).

Kilpatrick (1987) also commented that 'all five consequences fit the constructivist stance, but they appear to fit other philosophical positions as well' (p. 12). It appears that the same comment could be made about most of the advice given by Steffe and Confrey, and some practices suggested by them have been widely recommended by mathematics educators who follows Piaget, Ausubel or Vygotsky's ideas, as discussed early in this chapter.

When it comes to **learning**, constructivism uses the metaphor of knowledge being actively constructed by the learner, opposed to the metaphor of knowledge being transferred during teaching (Kilpatrick, 1987, p. 14). Another definition for learning was found, that complemented the one presented at the beginning of this section, given by von Glasersfeld (1983) which reinforces the adaptive character of learning: Bauersfeld (1988) stated that

'learning is characterized by the subjective reconstruction of societal means and models through the negotiation of meaning in social interaction and in the course of related personal activities' (p. 39). Both definitions are based on the second principle of radical constructivism, which denies the possibility 'to come to know a reality outside ourselves. Instead, all we can learn about are the world's constraints on us' (Kilpatrick, 1987, p. 9)

Kilpatrick (1987) said that 'as a theory of knowledge acquisition, constructivism *is not a theory of teaching or instruction* [italics added]. There is no necessary connection between how one views knowledge as being acquired and what instructional procedures one sees as optimal for getting that acquisition to occur' (pg. 11). For him 'the implication ...that certain teaching practice and views about instruction presuppose a constructivist view of knowledge is false' (pg. 12). This seems to be a reasonable explanation of why the role proposed for teachers by constructivism appears to have so much in common with theories of teaching and instruction. It would also provide an explanation of why the role of instructional materials in general, and books in particular, are not particularly discussed in the constructivist theory.

One exception is made by von Glasersfeld (1983) when commenting on written language. He challenged the usual view that '..."meaning" is conditioned by centuries of written language' (p. 51). According to his point of view, although 'written language and printed text have a physical persistence' (p. 51), the interpretation in which 'when we *understand* what we read ... we "grasped" the meaning of the printed words and we come to believe that this meaning was *in* the words' [his italics] (p. 51) is a 'misguided notion' (p. 52). Instead, he concluded that 'the compound of experiential elements' associated with a word 'cannot be anything but a compound of abstractions from the individual's own experience' (p. 53).

Supported by such a point of view, it is not surprising that advice on how to use printed materials in classroom situations is not so easily found in theoretical constructivists' papers. As a consequence of this lack of advice from the theory, most books aiming to give advice for mathematics teachers in the light of constructivism seem to consider printed materials as an improper tool to help teachers to promote learning in mathematics classrooms, as will be discussed further in chapter 4.

2.6. Summary

The main objective of this chapter was to make explicit the underlying theoretical framework for the present research. As Thompson (1992) said, even when they are not explicited, they are fundamental in any study. From the theoretical framework described in this chapter, it is important to highlight the following points, because they were essential in the processes of definition of research questions as well as in decisions about methodologies for data collection and analysis in the present work:

1- Ausubel and Gagné's educational theories are those more directly related to teaching. These theories discuss issues as progression of content, meaningful learning occurring during different instruction situations and the role of the school as a place to transmit accumulated culture. Without denying the importance of discovery learning and problem solving, they add importance to subject content knowledge and the role of the teacher as direct instructor. According to these theories, seat work is not the only activity that should take place in a mathematics lesson, and the teacher cannot be considered only as a classroom manager. The book cannot be considered only as a source of exercises, but it has to be considered as a reference material as well. It was from this point of view that the research questions were defined. These considerations were also quite important in the process of defining what data should be collected and were a fundamental background tool in defining how the data should be analysed.

2- Piaget and Vygotsky's theories provided the basis to understanding the child's learning process. Therefore, they were also an important background tool in analysing activities developed during the observed lessons. It was from the perspective gathered from these theories that the material to be used by the teacher was examined. In this sense, they were used to evaluate the suitability of instructional materials and learning activities.

3- Kilpatrick's (1987) point of view that Constructivism is a theory of knowledge acquisition and not a theory of teaching and instruction is adopted in the present research. As a consequence, it cannot be used to discuss the use of written materials in general and textbooks in particular. Nevertheless, as discussed in the following chapters, advice on use of material was offered to teachers in the light of 'applications of the Constructivist Theory'. In this sense, it could be useful to understand certain beliefs expressed by teachers in the sample. The misunderstanding of this theoretical approach is probably one of the underlying explanations for some radical advice offered during the eighties, which had to be challenged later, as exemplified by Good and Biddle (1988) when they argued that

> 'it seems reasonable to question the urging of some mathematics educators that the teacher's role be limited to only that of raising questions and allowing students to discover mathematics We suspect that most students will benefit from high-quality teachers statements about mathematics content [author's italics]' (pg. 132).

CHAPTER 3

REVIEW OF LITERATURE: RESEARCH RESULTS

The textbook continues to be a major influence on the classroom in many cases it still effectively determines the curriculum. How texts are written and selected for classroom use is therefore of paramount importance in curriculum development.(Howson, Keitel and Kilpatrick, 1981 - pg. 61).

The main interest of the present work is on how written materials have been used by teachers when they give such material to students for mathematical work. In this chapter a review of some research in this field is presented. Although it is possible to find in literature several references under the general headings 'textbooks' or 'didactic written materials', most of them are analyses of the books themselves or an analysis of some of their features, such as readability or illustrations. Very little research could be found on how teachers have been using this kind of material in real lesson situations. The first sections in this chapter are dedicated to review such research, and it was decided to look for such sources from the beginning of the nineteen eighties.

On the other hand, research on teaching could provide some useful directions for the present work. Its review is presented in section 3.4. Although no specific piece of literature could be found on how teachers make their decisions on textbooks (or other written materials for classroom work), research on teacher's decision making can be used to provide a framework to explain the choices made by the teachers in the sample of the present study. Research focused on the influence of teachers' beliefs in their classroom decisions is addressed in this section. Finally, in section 3.5 critical suggestions for future research on teaching found in the reviewed literature are discussed. These suggestions were detected mainly in texts which provided an overview on research on teaching, and had influenced the directions taken in the present work.

It is important to notice here that although research projects focusing on textbooks are rare, there are a considerable number of books and articles focusing on advice for teachers on how to use written materials for classroom situations. This literature will be addressed in chapter 4.

3.1. Surveys on How Teachers Use Textbooks

The first objective considered when it was decided to look for surveys on the use of resources, especially printed materials, was to obtain some clues on how teachers actually use textbooks. Unfortunately, not many surveys could be found including those for England, and even less if only secondary schools are considered. The findings in such surveys were obtained through teachers' answers to a questionnaire and research methodology did not include any kind of validation for such answers by for example classroom observation. As there were so few surveys, they usually are quite general. The emphasis in them is on time spent using printed materials or the amount of work done by the class based on them. The surveys did not consider different ways of using printed materials or what teachers who were not heavily dependent on such materials used instead.

The A.P.U. Report.

The *Assessment of Performance Unit* [A.P.U.] (1982) report presented the results of a survey in England, Northern Ireland and Wales on the use of textbooks in primary school classrooms. A questionnaire was answered by a large sample of primary teachers in these countries, with 96.5 percent returned. Figure 3.1 below, reproduces table 9.3 in the A.P.U report (p. 9.6), which shows the answers to a question about how much of a pupil's mathematical work was based on the set of textbooks used by the school.

Answer Given by Teacher % TIME	Range Covered by Answer % TIME	Percentage of Pupils	Cumulative Percentage Of Pupils	Cumulative Range % TIME
90% or more	85 - 100%	19%	19%	85% +
80%	75 - 84%	18%	37%	75% +
70%	65 - 74%	18%	56%	65% +
60%	55 - 64%	15%	71%	55% +
50%	45 - 54%	16%	86%	45% +
40%	35 - 44%	6%	93%	35% +
30%	25 - 34%	3%	96%	25% +
20%	15 - 24%	2%	98%	15% +
10% or less	0 - 14%	2%	-	-

Figure 3.1.1: Percentage of Pupils' Mathematical Work Based on Text Books in Use (APU, 1982)

The authors commented on the figures presented in the table saying that:

> The picture obtained is that for the majority of pupils the greater part of their mathematical work was based on the scheme in use. Three-quarters of pupils spent over half of their time on mathematics work based on the scheme, and, on average, two-thirds of each pupil's mathematical work was based on the scheme in use.

> Clearly, the scheme in use exerts a great influence on the mathematical diet provided for the pupil, since work based on it accounts for such a large proportion of pupil's mathematical experience in many cases. ... (p. 9.6)

The survey also gives us a list of the most popular series for primary mathematics at the time. They were: Alpha/Beta Series (30%), Fletcher Series (15%), Scottish Mathematics Group Series (12%), School Mathematics Project [S.M.P.] Series (9%) and Making Sure of Mathematics Series (6%). There were regional variations, but it's interesting to notice that all these series were published at least 10 years before the survey took place and the great majority of schools had been using the same series for three years or more.

As it was not expected to find many primary teachers at the beginning of the 80's with special mathematics qualifications, one could expect instead the book series, and in some cases the teacher's guide, to be the bases on which the curriculum was developed. The word 'scheme' is being used in the A.P.U. Report with the same meaning 'textbook' has in the present research. As the table reproduced above is about mathematics lessons, it seems clear that, as Howson et al (1981) said, textbooks are the major influence on classroom work, at least where primary mathematics lessons are concerned.

By the end of chapter 9 of the A.P.U. report, where the results of the primary survey were presented, the authors suggested that the 'pattern of use is a complex one' (p.9.21). They also commented that a 'more detailed examination would appear to be needed, in order to produce a fuller picture of textbook use...' (p. 9.21). The areas for further investigations recommended are: 'practice in selective use of textbooks', 'to discover which sections are omitted altogether from the pupils' mathematics and which sections are replaced by material from other sources'. They also admitted that such a survey was not easy to administer and should be carried out 'over only a restricted range of books' (p. 9.21)

Unfortunately the report does not develop the same kind of survey for secondary school (it gives priority to a survey in opportunity of learning specific mathematical topics instead). The report of the National Curriculum Evaluation, analysed in section 3.3 provides more recent information on these matters.

The I.E.A. Study of Mathematics.

In the I.E.A. Study of Mathematics II (Robitaille and Garden (eds.) - 1989), England and Wales[code: ENW] were jointly included, among 19 systems of education (which were, with respective codes: Belgium [Flemish] [BFL], Belgium [French] [BFR], Canada [British Columbia] [CBC], Canada [Ontario] [CON], Finland [FIN], France [FRA], Hong Kong [HKG], Hungary [HUN], Israel [ISR], Japan [JPN], Luxembourg [LUX], Netherlands [NTH], New Zealand [NZE], Nigeria [NGA], Scotland [SCO], Swaziland [SWA], Sweden [SWE], Thailand [THA] and United States of America [USA]) in an international study which was an

attempt to advance mathematics education through international comparisons of curricula, aims, attainments and attitudes of students and teacher in a wide variety of educational systems. The codes presented here were used throughout the report.

The results of this study were published in several volumes, the second one, Werry (1989), talked about the results of a survey on use of resources, textbooks included. Although it was not possible to obtain complete information about the criteria of sampling by reading the book, the editors said that the study on use of resources was based upon a questionnaire answered by about 6,000 classroom teachers and 4,000 principals or heads of mathematics departments in the schools included in the sample among 1980 and 1982. For other purposes, achievement tests were also administered to over 125,000 students divided into two age groups (populations A and B).

system	textbooks	published workbooks	individual material	visual material	own teacher material	published tests	own tests
BFL	****	**	*		***	*	***
BFR	***	*	*		****		***
CBC	****	**	*	*	****	*	***
CON	****	**	*	*	***	*	***
ENW	****	**	*		***		***
FIN	***	*			***	*	***
FRA	****	**			***		***
HKG	***	**	*	*	***	*	**
HUN	****	****			***	**	***
ISR	***	**			***		***
JPN	****	**			**	*	**
LUX	***	**			***		*
NTH	****	*			**	*	***
NZE	****	*	*		***	*	***
NGA	***	*	*	*	***	*	**
SCO	****	**	*		***	*	**
SWA	****	***	*	*	**	**	**
SWE	****	*	*		**	**	**
THA	***	**		*	***	*	***
USA	****	**	*	*	****	**	***

Figure 3.1.2: Use of teaching resources (Population A)

In the table, four stars may be interpreted as fairly frequent use of the resource by teachers, and no stars as isolated or infrequent use of the resource.

Figure 3.1.2 reproduces table 3.6 (pp. 53-54) from the report. For each of six different kinds of resources, teachers were asked to indicate the frequency of their use. To get a measure of the extent of the use of one resource, the percentage of teachers responding "often" was multiplied by three and added with the percentage of teachers responding "sometimes". The results were transformed into the five-point scale (from 0 to 4) shown.

Although Population B table is not presented in the text, the following comment lets us know that the situation in that case is almost the same:

> Population B teachers show the same broad level of use of resources: widespread use of published textbooks and workbooks, and their own teaching materials and tests, with little use, even less than Population A teachers on the whole, of individualized material, visual material or published tests.(p. 54)

From this study, it seems clear that teachers from all the systems have reported the use of textbooks (containing both explanations and exercises) as one of the major resources. Notice, however, that teachers also considered their own teaching materials and tests as important resources. Published workbooks seem to be relevant in some systems as well. Published tests are not widely used in any of the systems, although teachers in some of the systems, such as Hungary, have reported using them a little more often. In all researched systems, teachers do not find a great use for the kind of materials from which students may work individually, without a teacher's intervention, or for visual materials such as charts, posters, films, video, or overhead transparencies. Nigeria is the only country where more than 10 percent of teachers often use individualised material. Nigerian teachers also recorded the highest percent, 7 percent, of teachers using visual material often' (p. 54).

As far as England is concerned, this international study not only reinforces the picture the A.P.U. Report gave for primary mathematics but also adds information about the use of textbooks in secondary mathematics. It seems to be the same as for primary schools: secondary schools teachers also use textbooks as sources of instruction and exercises.

Other Surveys Referenced in the Literature.

During the literature search related to the use of printed materials for classroom mathematical work, several references to surveys that were developed in the United States of America on this matter were found. For example: Howson et al.(1981) refers to some of them (as the survey supported by the National Science Foundation, 1978) and concludes 'Commercial textbooks in USA have set up elaborate networks for promotion and distribution of their products. Teachers in USA depend heavily on textbooks. This dependence has roots that go back to the introduction of free public education into a frontier society: the textbook was used to compensate for a shortage of well educated teachers...'.

Also Woodward (1987) refers to many surveys on use of textbooks carried out in USA during late 70's and early 80's, not only in Mathematics but covering almost all disciplines. He summarises their findings: "Formal qualitative and quantitative studies in the US have shown that instructional materials especially textbooks, structure up to 90% of instructional time and personal observations in US elementary and junior high schools

confirm that many teachers are highly dependent on teacher's guide and textbooks to organise day-to-day instruction. And the likelihood is that, whether or not textbooks used in a classroom fit a curriculum, what is contained in the textbook is what many teachers teach and students learn." (p. 511)

Because this research is especially interested in what is happening in England, it has been decided not to go further in these surveys about the USA, but it is interesting to notice that textbooks are also considered a main resource used by teachers there.

3.2. Other Research Results.

The particular interest of this research is related to the actual ways secondary mathematics teachers have been using not only commercially produced textbooks but also their own written materials when they give them to the students for classroom work. Two studies were particularly useful when defining research questions and research methodology for the present work. One of these research reports includes results relating to secondary education, although not only mathematics, while the other one relates only to primary education and prospective teachers.

Textbooks and Future Teachers [Ball and Feiman-Nemser's Study, 1988]

The Ball and Feiman-Nemser (1988) study was originally interested in exploring what prospective primary teachers learn in preservice teacher preparation in the U.S.A.. The adopted methodology was to follow six elementary education students through two different two-year undergraduate teacher education programs. 'Although we had not specifically set out to investigate that matter, the issue of textbooks and learning to teach emerged in the course of the inquiry' (p. 402).

The article presented the analysis of the answers to three questions:

> 1. What did the teacher education programs that we documented convey about textbooks, planning, and curricular decision making?
> 2. What did the prospective teachers in our study come to believe about the use of textbooks, about planning and curriculum decision making?
> 3. What did the prospective teachers do with textbooks and teachers' guides during student teaching?"(p. 403).

The programs documented (A and B) were part of an effort to reform teacher education. The advice given to student teachers on use of textbooks was the same, although the reasons for such advice could vary from one course to another.

To answer the first and the second questions, the researchers followed the students during the courses that gave advice on use of textbooks and interviewed the six students they had been using as informants after such courses, gathering the following information:

> Despite their ... differences, both programs explicitly communicate that textbooks should be used only as a resource, that following a textbook was an undesirable way to teach. Both programs encouraged teacher candidates to make their own decisions about content and instruction...Abandoning their common sense notions about textbooks, teachers candidates in both programs came to see textbooks in terms of theirs program's ideology. While both programs urged teacher candidates to examine the content of the textbooks, neither program pursued a critique of the subject matter content. Nor were other critical perspectives explored, such as those

> focused on bias. Instructors in both programs seemed to assume that the teachers candidates knew their subjects well enough to decide what to teach and how to go about it, and that curricular and instructional decisions were to be individually made and justified. Overall, our student informants developed the strong impression that their personal ideas and knowledge were a better source of content than anything in the textbook or teachers' guide. (p. 405)

To answer their third question, the researchers followed the student teachers during their student teaching and present the following comments:

> During student teaching, five out of our six student informants were placed in settings with cooperating teachers who used textbooks as the core of their reading and mathematics teaching. In spite of what they had been taught in their courses, the student teachers in both programs ended up using textbook programs to teach reading, math, science and social studies. Some student teachers felt pressed to maintain the established classroom practices. Others were simply overwhelmed by the responsibility of teaching the entire day, and resorted to textbooks as a reasonable way to manage, or at least survive, the demands.
> Just following the text presented unexpected problems for the student teachers in both programs. Some discovered that they were unprepared to use textbooks and teachers' guides to teach subject matter. Others followed the teachers' guides rather mechanically, moving through activities without really understanding what they were doing. Not sure how to adapt the textbook material appropriately, their modifications sometimes distorted the point of the lessons....
> Whether student teachers used textbooks or departed from the teachers' guides to create their "own" lessons, they often did not understand the content they were teaching and did not seem to get the point of the lesson. In a few instances, however, the teaching suggestions in the teachers' guides seemed to provide a scaffold for student teachers' efforts, helping them understand more about the topic and how it is learned. The guides showed some ways of organizing content for instruction and offered activities and questions that helped these novices know how to proceed...(p. 407)

This report raises some interesting points: In spite of all the advice these teachers received during their preservice courses, they all ended up using textbooks not only as a source of activities, but as an organiser of the content of the lessons and the curriculum. The researchers present some reasons for this, such as: (a) lack of knowledge of the content to be taught, (b) lack of confidence by inexperienced teachers in developing their own lessons and (c) pressures due to school organisation and textbook adopted.

There is no reason to believe that prospective mathematics secondary teachers are receiving different advice from their teacher trainers from that received by their colleagues in primary education. Although the authors do not discuss the reasons why advice given is against textbooks, the report shows that student teachers understood it only as a severe criticism and built the idea that textbooks should not be used in any case. Using the authors' words: 'Can teacher education teach prospective teachers to use textbooks without making

that the ultimate goal?...Perhaps they can be oriented toward learning from teacher's guides and other curriculum materials...' (p. 422).

Ball and Feiman-Nemser point to the use that can be made of research in use of textbooks in teacher training. As mathematics educators, it is possible to improve the quality of advice given to student teachers by including some research results on how experienced teachers have been using textbooks and other printed materials for their lessons. These experienced teachers probably have been through the same kind of situations discussed in this article, and their solutions to the problem can help other teachers to find better ways to use printed materials for secondary mathematics lessons. As there are some teachers that have chosen not to use textbooks, such research should also include some information on what experienced teachers who do not rely on textbooks as a major source of content have been doing instead and what kind of results their students have been obtaining.

A Questionnaire on Use of Textbooks [Davey's Study, 1988]

Davey (1988) reported a study involving 90 teachers, each with more than two years of teaching experience and enrolled in several graduate education courses. These teachers were asked to answer a questionnaire on how they normally use their textbooks. Forty five such teachers were primary teachers and the other 45 were specialist teachers working in secondary education (14 English teachers, 11 social studies or history teachers, 10 science teachers and 10 mathematics, foreign languages, home economics or industrial arts teachers).

SURVEY ITEMS	45 elementary teachers	45 secondary teachers
I give the students time to read silent in class	2.61	1.75
I use one textbook primarily	2.26	2.63
I teach students how to use the textbook	1.69	1.75
I give independent homework assignments from the text	2.29	3.10
I have the students answer questions at the end of the chapter	2.21	2.60
I use different texts with different students	2.25	1.60
I rely on text information for my instruction	2.41	2.35
I overview the text selection before giving an assignment	3.71	2.90
I ask students to read from the text orally in class	2.31	2.35
I change texts when I see students cannot read them well	2.42	1.82
I expect students to read most of the text.	2.09	2.74

Figure 3.2.1: 90 teachers' ratings of textbook uses (mean frequency)

Eleven statements were given to the teachers who were asked to mark their responses on a 1 to 4 scale (1- rarely, 2- occasionally, 3- frequently and 4- most of the time). Three open ended questions about the use of textbooks and two questions relating to teacher's attitudes towards texts were also given. The results are shown in figure 3.2.1 and figure 3.2.2, reproducing tables 1 and 2 (pg. 24) respectively.

To better understand these tables, it is important to observe that: 1- The questions were initially open but, whenever two or more teachers provided a similar response, a category was formed for the analysis (listed in the table above). 2- Whenever a teacher provide a response to a question that could be included in more than one category of answer, each category received proportional weight (e.g., with three responses, each counted 1/3 of a response in its category). 3- Responses that were impossible to interpret or those with no answer were not counted.

Survey Questions and Categories of Free Responses	percent of elementary teachers	percent of secondary teachers
(1)How do you typically use your textbook?		
-to supplement instruction	33	27
-for students discussion on group work	30	10
-as a source for activities	17	17
-for assigned reading	7	20
-as a basis for my lectures or for my instruction	13	13
(2)What things can be done to make the text more readable		
-use a variety of sources	23	3
-preview and review vocabulary	20	10
-rewrite it	23	23
-oral reading	10	-
-highlight main ideas	7	7
-use graphics, activities	13	3
-use study guides	-	20
(3)What things keep you from the most effective use of your textbook?		
-time constraints	37	10
-student characteristics	23	27
-requirements of the system	17	10
-limitations of the materials themselves	13	13
-my limitations (i.e. lack of knowledge)	3	7
(4)What do you like about your textbook?		
-content	23	37
-organisation of ideas	3	20
-visual, graphics, print	33	3
-end of chapter questions and activities	13	2
-readability	20	7
-overall: everything	7	10
(5)What do you dislike about your textbooks?		
-content	20	33
-organisation of ideas	20	7
-end of chapter questions and activities	3	10
-readability	23	7
-overall: everything	33	23

Figure 3.2.2: Teachers responses to open ended questions

Although the number of secondary teachers in the sample is small, and the number of mathematics secondary teachers even smaller, these tables show some interesting data: Secondary teachers seem to use one textbook primarily and students are supposed to do their homework based on it, as well as being able to read most of the text and do the exercises and activities at the end of each chapter. The mean frequency for 'I use different texts with different students' is the lowest for secondary teachers in table 1, also the mean for 'I change

texts when I see students can't read them well' is low, suggesting that these teachers do not change texts very often for this reason.

In the view of picture presented in table 1, it could be expected that the textbook would be highly considered by the teachers. However, looking to the answers these same teachers gave to questions in table 2, it seems that the text is not so important, and it is considered only as a supplement to instruction for a large number of teachers. It is seen as a main source of activities by only 17% of the secondary teachers in the sample. These teachers also chose 'rewrite' as the most appropriate way to make the text more readable, which seems a contradiction with the former position in which they said they did not change texts. They also claim that the student's characteristics are the most important factor in keeping them apart from the most effective use of their textbook, making it difficult to understand why they do not change texts.

Secondary teachers seem to consider content as the more important factor in a textbook. Thirty seven percent of the sample chose the content as the main reason why they like their textbook, while 33% of the same sample chose it as the main reason why they do not like their textbook (as they cannot be the same teachers, 70% of the sample consider the content of the textbook as a factor of major importance: whether as a reason to like the textbook or to dislike it)).'Thus, the content's accuracy, scope and appropriateness to the curriculum appeared to be the most important textbook factor for them [secondary teachers]'(p. 344). On the other hand, readability seems not to be considered so important, and only 7% of the sample chose it as the main reason they like their textbook, while another 7% of the sample chose it in the opposite direction (14% of the sample consider readability as the more important factor when working with a textbook given to students).

There are several differences between the answers given by primary and secondary teachers. For example, content seems to be considered much less as the main factor why primary teachers like or dislike their textbook than it is for secondary teachers, while readability becomes more important when considering the answers of the primary teachers. Comparing the mean frequency that primary teachers say they would use different texts for different students with the one for secondary teachers, it appears that primary teachers are more flexible than their secondary colleagues.

While studying this report, it became clear that the information was incomplete. Several questions are not answered when looking to teacher's opinions about their use of textbooks and a whole new set of doubts arise. One cannot be sure how important the textbook really is in day-by-day classroom work because information gathered from different questions seems to contradict each other. Access is not given to each teacher's complete answer, to verify if there are any contradictions and one just gets that impression because one is looking to means and percentages, instead of looking to the complete opinion of a teacher (for example: it is mathematically possible that all the teachers that blame student

characteristics as keeping then from the most effective use of their textbook do change texts when they see the students cannot read them). On the other hand this report is certainly a good source of questions to verify how important the textbook is in day-by-day classroom work.

Davey herself suggests that this research needs to be complemented and offers some guidelines: 'Future research should incorporate classroom observation to validate conclusions drawn from this study' (p. 345). If one wants to research the actual ways teachers have been using textbooks in classroom work, lessons have to be observed, otherwise one could end the research work with a picture of teacher's opinions about the usage of textbooks instead of with a picture of what is really happening in mathematical secondary lessons.

3.3. The 'Evaluation of the Implementation of National Curriculum Mathematics at Key Stages 1, 2 and 3' Report.

As described by Askew et al. (1993) , a proposal for a project designed to address issues and questions related to the evaluation of mathematics in the N.C. was submitted to the National Curriculum Council by the Centre of Educational Studies, King's College, University of London, including collaboration with staff from the University of Birmingham and the Cambridge Institute of Education. Four areas had been identified by H.M.I. and the N.C.C.'s early monitoring as areas of especial concern in the implementation of the National Curriculum for mathematics:

(1) Difficulties encountered in the implementation of selected topics.

(2) Inappropriate progression in the PoS (programmes of study) and the ATs (Attainment Targets).

(3) Implementation of AT1 (MA1),'Using and Applying Mathematics' (UAM).

(4) Effectiveness of the PoS for in-school mathematics.

The aim of the study was to 'pursue the four areas of concern through a connected set of investigations, denoted respectively as Studies 1-4' (Askew et al., 1993). Each Study was related to one of the areas of especial concern above, and can be briefly described as:

Study 1 focused on identifying factors that inhibited (or facilitated) teaching in those areas of mathematics (Algebra, Shape and Space and Handling Data) which have been identified as difficult to implement.

Study 2 looked at sequencing of statements of attainment, and consistency in the implied developmental model for curriculum planning.

Study 3 focused on aspects of teaching for MA1, 'Using and Applying Mathematics'.

Study 4 considered the nature of school and teacher planning and review procedures, especially the extent to which teachers were basing these on National Curriculum documents. In this study, the role of published schemes and other materials as mediators of the National Curriculum were emphasised.(Askew et al., 1993)

Different methodologies were applied to these studies: Studies 1 and 4 started from a national survey questionnaire and complemented data with in-depth interviews with teachers, advisers and publishers. Study 2 used methodology based on monthly teacher's group meetings, with the teachers also participating in classroom based research in their respective schools. Finally, study 3 used methodology based on case studies, using participant observation and interview (Askew et al. 1993).

Although Study 4 is the one of particular interest for the present research, interesting results were obtained in all other studies. Johnson and Millett (1996) summarised the main ones from Askew et al.(1993). Some examples are:

1. It was noticed that, in contrast to earlier monitoring, shape and space (MA4) was not perceived as an area that was particularly difficult to implement.(Study 1)
2. Despite the many concerns resulting from the investigation, the teacher groups decided that it was not desirable nor necessary to rewrite the curriculum immediately. (Study 2)
3. Concern was expressed that the national tests, with short and mainly written items, could lead to a narrowing of the curriculum. (Study 2)
4. MA1 appeared often to have been interpreted in ways which fitted with current practice and presented minimal need for change. (Study 3 - 'teachers' interpretation and implementation')
5. Although questionnaire responses indicated a high degree of planning for MA1 (and a correspondingly high percentage of classroom time spent on this AT), interview data indicated that there was little specific planning for this AT. At KS3, what was done was often in terms of providing contexts for assessment rather than for teaching. (Study 3 - 'teachers' interpretation and implementation')
6. MA1 is a mean of both applying and developing understanding of the content of MA2-5. (Study 3 - 'promising practice')
7. The data generally confirmed the findings obtained in earlier monitoring studies - teacher's perceptions reported by the evaluation were that their difficulties lay in three main areas; those of 'Using and Applying Mathematics' (MA1), handling data (MA5) and Algebra (MA3).(Study 1)

Johnson and Millett (1996) also stated that 'the findings from the four interlinked studies ... were drawn together to address a broader set of themes and issues' that 'went beyond those identified in the initial proposal' (p. 47). These findings led to the presentation of 17 Recommendations. Although it is beyond the scope of the present review to comment on all of these recommendations in depth, it was noticeable that some of the recommendations recognised the importance of effective materials to help teacher's planning and day-by-day classroom work. For example:

> **Recommendation 14:** Provide advice and guidance to publishers on the order's intentions and requirements, and on future developments. Provide publishers with a review plan covering a period of 10 years, which will be maintained, so as to enable them to plan the preparation of new materials. Give sufficient notice between any changes to the Order and their implementation, so that materials can be designed, written and trialled.
>
> **Recommendation 7:** Assist teachers to interpret SoAs by providing guidance other than that of the test items. This should be in the form of exemplar classroom material, including pupils' work, including some which integrates MA1 with other attainment targets. This would also provide guidance for publishers, and for the teams writing the tests.

The findings of Study 4 are directly connected with the present work. This is not because planning is one of the main interests in this research, but because one of the 'major findings of the planning study [is] that the use of commercial mathematics schemes is a major factor in teachers' planning' (Millett and Johnson, 1996). A summarised description of the study: (1) methodology, including the questionnaire - its development, sample, and respondents - and the interviews - subsample, interviews schedule and post-interview procedures, and (2) findings can be found in Johnson and Millett (1996, pp 32-44). The outcomes of particular relevance related to use of commercial schemes (or textbooks, as they are called in the present research) were summarised as:

1. the major role in planning for many teachers was taken by the commercial scheme - that is, the degree of (commercial) scheme use and the way that the scheme was used were major influences on the way in which teachers planned their teaching for mathematics;
2. some teachers relied almost entirely on a commercial scheme for differentiation, with varying degrees of intervention;
3. the use of commercial scheme increased through the key stages, from KS1 to KS3;
4. with the above said, it is also the case that many teachers used a commercial scheme as only one of a range of planning resources - these teachers also made use of N.C. documents, school schemes of work and other non-scheme mathematics resources. (Johnson and Millett, 1996, p. 44)

Figure 3.3.1, from Millett and Johnson (1996), described the percentage proportion of pupils' work taken from a commercial scheme. The table was adapted from Askew et al. (1993) and reproduces the answers to question 15 in the questionnaire, about the use of textbook for classroom work. It can be seen that the textbook plays a major role, especially if secondary education is considered, with about 80% of the teachers in KS3 using the textbook for more than 50% of classroom work.

	n	0	1-5	6-20	20-50	51-80	> 80
KS1	215	11	15	19	23	27	6
KS2	282	4	6	7	24	49	10
KS3	237	4	4	5	8	41	38

Figure 3.3.1: Proportion of pupils' mathematics work done from a commercial scheme (question 15) (%). From Millett and Johnson (1996, p. 57)

Askew et al. (1993) used the interlinked data from studies 1 and 4 to produce another study, dedicated to the role of commercial schemes for in-school mathematics. This study was based on the interview data, complementing the questionnaire data. The teachers were classified as 'planners', but the categorisation presented also implied a determined use of textbooks for classroom work. Millett and Johnson (1996), described this classification as:

Scheme-driven planners: All interviewed teachers in this category were from KS2 and KS3. These teachers would rely on the commercial scheme for planning and the use of N.C. documents would be low, and mainly for 'checking coverage'. Their classroom work is described as 'pupils in the classroom of a scheme driven planner would be likely to be working individually through scheme books or cards' (p. 59). These teachers used the scheme for more than 50% of classroom work.

Scheme-assisted planners: 'These teachers used the scheme (often very frequently) to fill what they saw were their needs and the needs of pupils in their class' (p. 56). The group of interviewed teachers consisted of three KS1 teachers, six KS2 and two KS3. For these teachers, 'the schemes appeared to be used both for individualised work and for group work, and sometimes in both ways by certain teachers. There were indications that number work was more likely to be conducted in an individualised, ongoing way' (p. 60). Seven of these teachers used the scheme for more than 50% of classroom work, one for 21-50%, and one for less than 20%.

Low-scheme planners: 'These teachers used a scheme (or schemes) for less than 20 per cent of their work (often to a very limited extent) and on the whole selected from a variety of resource materials. They used a school/departmental scheme of work, their own scheme of work, the NC documents or a combination of these in their planning' (p. 60). The group consisted of six KS1 teachers, one KS2 and one KS3. 'Pupils in the classrooms of scheme-assisted planners and low-scheme planners would be likely to be working in attainment groups on pages or exercises selected from the scheme by the teacher, or on work which the teachers had themselves prepared. Some pupils might be working in an individualised way through selected sections of the scheme' (p. 60).

Finally, several issues related to the use of commercial schemes emerging from the data were discussed in Millett and Johnson (1996). A summary is presented here, not only because they can be used as parameters for the present research but also because some of them can be confirmed by research based on classroom observation. The selection of issues was made in the light of the present research. There were other issues highlighted in the article, which are not going to be considered here. The issues are presented here in the same order in which they appear in Millett and Johnson's article.

1. 'considerable numbers of KS3 teachers in particular shared some of the characteristics of the *scheme-driven* teachers who were interviewed, with 56 per cent of KS3 teachers falling into the ... scheme-driven planners' (p. 62).
2. 'when [the teachers were] asked to identify reasons for these difficulties [MA1, MA5 and MA3], both questionnaire and interview data indicated several reasons which related closely to scheme use [lack of availability of resources, lack of suitable activities in probability and inadequate teaching materials in handling data] ... the areas of UAM

[Using and Applying Mathematics],handling data and probability were mentioned most frequently as being not well covered by the commercial scheme...' (pp. 62-64)

3. 'some teachers made extensive and sometimes uncritical use of a commercial scheme, and were using this scheme as a mediator of the NC.' (p. 64)
4. '*scheme-driven* planners [interviewed] did use a narrower range of resources than both *scheme-assisted* and *low-scheme* planners.' (p. 64)
5. 'all schemes appear to have been 'tarred with the same brush', by being discussed together. It is fully recognised that some commercial schemes are designed to be 'all the teacher needs' whereas others specifically place the teacher in the central role ... There were indications that some schemes were more likely to be used as core materials than others, which were more often used as one of a number of resources. However, *whatever the intentions of the publishers, it is the use made by the teachers of commercial materials which affects the mathematics experienced by pupils in the classroom* [italics added]'. (p. 65)
6. 'several teachers in the interviews mentioned that the schemes ... did not cover well certain areas of mathematics. It seemed that 'not covering well' could relate to content or presentation, or both.' (p. 66)
7. 'contact with the publishers of mathematics materials indicated that they themselves had identified areas where supplementary materials were needed and these were being produced ... the lack of materials which support the view of UAM as a vehicle for the other main content areas still remained a source of concern. It was also suggested that financial constraints may have affected the ability of schools to purchase the most up-to-date materials'. (p. 67)
8. 'differentiation within the mathematics taken from a scheme seemed to be established either by having pupils working individually through scheme books or cards ... or by teacher intervention in selecting work from the scheme at the appropriate level, usually for attainment groups'. (p. 68)
9. 'all the KS3 interview teachers working with mixed-ability classes were using the strategy of individualised work'. (p. 68)
10. 'pressure to use a commercial scheme ... is both internal and external. Internally it may stem from a feeling of inadequacy about mathematics ... anxieties about lack of experience or lack of subject knowledge might well be a reason for turning to commercial materials... low scheme use may not necessarily be associated with good practice... Externally pressure may come from parents, from other teachers and from the children themselves ...' (p. 72)
11. 'there may be less co-operative work in a *scheme-driven* environment, with a consequent lack of development of subject knowledge'. (p. 73)

12. 'until the teachers can see the value of alternatives to the commercial schemes upon which they depend, there is not likely to be significant change in their use'. (p. 73)

13. 'there are many other issues related to the use of commercial schemes or textbooks which deserve attention; for example, the use of research evidence in the development of schemes and the possibilities of encouraging informed and critical use of schemes during teacher training.' (p. 73)

Millet and Johnson (1996) seemed to adopt a point of view in which the value of the commercial schemes is downplayed, although there was no reported research to support it. Nevertheless, the issues raised by Millet and Johnson (1996) are important. It is true that more research is needed on how printed materials are used for classroom work. Using their words: 'In the light of many concerns expressed, it is somewhat surprising that, while it is generally accepted that teacher's use and dependence on this medium [the textbook] is a common and potentially highly important phenomenon, the implications of this situation have had limited attention in the mathematics education research' (Millett and Johnson, 1996, p. 71).

3.4. Research on Teacher's Decision Making.

As it would be impossible to review literature on teaching research as a whole in this section, it was decided to give preference to those articles which provided an overview of research on teaching. According to Romberg and Carpenter (1983), research on teaching has shifted from a paradigm in which teachers were seen mainly as managers of classroom activities to another, which 'assumes that the teacher behaviors ... are... results of purposeful, reflective plans...' (pg. 864). Although the initial focus of this section is on teacher's decision making, it will become clear that researchers started to consider teachers' beliefs and perceptions as important tools to explain their decisions.

Cooney (1988) described teaching as 'an interactive process', in which conscious decision making is needed, not only beforehand, during the planning of the lessons but also 'on the stage'. Summarising the findings in this particular issue of research on teaching, he pointed out that 'several aspects remain constant: teachers gather and encode information, generate alternatives, and select a course of action' (pg. 273). According to him, there are different types of decisions made by teachers: some related to content development, others related to managerial aspects of the classroom environment and yet others related to affective concerns, although 'in the real world of the classroom, classification schemes are seldom clearly exhibited' (pg. 275).

Cooney (1988) presented a review of the literature on research on teacher's decision making during the seventies: these pieces of research were concerned with issues as external factors affecting teacher's decisions and alternatives, and classroom issues as time allocation during mathematics lessons, time allocation for different topics of the curriculum, etc. By the end of this article, he had provided an explanation on why research on teacher's decision making has become more and more interested on teacher's beliefs. He said that another aspect that can be considered is the reflection on 'why certain alternatives are selected. Value judgements, perceptions about what constitutes the teacher's role, and what constitutes mathematics...' are some of the factors that, according to him, can provide 'additional insights into the teaching process' (pg. 285).

In an article presenting a critical synthesis of research on teacher's beliefs and conceptions written in the beginning of the nineties, Thompson (1992) indicated that research on teaching during the eighties started to be very much concerned with these issues. She stated that the interest 'was fuelled by a shift in paradigms for research on teaching', from a process-product paradigm, to a 'focus on teacher's thinking and decision-making processes'. According to her, these led to an interest in identifying and understanding the composition and structure of "belief systems and conceptions" ... underlying teacher's thoughts and decisions' (pg. 129).

Instead of trying to define 'beliefs', Thompson (1992) stated that 'researchers have assumed that readers know what beliefs are' (pg. 129). She discussed the differences between beliefs and knowledge presented in the literature. Although she admitted that a close connection between them exists and 'distinctions between them are fuzzy', she argued that

> ... a characteristic of knowledge is general agreement about procedures for evaluating and judging its validity; knowledge must meet criteria involving canons of evidence. Beliefs, on the other hand, are often held or justified for reasons that do not meet those criteria, and, thus, are characterized by a lack of agreement over how they are to be evaluated or judged. (pg. 130)

From her point of view, a belief system is 'a metaphor for examining and describing how an individual's beliefs are organized'. She stated that 'beliefs systems are dynamic in nature, undergoing change and restructuring as individuals evaluate their beliefs against experiences.'(pg. 130). Finally, 'conceptions' are 'viewed as a more general mental structure, encompassing beliefs, meanings, concepts, propositions, rules, mental images, preferences and the like.'(pg. 130).

Thompson (1992) summarised in her article studies of mathematics teachers' beliefs that focused on beliefs about mathematics, beliefs about teaching and learning, or both. She argued that 'the diversity of purposes, methods, designs, and analytical frameworks used by the researchers has led to great variability in how teacher's conceptions have been described' and that research on teachers' beliefs and conceptions should not be looked at 'in isolation from research on mathematics teachers' knowledge' (pg. 131).

As none of the research pieces discussed in her paper is strongly connected with beliefs about the usage of written materials, nor could any of them provide a framework in which a categorisation of teacher's beliefs on this issue could be fitted in, reviews of these articles are not presented here. Nonetheless, it became clear from her article that this type of research can be used as a framework for the present research, as teachers' beliefs on how written materials should be used for classroom work can determine their practical decisions. As Thompson said, although it remains unclear what 'this line of research has to contribute to mathematics education' (pg. 141), the connections and contributions to research on mathematics teacher education and research on teaching and learning are relevant, as several aspects are being re-examined in the light of research on teachers' beliefs and conceptions. Another contribution of this type of research highlighted by Thompson is 'the importance of teacher reflection as a vehicle for knowledge growth' (pg. 142).

As far as practical implications are concerned, Thompson (1992) argued that research on teachers' beliefs has strong implications for teacher educators and staff developers (pg. 143). According to her, it allows researchers to begin investigations on 'how teachers learn from their experiences in classroom as they interact with the students and subject matter' (pg. 143).

Thompson's point of view about the importance of integrating research on teachers' beliefs with other aspects of research on teaching is also indicated in another review of research on mathematics teaching, by Koehler and Grouws (1992). They commented on the growing level of complexity in teacher research. It was argued that research in teaching should consider not only classroom processes and teacher knowledge of mathematics, but also teachers' knowledge about students learning and pedagogy, and teachers' beliefs about teaching and mathematics. On the other hand, pupil characteristics and attitudes (as well as teachers' perceptions of these characteristics) should be taken into account, as they can influence students' outcomes (cognitive and affective ones - pp. 117-118). They presented a review on research projects classified into this level of complexity. The model of research proposed in the work of Fennema, Carpenter and Peterson (1989a and 1989 b) and the expert-novice paradigm considered by Leinhardt (1989) and Livingston and Borko (1990) are of special interest for the present research.

In an article considering the 'translation of ... [findings from research on teaching and learning] knowledge into educational practice' , Fennema et al. (1989 a) presented the outcomes of a research project which implemented educational change through curriculum development (Fennema et al, 1989 b). On the basis of previous research investigating the cognitive processes of teachers, the paradigm for that research considered the teacher as a 'reflexive, thoughtful individual', who plans for instruction in a 'rich variety of ways and that these plans have real consequences in the classroom'. It was also considered that teachers' 'beliefs, knowledge, judgements and thoughts have a profound effect on the decisions they make, which in turn determine to a large extent what students learn in their classrooms.'(1989 a, pg. 176).

The model proposed by Fennema et al. (1989 b) is presented in figure 3.4.1. It shows classroom instruction based on teacher's decisions. It also considers that the main influences on these decisions are teachers' beliefs and knowledge as well as students' behaviour. It is also interesting to notice that the proposed model considers teachers' knowledge and beliefs system as dynamic, as well as their mutual influences .

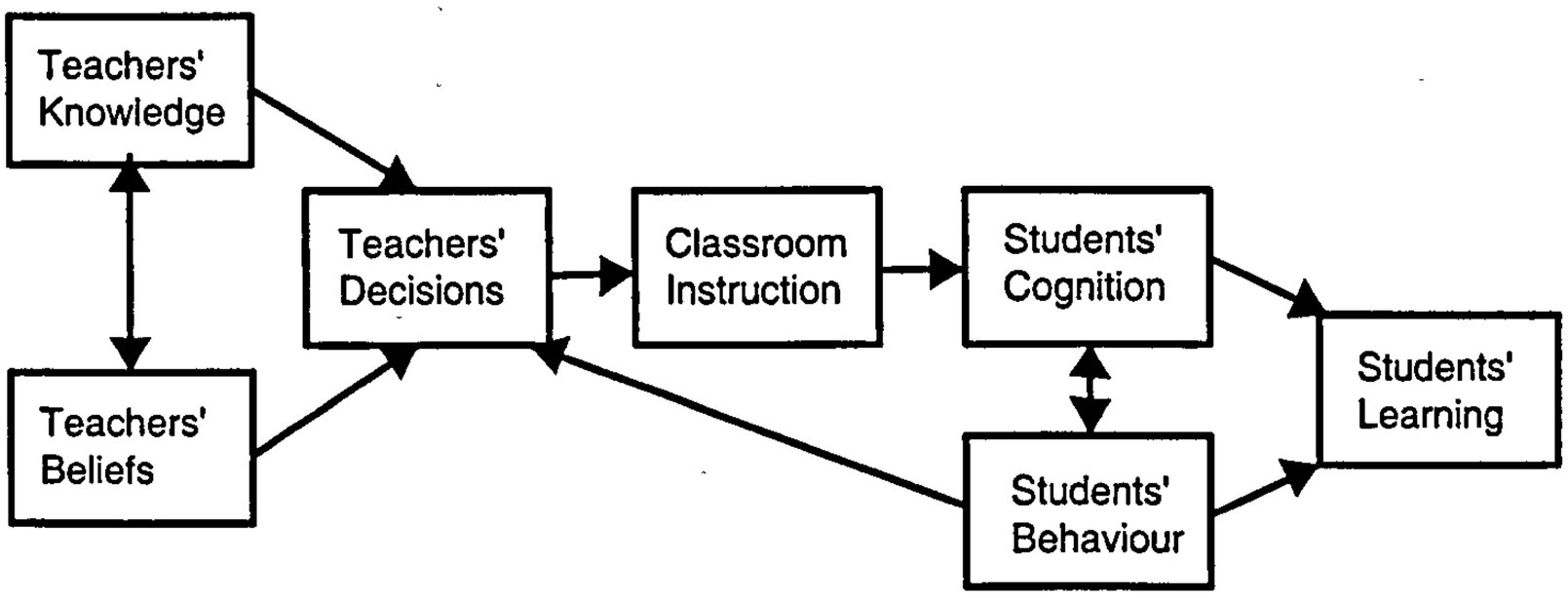


Figure 3.4.1. Research Model for Cognitively Guided Instruction
From Fennema, Carpenter and Peterson (1989 b), pg. 204.

The article by Fennema et al. (1989 a) described the main findings of the Cognitively Guided Instruction (CGI) Project, commenting that 'before the workshop, teachers had a great deal of knowledge about their children's thinking', but they had not integrated this knowledge into their decision making, so they were 'less able to implement instruction and built on their beliefs'. They also comment that 'knowledge and beliefs are interconnected in a way that is strongly linked to teacher's instruction and to students' learning of mathematics' (pg. 185).

The works of Leinhardt (1989) and Livingston and Borko (1990) were integrated by Koehler and Grouws (1992) with other research projects into a section dedicated to the *Expert-Novice Paradigm* (pp. 120- 121). In this section, the authors explained that, in this type of research, the 'underlying philosophy is actually similar to the process-product paradigm in those two categories of teachers are observed ... with the intention of identifying the qualities and behaviors necessary for successful teaching'. On the other hand, they recognised that there are differences between these two types of research, in particular when the focus of the observation and the measure of success or effectiveness are considered (pg. 120).

To better understand Leinhardt's (1989) study, the concept of 'lesson segments' must be introduced. The author defined lessons segments as 'segments of social events in which various actors assume specific roles to accomplish specific tasks' (pg. 56). Koehler and Grouws (1992, pg. 121) pointed out that other authors in the literature have also identified and investigated lessons' segments (for one example quoted in the literature, see the concept of 'developmental portion of a mathematics lesson', defined by Good, Grouws and Ebmeier, 1983).

Leinhardt in his (89) study analysed videotaped lessons and interviews of four experienced teachers and two novice ones, in order to compare their performances. Different 'lessons' segments' were analysed, and the importance of the goals for each segment and the strategies used by the teachers to achieve the goals were compared. The findings from that research pointed out that experienced teachers have clearer goals, better strategies to achieve their goals and spend less time on transition moments (from one segment of the lesson to another), with clear indications that experienced teachers 'more consistently distributed their time among other lesson components' (Koehler and Grouws, 1992, pg. 121). On the other hand, novice teachers' lessons 'are characterized by fragmented lesson structures with long transitions between lesson segments, by frequent confusion caused by missent signals, and by an ambiguous system of goals that often appear to be abandoned rather than achieved' (Leinhardt, 1989, pg. 73).

In another study questioning differences between expert and novice teachers, Livingston and Borko (1990) were specially interested in the pedagogical reasoning demonstrated by these teachers. They defined 'schema' as 'an abstract structure that summarizes information about many particular cases and the relationship among them' (pg.

373). Their study had confirmed that experienced teachers 'have more elaborated and more interconnected schemata than novice teachers' (Koehler and Grouws, 1992, pg. 121). Another interesting finding of the research is that 'experienced teachers became in some ways like novice teachers when teaching new content, which underscores the important influence of the knowledge of the content being taught'/ (Koehler and Grouws, 1992, pg. 121).

Writing for teachers, Good and Brophy (1997) presented another review of research on teaching, with emphasis on 'classroom life' (pg. 1). They highlighted that teachers' decisions are not always taken beforehand, as it is often necessary to make decisions during lessons. In such situations 'teachers must often make quick decisions while using incomplete information' (pg. 1). As far as use of textbooks is concerned, they suggested that

> Many educators have contended that textbooks define the curriculum. However, recent research challenges this simplistic view and suggests that teachers act as decision makers, modifying the curriculum in relation to factors such as teacher's beliefs about students' aptitude, their instructional intentions, and their subject-matter knowledge. If teachers influence the curriculum, then their decisions about curriculum help to determine performance expectations for students, just as teachers' behaviors and activity structures do.(pg. 105).

They offered no references for research projects related to the use of textbooks in classrooms. On the other hand, they raised another important issue: the importance of teachers' performance expectations. Starting from a review of research projects dated from the beginning of the nineties, Good and Brophy (1997) stated that expectation research can be expanded in many ways ... besides affecting students' achievement, teacher expectation can be expected to affect student's attitudes, beliefs, attributions, expectations, motivational patterns and classroom conduct.' (pp. 104-105). They also suggest that it would be profitable if future research 'attempt to integrate study of teachers' decisions ... with study of teachers' expectations for students (how much students are likely to learn, etc.)' (pg. 105).

More recent examples in research projects on teachers' beliefs and concepts about mathematics can be found in the literature. Philippou and Christou (1997) reported an attempt to relate 'teachers' conceptions about mathematics and its pedagogy and students' achievement' (pg. 4-9). Using data gathered in the IEA international study (Robitaille and Garden, 1989), teachers from different countries were grouped according to their students' scores in the tests. Teachers' conceptions about the nature of mathematics, the learning of mathematics and the teaching of mathematics were compared among the different groups.

Askew et al. (1997) reported a study intended to identify effective teachers of numeracy in primary schools in U.K. Data were gathered from different sources, including interviews and observation of lessons (three lessons for each case study teacher). They justified that it was important to use these different sources, because previous research had already shown that 'teachers may have adopted the rhetoric of 'good' practice in teaching mathematics without changes in their actual practices' (pg. 2-25). The identification of

effective teachers was made by measures of their students' outcomes, 'in a specially designed test ('tiered' for different age ranges) of numeracy' which was applied twice for each group of students. 'Average gains were calculated for each class, providing an indicator of 'teacher effectiveness'... ' (pg. 2-26).

The analysis of the data considered three models of sets of beliefs 'as important in understanding the approaches teachers took towards the teaching of numeracy' (pg. 2-27): (1) connectionist: 'beliefs based around both valuing pupils' methods and teaching strategies...emphasis on ...connections within mathematics'; (2) transmission: 'beliefs based around the primacy of teaching ... mathematics as a collection of separate routines ...'; (3) discovery: 'beliefs clustered around the primacy of learning ... mathematics as being discovered by pupils'. (pg. 2-27). It was found that all teachers in the sample who were classified as 'strongly connectionist' were also classified as 'highly effective'. On the other hand, all teachers classified as 'strongly discovery' or 'strongly transmission' were also classified as 'moderately effective'. Finally, those teachers whose beliefs showed 'no strong orientation' distributed themselves into the three categories of effectiveness, the majority of them being considered as 'effective'. (pg. 2-30).

Askew et al. (1997) discussed these results suggesting that 'orientations towards teaching mathematics need to be explicitly examined in order to understand why practices that have surface similarities may result in different learner outcomes'. They also commented that 'in a sense the connectionist approach is not a complete contrast to the other two but embodies the best of both...' (pg. 2-32).

3.5. Other Relevant Issues Related to Research on Teaching

The latter two examples on research of teachers' beliefs raised an important problem, that seems to remain unsolved: how to test students' outcomes? Good and Brophy (1983) stated at the introduction of their summary review on effective schools that 'standardized tests scores cannot be equated with effectiveness per se' (pg. 570). Discussing the topic further, and commenting on 'tests specially designed', Romberg and Carpenter (1983) stated that 'studying teacher's effectiveness with dependent measures tied to special teaching units may not be a fair characterization of teaching over the long haul, because different teachers may not cover the same units during the year' (pg. 861). Nevertheless, works developed by the second half of the nineties still used 'standardised tests' or 'specially developed tests' to assess students' outcomes, indicating that testing students outcomes is still a problem when research on teaching is considered.

Romberg and Carpenter (1983) also stated that test for 'multivariate outcomes' as another problem to be solved. They suggested that 'although students' outcomes are indisputably the ultimate dependent variable in research on teaching, researchers need to question what outcomes are to be examined, and how data are to be gathered and interpreted' (pg. 861). Discussing the matter, Brophy and Good (1983) argued that 'what constitutes "teacher effectiveness" is a matter of definition, and prefer to use the 'more neutral term "teacher effects" ' (pg. 328).

Brophy and Good (1983) presented a review on research on teacher's effects, which raised important issues to be considered in the present work. They stated that, 'despite the importance of the topic, there has been remarkably little systematic research linking teacher behavior to student achievement' (pg. 329). It is also argued that this type of research should include classroom observation, and this can be expensive, if large field studies are to be developed.

Summarising process-product research relating teacher behaviour to student achievement, Brophy and Good (1983) presented a critical revision of their findings. It is important to notice that these results are correctional, and there were no cause-effect conclusions. Some relevant correlations for the present research, concerned with Junior High level are listed here.

- 'Achievement was associated positively with active group instruction in the subject matter and negatively with emphasis on nonacademic activities, poor organization or classroom management, or approaches in which students are expected to manage their learning primarily on their own' (pg. 339).
- 'Teachers who got the most gain in high-SES classes motivated by challenging and communicating high expectations to their students ... In contrast, the teachers who got the most gains in low-SES classes motivated primarily through gentle and positive encouragement rather than challenge or demandingness...' (pg. 342).

> - In mathematics classroom organization and instruction associated with achievement ...The more popular mathematics teachers not only had good relationships with their students but were academically stimulating and demanding...The more successful ... rated highly as classroom managers ...Measures of the amount and quality of instruction were even more directly related to achievement... The more successful teachers taught more actively, spending more time lecturing, demonstrating, or leading recitation or discussion lessons. They devoted less time to seatwork, but were more instructionally active during the seatwork time they did have, being more likely to monitor and assist the students rather than leave them to work without supervision....The more successful teachers asked many more questions ... In general, unlike the primary grades where it is essential to take time to work with individuals during (small group) lessons, in the upper grades it is more important to keep (whole class) lessons moving at a brisk pace... teacher appears competent, confident, credible, enthusiastic, receptive to student's input, and clear in presentations. (Brophy and Good, 1983, pp. 343, 344)

Notice that the authors made clear that 'the notion that there are "generic" skills that are appropriate and desirable in any teaching situation' was not supported by the data collected in these research projects. They stated that 'although certain abstract principles appear to be universal (e.g. match difficulty level of content to students' present achievement levels), few if any specific, concrete teacher behavior are "generic" correlates of achievement' (pg. 350). On the other hand, they made clear that several aspects of teacher's 'personality' can affect students' outcomes. In their conclusion, they offered some hints why this is so, for example, they highlighted the importance of 'preservice and in-service teacher education in both subject matter and pedagogy' (pg. 370), pointing in the same direction as Livingston and Borko (1990), when they recognised that experience, and both content and pedagogical knowledge are important factors for effective teaching.

Brophy and Good (1983) also stated that 'what appears to be just the right amount of demandingness (or structuring of content, or praise, etc.) for one class might be too much for a second but not enough for a third class' (pg. 370). This factor has to be considered if the same teacher is teaching groups of students of different ability levels or even mixed-ability groups, as it would be the case in many comprehensive secondary schools in the U.K.

The selection of activities was also considered, among other decisions made by the teacher, one important factor that can distinguish a teacher that can produce better achievement (pg. 370). Once more, the consequences of this kind of research for teacher education were emphasised, when Brophy and Good (1983) stated that 'rather than trying to translate it [the information from research findings] into rigid or generalized prescriptions, teacher educators should present this information to teachers within a decision-making format that enables them to examine concepts critically and adapt them to the particular contexts within which they teach' (pg. 370).

The importance of classroom observation is stated in another article (Good and Biddle, 1988), in which the authors showed their concern about reform movements in the

U.S. that were not taking into account enough research evidence (pg. 115). They argued that past reforms were characterised by 'tendencies to define problems without careful documentation and to generate solutions without the aid of observational data' (pg. 117). According to them, this could lead to teachers not being 'involved in the development of these new materials, ... [materials] not developed with the classroom in mind' (pg. 119). It is interesting to notice that this type of behaviour among teachers was observed in Askew et al. (1993), discussed in section 3.3.

Good and Biddle (1988) also suggested that future research should identify teachers' beliefs and 'determine how these beliefs interact with teachers' views of intelligence and classroom learning strategies that are appropriate to particular students' (pg. 125). The importance of classroom observation was stressed once more when the authors claimed for 'more information about the classroom contexts and particular combinations of teachers and students that make the program more or less effective...' (pg. 131).

On the inherent limitations of classroom observation, Good and Biddle (1988) highlighted the following points: (1) 'it is expensive, labour-intensive business, and tends to limit the size and representativeness of samples of classrooms, teachers, pupils and classroom events studied' (pg. 137), (2) 'even when the interest is focused on teachers' behaviors or on short sequences of teacher-pupil interaction, ... the sample is nearly always drawn for convenience and not randomly' (pg. 138).

Although the majority of research projects reviewed in these two last sections came from the U.S., recent publications showed that there is international concern with the differences between 'said and done' when research on teaching is considered. Some recent examples: Laborde (1996) commented that 'identify current practices ... is a difficult task because there is usually a gap between what is said about teaching practice and what is really done in classrooms' (pg. 506). In the introduction to their article, which attempted to recognise world-wide '*very widespread practices* while at the same time giving a place to *practices that are rarer*' [authors' italics] (pg. 565) Bodin and Capponi (1996) stated that 'the question "How do you teach?" is almost never asked' (pg. 565). They comment further on the matter, by saying that

> ...Studies concerning the *what* and *why* of mathematics are quite common. You can still find studies on *who* (who teachers? who is being taught?), or *with what* (what equipment, what textbooks, etc.) but as to *how*, global studies are very thin on the ground ...As you have realised, we mean *how* in the descriptive sense, how teaching is done, not how it ought to be done... [authors' italics] Bodin and Capponi (1996, pp. 566-567)

Another problem that has to be addressed in the present research was raised by Hoyles (1988) when she argued that research findings 'seems to indicate that factors other than teachers' views of mathematics more crucially affect teaching behavior; in particular school context factors, ..., texts used, and the 'philosophy' within a mathematics department (pg.

155). She stated that 'school mathematics (as opposed to mathematics) cannot be examined independently of educational discourse' (pg. 155), and argued that 'a good indication of what constitutes school mathematics can be obtained by taking a look at school textbooks and syllabuses, which exhibit a very high degree of standardization across different countries' (pg. 156).

In a sense, Hoyles' (1988) assertions complement the views already discussed in the present chapter: even if the mathematics teacher is seen as a 'reflexive, thoughtful individual' (Fennema et al., 1989a. pg. 176), the real constrains of school activities have a clear influence in his/her decisions, and one important factor to be considered is written materials for classroom work, textbooks in particular, as highlighted by Howson, Keitel and Kilpatrick, 1981, pg. 61). Robittaile et al. (1993) complemented these considerations when, based on previous research and on their own international study, stated that teachers have to fulfil several functions when implementation of the curriculum is considered. Among these functions, teachers have to interpret and adapt curriculum materials to their personal view and approach to teaching, as well as their view of the needs of their specific classroom situations. (pg. 28).

Taking one step further, Clarke, Clarke and Sullivan (1996) also acknowledged that there are factors that constrain the teacher's role in mathematics curriculum development, and argued not only that 'teachers need to be supplied with appropriate resources (text or student materials, teacher support material, relevant technology..)' but also that 'teachers need time: time to plan, ... to meet together, ... assimilate new content and pedagogy ...' (pg. 1207).

If on the one hand, teachers need materials for classroom work, on the other, Kang and Kilpatrick (1992) argued that authors 'usually write the textbook from a teacher's position' because 'they cannot intervene directly in the communication between teacher and student' (pg. 5). Love and Pimm (1996) considered that mathematics textbooks 'are not in general conceived to *replace* a teacher, and have always been written with a greater or lesser sense of the likelihood of mediation by a teacher [authors' italics] (pg. 385). Discussing further, Love and Pimm (1996) indicated that textbooks are not written only for students, even if the texts 'appear to be addressed directly to the student: "appear to be", because there is almost always a ghostly presence of the teacher' (pg. 385). They argued that texts are written for teachers as well, because 'texts frequently exist in a wider context of other resources, including teacher guides on their use in class' and 'with the many textbooks 'schemes' that consist not of a single book, but include various other resources ... the teacher is indeed seen as an essential presence'. (pg. 385).

Love and Pimm (1996) presented some benefits of the textbook 'as seen by teachers': continuity, rich variety of ideas, solve management problems, help to implement the curriculum are some of them (pg. 398). They also commented that 'while teachers'

perceptions have received some attention, there is a dearth of research into the use of texts in class' (pg. 397).

To conclude the present review on relevant issues for the present work carried out by researchers, notice that, as far as use of written materials for classroom work is concerned, it has to be considered that research is in its 'early stages', as outlined by Kilpatrick, 1977, pg. 98, especially if observational data are included. Thus, the present work had to take into consideration advice for 'relatively primitive' research, as refered by Good and Biddle (1988, pg. 137) in the literature when deciding on methodological issues.

When summarising their review on research on teaching, Romberg and Carpenter (1983) said that, at that time, this type of research appeared to be just beginning. They argued that 'in the absence of a paradigm or set of organizing principles, all facts ... are likely to be equally relevant' (pg. 865). Nevertheless, they stressed the importance of 'early fact-collecting' as an essential step on science development (pg. 865). The same sort of advice was also found in Kilpatrick (1977), when discussing the 'early stages of research' into teaching. He argued that it was necessary to begin 'with careful descriptions of a specified set of teachers teaching a specified set of topics to a specified set of pupils' (pg. 98). On the other hand, discussing the limitations of observational research, Good and Biddle (1988) claimed that

> 'the fundamental task of conducting classroom research is to generate and test plausible theory concerning teaching. When relatively primitive, such theories consist of terms that represent classes of observable events and propositions that summarize observed relationships among them.' (pg. 137).

CHAPTER 4

LITERATURE ON ADVICE FOR TEACHERS ON WRITTEN MATERIALS

FOOD FOR THOUGHT	
JUNK FOOD *There is a lot of about* *All the preparation is done for you by someone else* *The instructions for use are simple and laid out in steps* *It is superficially attractive but turns out to lack flavour* *It does you little good; it tends to pass through quickly* *All the real nutrient is removed and substitutes have to be added*	***JUNK MATHEMATICS*** *See most of school textbooks* *This is done by the author or teacher - all the nasties are removed* *See most textbook questions* *It looks well structured and appears logical, but is dull and lacks in substance* *Pupils are unable to retain or apply it in new contexts* *It offers no real life situation but invents and contrives them*
DANGER: HEALTH WARNING ***Junk mathematics can cause seriously damage to your pupils.***	

The Mathematics Centre, West Sussex Institute of Higher Education - "Better Mathematics" (1987).

This chapter continues this work's review of literature. Although it was not possible to find many pieces of research on how teachers are actually using textbooks, the same does not apply to publications advising teachers on this matter. Section 4.1 presents a summary of publications in England directed at teachers. Section 4.2 adds more information by discussing some other books that teacher trainers have been advising their P.G.C.E. Mathematics students to read. Section 4.3 presents some reviews of printed materials published in specialised journals dedicated to mathematics teachers. Finally, in section 4.4, published matters on written materials in the United States are discussed to complete the picture, as some interesting points of view were not yet focused.

As in Chapter 3, it was decided to look for such sources from the beginning of the eighties. Nevertheless, some important information would be missed, for example: some of the more popular secondary mathematics textbooks in this country were first published at the beginning of the seventies. So, it was necessary to look back for specialised journals starting from 1970.

4.1. Advice for Teachers on Printed Materials.

Teachers in secondary schools in England have access to a series of books specifically written to help them in their classroom work. During the last 16 years, two major influences were found, as far as mathematics education is concerned: the first one, called *'Mathematics Counts'* (Cockcroft (ed.), 1980), is also known as *'The Cockcroft Report'* and seems to have been quite well received at the time of its publication and ever since. It is possible to find *The Cockcroft Report* quoted or included as a reference in a great number of books published to advise mathematics teachers since then. It is also quoted in the other influential series of documents published from the late 80's - *The National Curriculum* Documents.

Besides these two major influences, other publications addressed to secondary mathematics teachers with the objective of helping them in their classroom work can be found. As a large number of teachers have access to them, due to their widespread availability in schools, it is possible that teachers' opinions about printed materials as well as the way they have been using written materials when giving them to their students for mathematics work could be influenced by suggestions made in such books.

The Cockcroft Report.

'Mathematics Counts' presents a summary of the existing situation in Mathematics Teaching in the late 70's and early 80's and offers guidelines to be followed when generating a new general policy for Mathematics Education both in Primary and Secondary Schools. Its suggestions have been considered since its publication. As examples of its influence, it is possible to quote the importance given nowadays in England to calculators (and computers) in the secondary mathematics curriculum and the widespread publication of 'topic books' in mathematics during the past 15 years. Both topics received special attention in the report: *Use of Calculator* is one of the headings in the *'Foundation List of Mathematics Topics'* (§ 458) and the publication of new topic books is encouraged either as a textbook alternative or complement when the report talks about *The Use of Books* (§§ 312 to 314) for primary mathematics:

> *...Although some books of this kind are available, more are needed* [their italics]; suitable topics would be the mathematics used in everyday life, the exploration of shape, communication by means of graphs and diagrams, the history and development of counting, calculation and measurement, and links between mathematics and science or art. More books of puzzles, problems and suggestions for investigations are also required. (§ 314).

Mathematics Counts offered some important guidelines for mathematics teaching in primary and secondary schools, but very little was said in this report about the use of

textbooks in mathematics lessons. For primary education, the advantages and disadvantages of using textbooks were established in three paragraphs about use of books:

> Even though a child may without difficulty be able to read what is written in a mathematics textbook or on a work card, he may well feel great difficulty in learning an unfamiliar piece of mathematics from the written word. This is likely to be the case however careful has been the choice of the language which is used. The ability to learn mathematics from the printed page is one which develops very slowly, so that even at the age of 16 there are few pupils who are able to learn satisfactorily from a textbook by themselves. At the primary stage new topics and concepts should always be introduced by appropriate oral and practical work and the necessary links with what has gone before established by discussion.
> Nevertheless, textbooks provide valuable support for teachers in day-by-day work of the classroom. They can provide a structure within which work in mathematics can develop and provide ideas for alternative approaches. They can be a source of exercises which have been carefully graded and are likely to provide revision exercises at suitable intervals. Accompanying teacher's manuals may suggest other kinds of work which should be undertaken alongside the exercises in the textbook and indicate ways in which the topic can be developed further for some pupils. However, it is always necessary to use any textbook with discrimination, and selections should be made to suit the varying needs of different children. It may be better, too, to tackle some parts of the work in an order that is different from that in the book or to omit certain sections for some or all children. It should not be expected that any textbook, however good, can provide a complete course or meet the needs of all children; additional activities of various kinds need to be provided.
> By the middle junior years some children are skilled readers and have become accustomed to acquiring information from books. Although the printed word is seldom a satisfactory mean of introducing new mathematical concepts, the same limitation does not apply to the use of mathematical problems, puzzle and topic books, and books of this kind should be available in the classroom or school library. Their use can enable children to realise that mathematics is a living subject which is full of interest and of use outside the classroom, and can also contribute to the children's overall mathematical development...(§§ 312 to 314).

Coming to secondary schools, the report says 'we put first responsibility for the production and up-dating of suitable schemes of work because it is by these means that the mathematics department makes clear its aims and objectives and provides guidance and help to its members as to the way in which these may be achieved.' (§ 510). In the same paragraph, the report reinforces the importance of a good scheme of work and complements the information about use of resources saying that 'It [the scheme elaborated by the mathematical department] should indicate the teaching resources available and state the procedures to be followed for routine matters such as the issuing of textbooks and stationery'.

It is likely that the decisions about curriculum and scheme of work will be left to the school mathematics department. There are no other suggestions on these issues made in the report, except for a general comment on *'Reference Material for Teachers'* (§ 616) where it is

emphasised that 'in both primary and secondary schools there should be a supply of reference books for teachers relating to the teaching of mathematics'. Suggestions of such books are also offered:

> These should include some of the publications of the professional mathematical associations as well as of the DES [Department of Education and Science], HMI [Her Majesty's Inspectorate], and Schools Council. There should be copies of any teacher's guides which relate to textbooks in use in the school and also a selection of mathematics textbooks other than those which are in general use, which can serve as an additional resource for teachers. ..(§ 616).

The report has recognised that it could be difficult for a student to master how to learn mathematics from the written word, but offers no suggestion on this matter for secondary teachers who should be working on improving such capacity with their students. It is also interesting to notice here that, in the same paragraph in which the responsibility is given to the mathematics department to elaborate the scheme of work, there is a quotation from the report on a survey by Her Majesty's Inspectors of Schools (1979) drawing attention to the fact that although in all the schools visited by them "there were written schemes of work of some kind, their quality and usefulness varied greatly" (§510).

National Curriculum Publications in Mathematics

When reading the National Curriculum's Attainment Targets and the Programmes of Study in Mathematics (DES, 1991), it was not expected to find any guidance on use of textbooks, because that was not the issue treated in this document. Nevertheless, it was expected that there would be some references to the ability to read mathematics from texts being a desirable attribute to be developed with students. It was not the case: references to several ways of using calculators and computers as tools for developing mathematical thinking in several levels were found, references to the importance of developing the students' ability to express themselves mathematically and even references to developing the students' ability in understanding data mathematically expressed were also found, but no reference to the development of the ability to learn mathematics from texts was found, except for an example of a complex activity: "explore independently a new area of mathematics" (level 10, AT1), that supposes such ability to be fully developed .

The guidance offered by the National Curriculum Council (1989, 1991) for the implementation of the National Curriculum in Mathematics was searched, looking for advice on the use of written materials for classroom mathematics. Here, as in the *'Mathematics Counts'* report, the responsibility for decisions on written materials is left to the schools' mathematics departments, saying that 'the programmes of study do not in themselves specify a mathematics curriculum in action... [they do not] require particular books or materials to be

used by pupils... these and other matters need to be addressed by schools when formulating policies and plans for mathematics' (p. B3).

Nevertheless, in the following paragraph, printed materials are mentioned as one of the resources a teacher is supposed to use. Some hints about what is expected from a mathematics classroom are offered as well as suggestions about what sort of resources they are expected to use:

> Activities should be balanced between different modes of learning: doing, observing,.... The overall scheme of work should include all the following types of activities [detailed only when books are quoted]:
> .Listening. ...
> .Reading (studying from textbooks or work cards; researching from topic books or reference books; comparing methods or solutions; gaining feed back from the computer monitor)
> .Writing. ...
> .Talking. ...
> .Reflecting..... (pg. B.9).

Resources and materials (but not specifically books) are also mentioned as possible useful tools in helping teachers to ensure continuity and progression of the content.

> Progression requires that individual teachers plan for a coherent and progressive experience of mathematics for pupils in their class. Different teachers and different schools who have responsibility for the same pupils over a period of time need to work together in order to ensure a continuity of experience for pupils.
> In determining how continuity can best be achieved, there are a number of matters that need to be considered:
> .schemes of work
> .mathematical language and conventions used in classroom.
> .assessment and recording systems.
> .resources and materials.
> .classroom organisation.
> .teaching and learning styles. (pg. C3)

With so little help in deciding the best ways of choosing written materials for mathematics classroom and so little official advice on the best ways to use them as tools to help the adaptation to the National Curriculum it is quite possible that commercial textbooks quickly produced to match the National Curriculum become quite popular in few years time and the fears of several Mathematics Educators become true, as Hart (1992) expresses them:

> ...If you use a textbook, and most mathematics teachers do, you will probably very soon have the national curriculum version, which matches very closely [the list in the attainment targets and programmes of study].

Curriculum Matters 3: Mathematics from 5 to 16.

'Mathematics from 5 to 16' (1987) is part of a series of books by Her Majesty's Inspectorate [HMI] providing advice for teachers. In this book, textbooks were generally criticised because of some faults traditional textbooks usually present: Problems that only give the exact amount of information required for their solutions and provide little challenge (pp. 4 and 16) and also because the way textbooks are generally written: providing rapid changes of experience, justified by the idea that many pupils cannot concentrate for a reasonable length of time (p. 6).

However, the authors recognise the importance of students being able to gather information from books and other sources: '... pupils need to be able to extract ... mathematical information not only from textbooks and workcards, but also from a range of other resources such as topic books, reference books, advertisements, catalogues,..' (p. 13). Advice in using textbooks for low attainers is also offered: 'Textbooks and workcard materials which are well suited to the needs of pupils in primary school are not, in general, suitable for pupils aged 15 year old, even if these are low attainers.'(p. 27) and once more the necessity of a good mathematics library in schools is emphasised: '... these will include a wide range of printed material, not only textbooks and worksheets but also supporting reference material and topic books...'(p. 43).

Better Mathematics

From all the books published for teachers reviewed here, *'Better Mathematics'* (The Mathematics Centre, West Sussex Institute of Higher Education, 1987). was the one that offers the more severe criticism of textbooks. The book suggests investigations and experiences as the better way to learn mathematics and also suggests that students must be given the opportunity to pose their own questions, reflect, invent and discuss. The 'poster' that opens this chapter was taken from this book, and it was located in a chapter that does not have resource materials as one of its issues. (Chapter 2: *Children Learning Mathematics*).

However, schemes of work developed by schools' mathematics department are the subject of a whole chapter (Chapter 6) and a special section (section A) is dedicated to published schemes of work. The term 'published scheme' is used to include the main series of books, teacher's guides, workcards and eventual supplementary material, such as computers' packages. The core of this section is a list presenting seven disadvantages of using a published scheme in 'close adherence'. There is no list of the advantages published schemes could offer to teachers (used (or not) in close adherence), so it can be assumed that the authors do not believe that there is any advantage in using textbooks except as a source of 'stimulus for mathematics enquiry in the classroom' (Chapter 6, section A). The disadvantages are:

> 1. Investigatory or enquiry elements of schemes are often *left out by teachers* [my italics], frequently because of 'syllabus pressures'. ...In trying to resolve this some

schemes have attempted to include enquiry as a compulsory component of their course. However, there are two problems with this.
(i) Because the more stretching investigatory elements of some schemes are not reached until initial material has been completed by pupils, it is often only the quicker children who get a taste of this kind of work. *All* children need and benefit from mathematical exploration..
(ii) When given greater status, and often in order to 'help' the teacher, investigatory elements can become prescriptive and limiting, precluding the teacher's and children's own questions and losing all sense of real enquiry or exploration....
2. New schemes are often chosen because they are seen to cater for a particular organisational classroom priority. However, the 'prices' that are paid for these priorities may well have adverse effects on the children's mathematical experiences. For example if the priority is to ensure that pupils can 'get on' independently of their teacher, ...[the scheme] may also lead to the mathematical content being unnaturally fragmented. ...
3. By their nature most schemes are answer-oriented, ...
4. Many mathematics schemes now include microcomputer software packages. There is, however, a danger that computers are not realising their potential as powerful vehicles for mathematical enquiry and development in the classroom. Although many of the programs within the packages can provide excellent starting points for exploration, some are no more than simulations of what are considered to be successful classroom activities. ...All too often the packages in general are used as 'rewards' and devices to 'keep the pupils occupied'.
5. The teacher's role can be undermined by a close adherence to ... a scheme ... Responding to the immediate needs of individual children, controlling material, deciding what should happen next and what pathways of learning should be encouraged are all essential parts of the teacher's role and cannot be generalised by an external system written for unknown children and teachers. ...
6. Frequently an adherence to a scheme is seen as a necessary support for so-called 'weak teachers'. However, even as a short term solution this can be unsatisfactory and often counter-productive. ... For all teachers, support and development of the kind indicated throughout this report are essential. ...
7. New schemes are often taken on in order to initiate change in static teaching situations dominated by exposition and textbooks. ... However, in many cases even when schemes have been taken on in this spirit they have failed to facilitate development.... Teachers begin to teach 'the scheme' rather than teach mathematics. (Better Mathematics, chapter 6, section A)

By the end of that chapter, a summary of the situation is presented. The authors summarised the list above, discussing also some problems concerning 'home-made' schemes: 'if not constantly reviewed, will suffer from the same problems as published schemes' and comments about teachers who had opportunities for further development: 'teachers have become more discerning and more demanding consumers of published material and equipment'.

Some recommendations are also made. The first being that already made in item (5.) above, the second suggesting that schemes of work should be continuously evaluated and

modified, with the whole mathematics department engaged in the process. The last three recommendations were:

> 3. Resources in the form of ideas, thematic and topic booklets, catalogues and timetables, books and periodicals, reference material, magazine and newspaper articles, games and practical materials including calculators and computers are needed. The spending of limited departmental funds on 'packaged' schemes should be seriously questioned by schools as this often means that no money remains for these other forms of resources and can lead to inflexibility.
> 4. It is essential that mathematics is recognised as a subject which has special accommodation requirements.
> 5. It must be seen to be part of the professional concern of all teachers to take responsibility for choosing appropriate materials which will encourage pupils to learn mathematics. It is this professional responsibility which needs encouragement and funds to develop. (Better Mathematics, chapter 6, conclusions).

It seems that the majority of criticisms made by the authors are not directed at the schemes (or books included in the schemes), but directed to the ways teachers have been using such schemes. It is common sense that teachers with better opportunities to develop their capacities are desirable, and probably such teachers would be able to use *any* resource material with criteria, ensuring all their students plenty of opportunity to experience different ways to learn mathematics. What the authors do not make clear is what is going to happen if the 'so-called "weak teachers" 'do not have a relatively 'strong' scheme to support them. Would such teachers be able to provide sensible and continuous experience for their pupils? Would such teachers be able to ensure progression of the content? Would such teachers be able to confidently use such a wide range of resource materials as suggested in recommendation (3)? The research developed by Ball et al.(1988) and discussed in chapter 3 of the present work responds negatively to such questions, at least where novice primary teachers lacking confidence are concerned. It is also important to note that 'Better Mathematics' was sent to every secondary school in England by the government.

It seems that rather than criticise schemes and textbooks mainly for the ways teachers are using them, it is more profitable to verify if there are experienced teachers who have been using them successfully. If it is the case that some uses prove to be efficient, inexperienced teachers could receive advice that would help them to use such schemes in better ways.

Using and Applying Mathematics.

Using and Applying Mathematics is an Association of Teachers of Mathematics [A.T.M.] publication (1990), destined to help teachers in adapting to the exigencies of Attainment Target 1 of the National Curriculum. The authors advise teachers on the best

ways of introducing investigative work and discussion in mathematics lessons and a few comments are made about textbooks under this heading.

Nevertheless, the issue "textbooks and investigative work" is quite important, and, instead of giving direct suggestions, the book offers a series of questions for teachers to discuss when they are interested in introducing investigative work in their lessons. Under the heading '*Organisation Questions*' it was possible to find the following series of questions:

> What textbook will you purchase?
> Or, will textbooks be inappropriate for investigational work?
> What topic books will you purchase?
> What teacher's resource books will you purchase?
> Do you have sufficient access to calculators? To computers? ... (Using and Applying Mathematics, 'Organisation Questions')

Another issue which was expected to be found more frequently is discussed: readability of a textbook. Under the heading '*The Importance of Discussion*', some problems related with reading and understanding mathematics from a book are mentioned, with the general purpose of convincing teachers of the importance of discussion in classroom:

> When new aspects of mathematics are introduced, particularly if it is done through textbooks, it is almost impossible for children simultaneously to comprehend new mathematical concepts and new sophisticated mathematical vocabulary...yet...we often expect children to master the two components together.
> The majority of mainstream publisher's mathematics resource books emphasise certain 'received' vocabulary (e.g. 'larger than'...) at the expense of other aspects of mathematics language. This can result in the teacher engaging the children in meaningless exercises from the textbook instead of in practical activities that involve them in using language in a real context....
> In addition to this, the language of mathematics textbooks raises the issue of complex linguistic constructions, especially with regard to the statements, including those of a conditional nature and the commands which permeate textbooks and many teachers 'conversations' with children.... (Using and Applying Mathematics, 'The Importance of Discussion')

Notice once more on how little advice is given to teachers on the use of written materials for classroom instruction. The few surveys found on the issue showed that the use of textbooks is widespread, and several books written in order to help teachers in day-by-day classroom work simply do not say enough about the use of printed materials in general, and textbooks in particular. Issues such as the evaluation and choice of textbook or the use of the teacher's guide as a resource for their lessons are not developed in these books. It is also surprising that books that offer severe criticism of textbooks do not at the same time offer a wide range of practical and immediate alternatives to teachers, and even more surprising is the fact that practically no suggestions are made on how teachers can better help children to master the ability of gathering information from a text. The advice given in such cases seems

to point to the use of other kinds of material instead ,and the postponing of the problem. It is not clear when the problem should be confronted.

Mathematics in the Sixth Form (HMI, 1982) gives us an idea of what had happened to students finishing their secondary school:

> Mathematics, in contrast to some other subjects, seems to have become accepted by students as an activity which can be pursued with the minimum of dependence on books, except as a source of examples. This situation may have come about as a result of teaching styles which encourage the student to rely heavily on the teacher as almost the sole source of knowledge. Textbooks are often written to provide an independent mean of learning but they are less frequently used for this purpose than they might be. ... HMI observed many lessons where the treatment of a topic did not differ greatly from what could have been read in the adopted textbook. The general (non-textbook) reading material in the library was rarely consulted by students... (p. 28).

It seems that at the time of the survey done by HMI the students had not mastered how to learn mathematics from a book during their secondary education, even when lessons are heavily based on the content of the adopted textbook. Has this situation changed? The evidence taken from what was reviewed suggests that there is no reason to believe so. Even if teachers nowadays are not lecturing anymore and are offering more opportunities for their students to work on their own during mathematics lessons (all books of advice suggest that), this independence seems not to include the ability to learn from books, as the programmes of study in the National Curriculum and the books of advice for mathematics teachers give so little attention to this matter.

4.2: Advice in Books Recommended by Teacher Trainers.

In this section, books that teacher trainers usually recommend for their students are reviewed. These books are part of the bibliography for P.G.C.E. students and also for teachers enrolled in a Masters Degree Course in Mathematics Education at two British Universities. In order to decide what books should be reviewed, suggestions from teachers who have been running the P.G.C.E. and Masters Degree courses at King's College London and at The University of Nottingham during the last few years were taken into consideration.

Several of these recommended books, such as *Children's Mathematical Framework 8-13: A Study of Classroom Teaching* (Johnson (ed.), 1989) are about research on children learning mathematics, but not related to textbooks at all, but there are others that include information and advice on the use of textbooks, and these are the books that are going to be discussed here.

Recommended Books Published Before 1980.

The books discussed under this heading are seldom recommended as course books for future mathematics teachers anymore, at least from the point of view of the teachers' trainers from whom advice was sought. Nevertheless, they were written as 'textbooks' for P.G.C.E. courses and none of the more recent literature analysed here was written with this specific purpose.

Instructional Materials (Shores, 1960) was written not only for mathematics teachers but for teachers in general in the U.S.. It is a book entirely dedicated to instructional materials and there is a whole chapter (chapter two) dedicated to textbooks. The author said in the introduction:

> As the teacher considers the various classes of Instructional Materials at his disposal, he must inevitably begin with the textbook. Traditionally the "T" book has performed in the classroom as the great organizer. It has been a code of law on basic content. As a common denominator, it has served to bind a class of pupils together and to the teacher... (p. 37)

Shores developed the outlines of criteria to evaluate textbooks: it includes content '... Text sampling here and there will reveal outdated information, omissions, or overemphasis on relatively minor aspects.'(p. 51); level of treatment or readability; sequence of the subject matter , format 'or physical make-up' (p. 52), authorship and bibliographies. The advice encountered so many other times was there as well:

> Effective teaching and learning utilize textbooks and other teaching materials in complementary combination. In such combination, textbooks can lead the learning

effort, supported by a variety of other materials - library books, magazines, graphics,...
It is also necessary to realize that (1) textbooks establish the course of study and (2) textbooks do not and should not determine the curriculum. Realistically, curriculum and materials interact. For this reason it is unwise to build a curriculum first, and then seek materials to implement it; nor is it desirable to develop a curriculum from a selected group of materials...(p. 53).

This chapter also included a list of functions of a textbook:

> ... to provide (1) course organization, (2) basic content, (3) a common denominator for a class composed of individual differences, (4) practice of intensive reading skill, and (5) opportunity to develop study habits...(p. 54.)

The authors of *Guidelines for Teaching Mathematics* (Johnson and Rising, 1972), another American book, also dedicated a whole chapter to mathematics textbooks (chapter 22). At the introduction of this chapter, the authors listed the reasons why they considered that 'the mathematics textbook has a unique role in the classroom' (p. 370).

> Direct experience, visual aids, and classroom instruction cannot provide all the instruction necessary. Some of this instruction must be covered by reference to a textbook.
> Teachers have too many pupils, preparations and extracurricular assignments to make it possible for them to plan and write complete units and daily lessons without the aid of a text.
> Mathematics requires a sequential study treatment, and the textbook provides a useful aid to this approach.
> For mathematics teachers with an inadequate background in mathematics and in the methods of teaching mathematics, the textbook is a substitute (albeit a poor one) for this background they lack.
> Many schools are limited in resources such as library books, concrete and visual learning aids, community resources, duplicating equipment; and so the text provides the basic, and sometimes the only resource.
> Learning mathematics depends on the mastery of concepts and skills. Students may grow in this mastery by performing the exercises of the text.
> Mathematics requires a storehouse of facts, theorems, formulas, and definitions to which reference can be frequently made. In this way, the mathematics text is as necessary as a dictionary or encyclopaedia in English or Social Studies. (p. 370).

In this chapter there was a section of special interest for our work: *'The Proper Use of Mathematics Textbooks'*. In this section, the authors presented suggestions 'for the proper use of a textbook for the typical mathematics class' (p. 373), and these suggestions can be summarised as:
1) A selection of the topics to be taught should be made from the text;
2) Topics that are not in the text and should be included in the course must be chosen from sources such as library books, pamphlets and other texts;

3) The 'competent teacher' must use different examples and different explanations from those offered in the text;
4) The text is a source of questions, exercises and reading material for the students and for the teacher;
5) The students should be given instructions in how to use the textbook;
6) The narrative or exercises of the text should be assigned with the students' differing abilities and needs in mind;
7) The assignment of textbook exercises must be done carefully and the student must understand the reasons why he/she is asked to do some exercises in class or as a homework;
8) Answers to some exercises must be provided for the students;
9)Textbooks explanations must be supplemented by instruction providing discovery exercises, references and local applications;
10) textbooks exercises must be supplemented;
11) textbooks tests must be supplemented;
12) The enrichment topics include in the text must be recognised as important as the explanations and exercises;
13) Additional textbook series and other resource books must be available for the teacher. (pp. 372-373)

These statements influenced the questions posed for classroom observations in the present research, because they are all related with usage of texts and can be applied whatever written material the teacher is using for his/her lessons. The questions can be used even if the teacher is using his/her own written materials instead of printed materials.

Notice that some of the advice found in more recent books was already presented in these books, but textbooks were considered much more valuable at the time of publication then they seem to be nowadays.

Recommended Books Published after 1980.

During the eighties, the issue 'textbook' starts to receive a 'new' treatment from authors of books written to advise teachers. The books discussed in this section will complement the picture of general advice given to teachers about written materials, presenting some more recent views on this matter. All the books reviewed in this section are somehow related with written materials, starting from a book dedicated to the problem of reading mathematics. Most of them are suggested reading for students teachers.

Children Reading Mathematics (Shuard and Rothery, 1984) reflects the authors' awareness of difficulties encountered by children when learning mathematics from a book, although they recognise that the majority of research projects in this area were developed in the USA, with very few research projects developed in England (p.157). The authors discuss themes such as graphic language in mathematics (chapter 5) and readability formulae and their limitations for mathematics texts (chapter 7). They also present the results of a research project in which the group of researchers rewrote a small part of actual textbooks in

widespread use in primary and secondary mathematics and compared the difficulties the children had using the original version of the text and the rewritten form(s). But the authors are quoted in this review when they discuss the difficulty and the importance of reading mathematics. They say:

> The purpose of mathematical text is to develop some form of mathematical thinking in the reader. Hence the manner of reading in mathematics must involve the reader in mathematical activity.
> Text is at great disadvantage as a teacher of mathematics: it cannot interact with the pupil. A live teacher can present information interactively, or ask a pupil to solve a problem and react immediately to his attempt. This is a serious difficulty, because mathematical writing aims to go further than merely communicating information; it tries to make the reader think, to help him to develop the way he thinks, and to enable him to do some mathematics (p. 154).

They also suggest some strategies teachers could use to improve the ability of their students to learn from a text:

> teacher who realizes the importance of reading for learning mathematics ... teachers might try an experiment in which half the class use paper and pencil while reading and the other half do not; the teacher could then observe which half had made better progress. Having pencil and paper is, however, only half of the battle: pupils also need strategies for using them effectively. These include:
> .asking themselves questions about the text,
> .trying to answer these questions in writing,
> .drawing diagrams,
> .investigating mathematical results related to those in the text.
> The style of write-in worksheets, and that used in some books, is intended to encourage interactive reading. However, the attempt to promote interaction may distort the syntax and layout of the text, and may introduce 'rhetorical' questions whose desired response is unclear. It is of more lasting value to the pupil to learn how to read mathematics. This is no easy task, and can be achieved only by long-term persistence in developing active reading skills...(pp. 155-156).

Teaching and Learning Mathematics 11-16 (Costello, 1991) presents an overview of topics related to teaching and learning mathematics: several research results, curriculum development, assessment, cultural and gender differences, new technology in mathematics education and how these topics are related with the National Curriculum are discussed in this book, extensively documented by a large bibliography. The book also includes a chapter on '*Professional Aspects of Mathematics Teaching*' where textbooks are discussed. The importance of the scheme in use is also pointed out when the author discusses themes such as mixed-ability grouping and individualised learning, and references are made to some of the most popular commercially produced schemes of work in this country. Describing the role of the textbook in England, the author says:

There is no shortage of material to help mathematics teachers decide what and how to teach, or for schools to buy to provide a comprehensive mathematical diet for their pupils. Publishing textbooks and schemes is a lucrative business. No sooner have series been completed to meet the needs of the GCSE examination than these begin to appear in revamped form to satisfy the requirements of the National Curriculum.

The way in which schools use textbooks varies, and there are fashions in this matter: different styles of text become popular. At one stage, school mathematics books were regarded as sources of exercises. The role of the teacher was to introduce topics and explain the methods to be used: only after this activity might the pupils be expected to work from the book. Some books of this kind are still published, and they have their uses. Others choose to include perhaps one or two illustrative examples before one may attempt to provide all necessary material for a complete mathematics curriculum.

Another interesting variable is the function of the teacher's book, guide or manual. In some schemes, this is nothing more than a book of answers. Sometimes, it is an annotated version of the pupils' book with notes and solutions. In its most sophisticated form, the teacher's manual may be very much larger than anything provided for the pupils, and contain all kinds of ideas and guidelines for lessons, as well as suggestions for further work and material.

Alongside the trend to publish elaborate schemes and series of textbooks which provide coverage of the curriculum in a neatly packaged way, there has been a very different movement in recent years to develop more open-ended, exploratory material which might encourage a less prescriptive style of mathematics teaching. Much of this reflects the view that pupils can benefit from greater freedom in developing, choosing and appreciating approaches to mathematics situations for themselves, rather than relying entirely on compulsory taught methods....(pp 79-80).

The author also presents the idea that 'the shift from text based whole class teaching to individual study from published schemes as perhaps the greatest change in the teaching of mathematics in 30 years' (p. 82) and suggests that '...when classes are based largely on an individual scheme, an investigation can be appropriate way of providing a whole class lesson or group work activities' (p. 83).

In chapter 10, *Mixed-Ability Teaching and Individualised Learning,* Costello discusses best ways of teaching a mixed-ability group. In this chapter some of the most popular commercially produced individualised schemes are discussed, together with the side effects of their use. The author says:

> Doubts about class teaching as a way of working with mixed-ability groups have led in recent years to an increase use of individualised learning schemes. The schemes may be based on a system of booklets, workcards, worksheets or a mixture of these. Pupils work at their own level and their own pace, so that all may be working on different topics or activities and, in theory at least, some may be years ahead of others. In practice, the order of the work has some flexibility, and is not devised as a linear progression. Careful planning and organisation are required, and, perhaps even more careful record keeping (p. 96)

Costello presents S.M.P. 11-16, the Kent Mathematical Project (K.M.P.) and the Secondary Mathematics Individualised Learning Experiment (SMILE) as examples of commercially produced individualised schemes in fairly widespread use, and adds the following comment:

> All these schemes have been planned with some care, so the teachers can expect to find them worthwhile and attractive: they are after all designed for commercial success. They may include a variety of additional material, such as equipment, sets of investigation cards, computer software or other supplementary resources. It all depends what you want.
> Inevitably, since the schemes were published some years earlier, the National Curriculum legislation has created a need for change, and publishers have responded to this. Because of the nature of this material, individualised systems have been classified into levels, not necessarily corresponding to age groups. There has been no consistency in the numbering of these levels across different schemes, and they have not matched the levels of the National Curriculum (pp 96-97).

As examples of side effects of individualised learning using such schemes, it is possible to quote:

> The teacher's contribution can degenerate into purely organisational matters concerned with which cards or booklets to work on, what equipment to use and so on... In this context, learning may cease to be a corporate or co-operative activity.
> Some considerations have been giving to the adequacy of individualised schemes in meeting the needs of the least able and most able pupils in mathematics. As far as less able pupils are concerned, published schemes vary their approach. The S.M.P. 11-16 material was not written to cover the entire ability range, and has not been promoted as suitable for the least able pupils...
> Using workcard - or booklet-based material with mathematically able pupils has inherent dangers. Pupils can, if they wish, choose to make life easy for themselves, taking the work slowly or adopting an over-relaxed approach. The material, most of which designed for a broad ability range, may not be especially motivating.... (p. 98).

As far as students on teaching practice and newly appointed teachers are concerned, when faced with the necessity of working with an individualised scheme, the author advises that:

> To start to work effectively, teachers need to become acquainted with the content and, even more critically, with the organisational aspects of the scheme. Students usually have the opportunity to do this by working with experienced teachers in the classroom, whereas the newly appointed teacher may find it difficult to arrange this. The essential preparation for teaching mathematics in this way lies in the process of familiarisation (p. 99)

It was interesting to note that, in a completely different approach from other books discussed here, Costello implicitly assumed that teachers do use textbooks and, at this moment (or at least in 1991) schemes such as S.M.P. 11-16 or SMILE, promoting individualised learning, are the most popular ones. It seems that although Costello considers that these textbooks were developed to allow 'pupils work at their own level and their own pace' (p. 96), the real situation is far from ideal, and advice given by the author in this situation changes to a different form: he advises against excessive individualisation and suggests some practical activities to balance the situation. Also included is some advice about the importance of preparing future teachers for these sorts of materials, because they will probably be working with them quite soon. On the other hand, very little is said about how to integrate other resources as complements to mathematics lessons.

Teaching and Learning School Mathematics (Pimm and Love (eds.), 1991) is another book concerning general issues related to the teaching and learning of mathematics. The book is subdivided in sections, and Section III: *Textbooks, Schemes and National Curricula* includes chapters in which the issues of textbooks and published schemes are tackled from different points of view. Introducing the section, the editors say:

> In the United Kingdom, the introduction of the various national curricula has exerted a powerful influence over the thinking of all concerned with the teaching of mathematics....both teachers and advisers have had to rethink their practices; meanwhile, the authors of textbooks and schemes have been busily producing new books or adapting current ones to their perceptions of the relevant national curriculum (p. 110).

Two chapters of this section present points of views on the National Curriculum. Another chapter presents a sociological analysis of S.M.P. 11-16 series, but this analysis does not include any references to the ways teachers have been using the scheme, so a review of this chapter is not included here. What is going to be discussed here are some opinions presented in the other two chapters of this section.

Fauvel (1991) is the author of an extremely enjoyable chapter in this section of *Teaching and Learning School Mathematics.* He presents a historical analysis of mathematics textbooks in England, starting from the sixteenth century (1543) - or even earlier in time if it is considered that the four books of Euclid's *Elements* (300 BC) are also mentioned by the end of the chapter. At the start, he describes the criteria he is about to use in analysing these texts. He says:

> The use of books in classrooms is a complicated matter. Whatever the overall structure of use - whether all pupils have copies or only the teacher does,...-they provide a fresh set of challenges, with both benefits and problems. A feature of mathematics texts which is useful to think about is the relationship between the author and the reader which is built into the book: how has the author set up the pedagogic interaction between text and pupil? What conception of the pupil is

> implied? What function is the teacher seen as serving? The way the text is written, and its pedagogical presuppositions, obviously constrain what the teacher, as one corner of the book/pupil/teacher triangle, can or should do in relation to the book's use in classroom (p. 111).

Fauvel is offering an interesting hypothesis to be verified: the strong influence of pedagogy of the book on the pedagogical aspects of mathematics lessons. One can have no doubts that the textbook influences the pedagogy of the lesson, but the strength of this influence is not clear. If it is true that it is strong, it can be expected that teachers using the same book work in a similar way (close to the way suggested in the Teacher Guide?). This research takes the position that it is not necessarily the case: instead, the position assumed in this work is that it is possible to find different experienced teachers using the same book in different ways, which means to say they have been adapting the book to their personal beliefs and their styles of teaching.

In chapter 12: *The Primary Mathematics Textbook: Intermediary in a Cycle of a Change* (Gray, 1991), the author emphasises the importance of textbooks as intermediaries between the changes mathematics educators (and other groups as well, including politicians) propose for the mathematics curriculum and the widespread implementation of such innovations, at least as far as primary education is concerned (p. 122).

Although Gray produces no research evidence, he presents the point of view that mathematics textbooks for primary schools have been increasing in importance over the last few years. He says:

> Traditionally, they [the textbooks] were no more than a permanent collection of suitable problems to support teaching. In their modern role, the function of primary mathematics textbooks has expanded to include attempts to:
> .help teachers to respond to the mathematical requirements of the society;
> .remedy and prevent weaknesses in children's levels of mathematical attainment.
> .provide a structured and sequential development of mathematics.
> .develop motivation through presentation and learning through understanding (p. 122).

However, the author also expresses the opinion that a textbook would probably be incapable of addressing all the needs of primary mathematics lessons, saying that:

> ...perhaps because of their wider function, textbooks do not always provide a coherent approach to the various elements of a broader mathematics curriculum. If a mathematical strand - for example, arithmetic - has a clearly defined hierarchy, textbooks can provide sound support. Curriculum elements without an agreed hierarchy - for example, shape - usually receive fragmentary treatment and teachers need to look for supplementary material. Sometimes it appears that mathematical structure and continuity lose out to other features, particularly the textbook's attempts to motivate children's learning (p. 123).

Finally, the author is quoted when he gives his opinion on the consequences on textbooks use following the advent of the National Curriculum:

> Publication of the national curriculum requirements in mathematics (1989) leaves us in no doubt as to the mathematical content identified for primary schools...The translation of these explicit requirements into action within the classroom may increasingly make teachers continue to turn to textbooks for topic definition, stage-by-stage mathematical progression, and mathematical activities for children. If history is anything to go by, writers and publishers will respond to the need (p. 123)

So, as far as primary education is concerned, it seems that, even if the advice given for teachers is pointing in other directions, there are mathematics educators who believe that teachers will still be using textbooks as their main resource for classroom work. This opinion is supported by the idea that in times of change, textbooks will spread innovations and present a practical way to implement such changes, even if this option is based on the author's views and interpretation of the new directions to be taken. It is possible to say that primary teachers are not mathematics specialists, but the pressures on secondary teachers to implement National Curriculum changes seem to be as strong as those affecting primary teachers, and there is no reason to believe that secondary teachers will act differently to primary ones.

4.3: Reviews of Textbooks by Mathematics Educators in Specialised Journals.

Reviews of textbooks in specialised journals were covered because it is possible that they could be influencing teacher attitudes towards textbooks. The time limit when looking for reviews on textbooks in specialised journals in England was changed, and extended back to 1970, for two main reasons:

1) The number of reviews published about textbooks in England in the last 15 years is quite low. The reviews in the specialised mathematics journals during this period of time are mainly about topic books or reference books. It seems that Cockcroft's (ed., 1980) suggestions were well accepted by the mathematics education community.

2) Some of the most popular textbooks in England were published at the beginning of the seventies, and mathematics educators' opinions of them at the time of their publication was of interest to the present research.

The following publications were searched: *Times Educational Supplement* (especially those with supplements dedicated to mathematics), *Mathematics Teaching* and *Mathematics in Schools*. All of them present a regular section of book reviews, usually written by mathematics educators and sometimes by mathematics teachers. Just to give an idea of how few textbooks have been reviewed during the past years, looking at Mathematics Teaching from the beginning of 1986 to summer of 1992, only one review of a textbook series was found.

A complete review of such material is not going to be presented, but there are some points it would be interesting to discuss here. All the more recent reviews talk about textbooks as possible resource books for teachers, rather than textbooks to be given to students as a basis for classroom work in mathematics during the school year. Some of the more recent reviews are about textbooks recently published to match National Curriculum attainment targets or levels, and their reviews reflect this approach. Some examples:

.Lee (1991) says about the textbook reviewed in her article:

> I have raised some reservations about the relatively limited amount of maths that has been presented, the mathematical foundations that are being laid, and the general pedagogical approach. ...I see it not solely (or even primarily) as a textbook, but rather as a suggestion of what a textbook, indeed what a pedagogical practice *could be*. This book has, in fact, increased my awareness of the limitations of the textbook mode of teaching,... (p. 58).

.Williams (1991) says about the textbook reviewed in his article:

> Mathematics in Action is a well written progressive course in mathematics...It merits the attention of any Head of Department looking for a new scheme....
> ...[the book] could provide the teacher with a useful source of easily accessed examples when using a different scheme. (p. 46).

.Waters (1992) says about the series of textbooks reviewed in his article:

Recognising the dangers of ignoring such facts of life, the authors of the N.C.C. guidance warn against trying to teach by levels. So, why should a publisher present a scheme whose title and structure appears to promote exactly that?
... The spread of topics appears to cover adequately the national curriculum attainment targets Ma 2 to 5. ...
It would certainly be possible to use parts of these books in a more realistic way than is implied by the titles. Some of the content overlaps more than one level and some questions could be extended to provide starters for groups of mixed aptitude and attainment. ...(p. 23).

On the other hand, the series published at the beginning of the seventies were welcomed by their reviewers at the time as innovative and useful textbooks that teachers could use as a basis for their lessons, especially if they provided extra material to complement the information given by the textbooks. Some examples:

.Sturgess (1971) says about the textbook series reviewed in his article:

> These books are extremely well produced. The type is clear: the pages not cluttered up with too much material: the diagrams first class and the illustrations relevant. The Teacher's Books are the Pupils' Books with answers and comments interleaved. I have used a lot of this material in a way similar to that suggested in these books with less able children and I know that it works.... I think there is no doubt that whatever criticisms one makes they are a major contribution to the textbook material available to teachers of mathematics. (p. 63).
> If my own area is anything to go by these will shortly become the most widely used texts in secondary schools in this country. This places an enormous responsibility on the trustees of S.M.P. to see that it really is a developing series and not static.... It could be done by issuing cheap topic books, supplementary work cards,.... This is the kind of material that many teachers produce for themselves. It is however extremely difficult to produce such additional work for unfamiliar content, and the pressures on the classroom teacher to produce this kind of material in all kinds of spheres are growing at an alarming rate.... (p. 64).

.About the same series of books, Reynolds (1972) says:

> This can be no ordinary review. Even before it is completed, there are indications that this series has become the most widely used secondary text in Britain; schools do not need to be told it is good, somehow the news has got around. ...
> But the dangers of such package are worth noting. It enables a weak, inexperienced teacher to do a competent job, but it also allows a lazy teacher to give up thinking about his work, and there's a good chance of getting in a rut. However, the rut is a much better one than some that have gone before....
> The philosophy of the series is class teaching: those who prefer unstreamed classes (or those who have to teach them!) may be interested to know of some work-cards, now being developed, which should be available in 1973....
> Basically, this is an excellent series: well written, ...But it's not perfect and so long teachers are aware of this and provide supplementary material, it should form the basis of a sound course for those schools that wish ... *examination.* (pp. 30-31).

Primary series were also published in the seventies, and their reviews were similar to those for secondary schools. They were welcomed for the innovations they presented,

criticised for some of their faults and recommended either as textbooks or resource books in accordance with schools' decisions. Some examples:

.Hewins (1972) says:

> ...[textbooks should] be capable of use by a non-mathematical teacher and still give children a progressive development of mathematical concept.
> There seem to be a limited number of books on the market which fit this definition, but the *New Oxford* series are a very welcome addition to the list. ...
> I found the whole series to be excellent material as a course book or an alternative source of material for the whole school, I would give my fully recommendation to the set... (pp. 58-59).

.Bolister (1977) says:

> The main contribution of the project materials, compared with other primary mathematics schemes currently available, lies on the one hand in the clear presentation and, on the other, in their production in form of individual assignment cards (with the disadvantages as well as the advantages of this). ... The challenge remains for others ... to show how exploration, experiment and the intrinsic motivation of mathematical activity can be integrated into a reasonably structured scheme for primary school children and their non-specialist teachers. (p. 55).

So, there was a clear change in the position of reviewers of textbooks during this period of time. Textbooks are considered much more harmful today them they were 15 or 20 years ago. This is probably related to a whole set of new educational theories that have appeared and influenced mathematics education during this period of time. What reviewers nowadays seem to forget is that, despite all those theoretical approaches, teachers still use textbooks.

4.4: Other Sources of Advice for Teachers.

Articles Published in Specialised Journals in the U.S.A.

The articles published during the last ten years in specialised journals in the United States reflected the awareness of American researchers and educators on how textbooks are a widespread resource used by teachers. Most of these articles are related to the quality of published textbooks as well as analysis of the content of the most popular ones. The more recent, published in the 90's, also include some expectation for changes in such books, influenced by the new directions in mathematics education.

In his article, prepared for the National Commission on Excellence in Education, Doyle (1983) discussed the character of academic work in elementary and secondary schools. The author is a specialist in classroom research and is also the director (or at least was at the time of the publication of the paper) of the Research on Classroom Learning and Teaching Program at the Research and Development Center for Teacher Education, University of Texas at Austin.

After presenting an impressive broad review of research done in the US during the late seventies and early eighties concerned with academic work in elementary and secondary schools (there are approximately 200 references at the end of the article), Doyle used their findings to support conclusions such as 'performance on academic work, especially in technical subject matter areas, depends on domain-specific knowledge, rather than general problem-solving strategies alone'(p. 169) and 'studies of the cognitive processes underlying academic work have revealed the enormously complex character of the operations and decisions that academic competence entails' (p. 170). Discussing the subject of complexity further, the author said: 'this complexity is much more severe, however, for young students and those who lack either the information or the skills required to understand tasks, process information in specific ways or decide when to use the strategies they possess' (p. 173).

In order to derive implications of such conclusions to Instructional Policy, Doyle also discussed the Classroom as the natural environment in which the academic work takes place. The role of the teachers as managers of such space is considered as well as the social nature of interactions that occur during a lesson and the evaluative climate of classes. The author also included the role of Instructional Materials in classroom environment. Based on previous research developed in the US, he stated that 'A large amount of classroom time is structured around printed materials. Indeed, many 'lectures' actually consist of a teacher going over content contained in a textbook. Moreover, in elementary and many secondary classes, students spend two-thirds of their time doing seat work with printed worksheets' (p. 180).

It was also clear that Doyle did not support the idea that it is possible to change the wide spread use of printed materials. Instead, he supported that 'clearly, more research is

needed on the cognitive demands of classroom materials and ways of making them more suitable for instruction because they play such a big role in academic work' (p. 181).Finally, the author concluded the article by presenting suggestions for improvement of the quality of academic work developed in elementary and secondary schools, showing special concern about the so-called 'low attainer students'. He summarised his ideas saying that 'major improvements in academic work clearly depend on further enquiry into the event structures of classrooms and how the work is accomplished in these environments' (p. 190).

Komoski (1985), who is (or at least was at the time of the publication of his paper) Executive Director of Educational Products Information Exchange Institute, Water Mill, New York, presents a historical - and economical - analysis of published textbooks in American Education, a summary of the usual criticism about textbooks as well as suggestions to improve the quality of such materials. During the historical analysis, he says:

> Despite occasional criticism, for almost 150 years now, textbooks have been the easiest, most economical and most convenient means of containing, articulating and managing the curriculum. The problem is that they are far from what those who are required to learn from them deserve...(p. 33)
> Because the major challenges of mass education were largely administrative in nature, the standardized textbook became much more an administrative tool and instructional organizer than a means of facilitating learning. Thus, learners who were either unprepared for or unmotivated to respond to that standardized textbook-based administration of instruction quickly became rejects of the system...
> Even efforts to develop materials that could be understood by most students took the form of textbooks-related options that clearly didn't really work for all students - particularly the ever-presented *workbook*.(p. 34)

This research takes the position that the situation in England has already changed from that presented in such analysis: the criticisms above are some of the most common ones found about the so-called 'traditional textbook' in this country, and several mathematics textbook publications since the beginning of the 70's were clearly planned to offer an answer to them. On the other hand, there are other kinds of criticism presented in this article that textbooks in England have been receiving systematically. These are:

> .Unchallenging or uninteresting textbooks with simplistic, formularized writing.
> .Textbooks that try to "cover" everything and hence "uncover" nothing in depth.
> .Unimaginative computer software with poor documentation.(pg. 33)
> Of course, no mere material - textbook, videotape, computer software, or videodisk simulation - can "teach".... Isn't a human teacher needed to do such teaching? And isn't a teacher the essential catalyst for learning?...This heavy teacher dependency on materials has - in large part - exacerbated the long-established administrator dependency on materials as well. It is a symbiosis that is not contributing to the health of the system. (Komoski, 1985, p. 35).

At the end of the article, several suggestions are made to meet the general goal of improving the described situation:

1. Make ... materials ... "fit" the central concern in the development and selection of instructional materials. This may sound like a recommendation calling for the "individualization of instruction". It is not. It simply calls for taking time and making the effort to put into the hands of the teachers and students' materials that can be readily understood and that communicate something of educational value in a well-written, well-researched and engaging manner.
2. Give teachers the time and the training to become more discriminating selectors and "fitters" of materials to student needs,...
3. Give teachers more responsibility for the selection of learning materials...
4. Give teachers the time and means for exchanging useful ideas and strategies for adapting the materials...
5. Develop a process for improving materials using regular and systematic *feedback from learners before and after* publication...
6 .Establish budgets, training programs, and purchasing and selection policies that recognize that development and use of better quality materials is going to take time and cost money. ... . (p. 37)

It is also suggested that instructional materials must possess "*instructional integrity and focus"*, meaning that:

Such material has a teaching and learning wholeness that is the result of:
1.A clearly apparent educational intention for the learners who are going to use it.
2.A coherence and currency of content that engages learners intellectually ...
3. Teaching and learning activities that engages the learner in mastering the intellectual content and processes in such way that the learner will be open - even eager - to learn more.
4. Mechanisms that help both the learner and teacher to continuously assess student mastery of the material and inform the publisher of how to improve the product's effectiveness.(p. 37)

So, instead of a general criticism implying that teachers must not use textbooks, the article presents clear suggestions on how to improve textbooks themselves using feedback from the schools. It also suggests better training for teachers.

Woodward (1987) presents in his article a criticism about textbooks in general: the author comments on illustrations and graphics, the excessive teaching of skills and the widespread use of worksheets that are not always integrated with the content, giving several examples found in textbooks from all subjects and levels. It also refers to the problem of how to cover the content, presenting a summary of the demands textbooks are expected to fulfil and their consequences:

Even though it is unreasonable to expect that textbooks should try to include everything that might be needed for a student to learn fourth-grade social studies or sixth-grade science - or for teachers to expect them to do so, textbooks do attempt to cover everything that might be wanted by teachers and administrators. As producers of materials for a national market, US publishers must include in their programmes the myriad skills and topics that are demanded by a decentralized, pluralistic and differentially powerful group of states and school districts. In addition, publishers

must attempt to meet the demands of the day, be they a consensus that textbook difficulty should be gauged by readability formulae, that the teachers' guides that accompany textbooks should contain precise and extensive directions for each lesson, or that skills-based textbooks should contain elaborate management systems. With the need to meet so many demands, it is hardly surprising that textbooks are imperfect learning materials. Short cuts are made; the unthinking application of readability formulae produces stilted and simplistic prose; the need to cover so many topics in a limited amount of space produces textbooks that emphasize breadth rather than depth. The effect of these pressures has resulted in the widespread concern about quality and accuracy of textbooks.(p. 522)

The advice given at the end of the article follows the general pattern we have found in the American articles:

> Clearly, if textbooks are to be improved it is essential that teachers, administrators and textbook selectors become more demanding and discriminating consumers. When this happens the market will indeed change and publishers will respond. In the meantime, perhaps teachers need to select textbooks on the basis of how well users (students) learn from particular textbooks....(p. 525)

Nibbelink, Stockdale, Hoover & Mangru (1987) present a study related to the number of problems offered by Mathematics textbooks and their grade of difficulty over the past 30 years. The article compares data on the number of problems included in textbooks influenced by different educational theories in Mathematics during that time and concludes that there was a change during the 80's from the previous situation during the 60's and 70's. It also includes some comments about late 80's textbooks in US:

> In our view, however, new concerns regarding the late eighties are arising. Never before have so many textbook publishers decreed so much about *how* problem solving should be taught,...we recall the centipede who, when thinking too hard about *how* it walked, could no longer do so.
>
> Another concern of late is that publishers may be trying too hard to be all things to all buyers. The new programs sport strands about computers, non-standard problems, calculators, careers, and exotic animals; bulletin board displays: accessories for gifted and accessories for not-so-gifted; outdoor math; indoor math; group experiments; and so on; to the point that a light pick-up truck may soon be a necessity to the sixth-grade teacher who wishes to haul the program home for the weekend. None of these accessories are bad, but the availability of so much may dilute any sense of focus for the mathematics curriculum. It may soon be possible for a class to spend a busy hour each day and never once get to traditional problem solving... (p. 37)

These criticisms of 'new' textbooks are quite different to those reviewed here. They raise the problem of misuse of excessive material. All the advice discussed so far criticised textbooks for not giving enough. This one looks at the problem from a different angle: How do teachers that have as many options as they want for classroom activities deal with so great an amount of material? Are their students really learning mathematics, even if they are kept busy most of

the time? Does the use teachers make of such materials include time for mastering the new concepts? From all that has been said about misuse of materials for unprepared teachers, one can only conclude that these are important concerns for a time where so many options are offered for teachers.

More recent articles point to a change in the situation presented here. There are mathematics educators in the U.S. expecting a change in textbooks to adapt to the new situation but not expecting that from now on teachers and students will rely less on textbooks. Kim (1993) summarises this idea saying that: 'Research publications ... powerfully urge a reform of mathematics education in the United States. The reform efforts should have many facets, including a review of content selection and organization in the textbooks. ... ' (pg. 125). Such a position may not be unanimous, but there are Mathematics Educators in the U.S. thinking about how textbooks can be used for classroom work from now on.

N.C.T.M. Publications

The powerful National Council of Teachers of Mathematics in the United States publishes several materials for teachers of mathematics: these publications include year-books treating important issues for mathematical lessons and specialised magazines and journals for teachers and mathematics educators researchers. N.C.T.M. has published the *Standards for School Mathematics* (1989), that has caused an impact in mathematics education in the U.S. and must be considered the main reason why so many changes are expected in the system during the next few years.

A complete summary of N.C.T.M. publications should not be expected here, but some of them have to be mentioned: a year book from 1973, dedicated to Instructional Aids in Mathematics (N.C.T.M., 1973); a pamphlet dedicated to evaluation of Mathematics Textbooks (N.C.T.M., 1982, 1987); some reviews of textbooks in N.C.T.M. journals before and after the publication of the *Standards* and certainly, the *Curriculum Evaluation - Standards for School Mathematics* (N.C.T.M., 1989) itself. It was possible to identify two different positions in such publications: before 1989, textbooks were generally treated as one of the most important resources teachers could use in classroom work; after it reviews have changed.

The main reason why the *34th year book* (N.C.T.M., 1973) and the publication '*How to evaluate Mathematics Textbooks*' (N.C.T.M., 1982, 1987) are mentioned here is because they are excellent sources to show how textbooks were highly considered before the publication of the *Standards*. Although a complete revision of such material is beyond the scope of this review of literature, they are certainly books to be taken into account if an attempt to evaluate textbooks is made.

The *Standards for School Mathematics* (1989) publication is presented as 'one facet of the mathematics education community's response to the call for reform in teaching and learning of mathematics ... Inherent in this document is a consensus that all students need to learn more and often different mathematics and that instruction in mathematics must be significantly revised' (p. 1). It presents general suggestions for improvement of mathematics education in US. Topics such as curriculum and evaluation are treated in detail. Ideas such as students being seen as active learners and opportunities for all students to develop all topics are emphasised through the text.

The authors do not refer to textbooks specifically, but to instructional resources for mathematics lessons. For grades from 5 to 8 the classroom materials suggested are:

1)Ample sets of manipulative materials and supplies (e.g.: spinners, cubes, scales, compasses)

2)Appropriate resource materials 'from which to develop problems and ideas for explorations' (pg. 67)

3)Calculators with functions for all the students.

4)At least one computer available at all times. 'Additional computers should be available for individual, small group and whole class use' (p. 68).

For grades from 9 to 12 the authors suggest 'changes in the instructional patterns and in the roles of both teachers and students' (p. 125) and there are no further comments on resources except for special attention being given to computer software and graphing calculators (p. 128) and advice to decrease attention to teacher and text as exclusive sources of knowledge, 'paper and pencil manipulative work...'(p. 129). The only explicit reference to textbooks found in the book is a criticism, when the authors are presenting the curriculum standard for grades 5 - 8:

> An examination of textbooks' series shows the repetition of topics, approach and level of presentation grade after grade... It is even more disconcerting to realize that the very chapters that contain the most new material, such as probability, statistics, geometry and pre algebra are covered in the last half of the books - the sections more often skipped by teachers for lack of time. The result is an ineffective curriculum that rehashes material students already have seen. Such a curriculum promotes a negative image of mathematics and fails to give students an adequate background for secondary school mathematics. (p. 65).

So, textbooks are not mentioned when the authors present the list of resources for primary mathematics education, they are not mentioned when secondary education is the theme and the only explicit reference to textbooks in the whole book is a criticism. It is not even possible to say that resources are not dealt with in the book. To give an example: calculators and computers are referred to several times as useful resources to be used in mathematics classrooms. From such evidence, one can only conclude that the authors do not consider textbooks a valid resource for classroom mathematics at any level anymore. From

the text, it seems that they should be replaced by other resource materials, but even the role of resource or topic books is not explicit in the text. Not even the 'usual' advice that textbooks can be used as a source of examples, exercises and activities was found.

N.C.T.M. also publishes a journal called *Mathematics Teacher*. This journal presents a regular section of reviews of publications. This section is organised in two parts: the first one refers to N.C.T.M. publications and the second one to other publications. It was expected that some advice on how to use such materials together with the general comments about the quality of the book itself would be found. Reviews in magazines were searched for 1991 and 1992 and it was not possible to add much information from this source, except for confirmation of the depth of the determination to comply with the *Standards*' recommendations. In several reviews affirmatives such the following were found:

> I sincerely recommend *Developing Number Sense* as a valuable tool as we work to implement the curriculum and evaluation standards. (Sanders, 1991)
> This book is a well-organized and thorough textbook for a traditional approach to teaching beginning algebra. The textbook does not adhere to the N.C.T.M.'s *Curriculum and Evaluation Standard* ... (King, 1992)
> This book does not approach the study of mathematics in a manner consistent with the N.C.T.M.'s *Curriculum and Evaluation Standards*. (Marchand, 1992)

but it was impossible to find much advice on better ways of using the books, except for strong recommendations for books somehow to 'fit' into the new approach recommended:

> The focus of the booklet is on strategies to promote this new approach. The introduction section gives good examples of the types of communication within a classroom that foster the development of number sense. ... (Sanders, 1991).
> Teachers will find exciting applications that they will want to use immediately. I used anagrams, ...in my high school classes. These problems were very well received...This book should be part of every mathematics teacher's library! (DeCoste, 1991).

4.5. Summary

Prior to the late seventies, the majority of publications recognised the textbook as a useful aid: the textbook was considered not only as an organiser of the progression of the contents but also as a good source of examples and exercises. This does not mean that textbooks were not criticised, but their 'negative' points were balanced by their 'positive' ones. New texts, which attempted to respond to the general points of criticism were warmly welcomed. By the late seventies, the use of textbooks began to be strongly criticised. The advice given tended to remove teachers from the use of a textbook as the main basis for their lessons. 'Topic books' became recommended and teachers (or the mathematics departments) became expected to produce their own material. The progression of the contents became a responsibility of the mathematical scheme of work, that should be developed by schools' mathematics departments. '*Mathematics Counts*' (Cockcroft, 1980) is probably the most influential example of such publications.

At the beginning of the eighties, a publication from Her Majesty Inspectorate: '*Mathematics in the Sixth Form*' (1982) pointed to the problem that students who were finishing secondary school and aiming for further education did not 'learn' how to learn mathematics from books. They seemed to rely on their teachers as the only source of knowledge and books were used only as a source of examples and exercises, even when the lesson 'did not differ greatly from ... the textbook' (p. 28).

Nevertheless, very few publications considered the problem. '*Children Reading Mathematics*' (Shuard and Rothery, 1984) is the only one found. The majority of publications for teachers during the eighties that consider the issues of texts and their use for classroom work, criticised textbooks in general, and advised teachers not to use them as a basis for their lessons. '*Better Mathematics*' (1987) is perhaps the most influential book of this kind. At the same time, very few reviews of textbooks could be found in the specialised magazines. Most of these reviews were about 'topic books' and the few textbooks reviewed were recommended not as textbooks, but as another possible source of examples and exercises. Note also that the publications reviewed here were not research based, and they produced neither evidence nor examples of schools that had successfully adopted the proposed changes. Some research in the actual ways teachers have been using textbooks for mathematics lessons is needed.

To conclude this summary, it is worth noticing that the more recent publications in England addressing the issue of textbooks seem to acknowledge that, despite all the criticism during the eighties, textbooks are still being used by schools, and in a great number of cases, used as the main source of classroom work. These publications recognise that the 'use of books in classrooms is a complicated matter' (Fauvel, 1991, p. 111) and tend to be more analytical than critical, considering the different types of textbooks that could be used by schools and the necessary adaptations to the National Curriculum.

CHAPTER 5

RESEARCH QUESTIONS AND GENERAL DESIGN FOR DATA COLLECTION

> There were indications that some schemes were more likely to be used as core materials than others, which were more often used as one of a number of resources. However, whatever the intentions of the publishers, it is the use made by the teachers of commercial materials which affects the mathematics experienced by pupils in the classroom.
>
> Millett and Johnson, 1996, pg. 65

The review of literature shows that very little research has been done on the way teachers (including secondary mathematics teachers) have been using written materials for classroom work (e.g. see Bodin and Capponi, 1996). It is also clear from the few research projects related with this matter that they are mostly based on questionnaires, making it possible that they are reflecting teachers' beliefs, without taking into consideration the pressures and constraints of the real classroom situation (Askew et al., 1993; Askew et. al, 1997; Hoyles, 1988; Laborde, 1996).

Nevertheless, there is a considerable amount of literature advising mathematics secondary teachers on how to use (or not use) texts for mathematics classroom work. Post 1980 publications are predominantly against the use of textbooks as a basis for instruction, as indicated in chapter 4. Although researchers seemed to be challenging these views in more recent publications (Hart, 1992; Robitaille et al., 1993; Clarke et al., 1996), these influences were not yet reflected in books specially written for teachers at the time when the data was collected.

Based on the existing surveys (Robitaille et al., 1989, Askew et al. 1993), one can expect to find many teachers basing a large amount of their classroom work on textbooks, but one can also expect that many of those teachers would be complementing the textbook with other printed materials and/or with their own produced written materials. One can also expect to find some teachers not using textbooks at all, but basing instruction on their own (or school's) selected printed materials or even on their own (or school's) written materials instead.

The underlying theoretical perspective to this research presented in chapter two presupposes the role of the teacher as that of an active instructor making decisions that have 'real consequences in the classroom' (Fennema et al. 1989 a, pg. 176). So, the general objective of this research project was to obtain a picture of how secondary mathematics teachers have been using written materials (commercially produced or their own) for classroom work. To obtain such a picture, it was intended to generate criteria based on

differences and similarities in ways of using these materials in order to obtain a categorisation of the ways teachers have been using written materials for mathematics classroom work. As stated by Romberg and Carpenter (1983), students' outcomes must be considered as an 'ultimate dependent variable' (pg. 861) in research on teaching, so it was also intended to verify if there is any relationship between teachers' decisions on use of written material and students' achievement of teacher's aims.

The purpose of this chapter is to introduce the research questions and the methodology adopted for data collection in the present study. Some of the chosen methodology's constraints are also discussed.

5.1. First Research Question

Based on the general objective stated at the introduction of this chapter, the first research question should be: Using certain criteria, is it possible to group different ways of using written materials into distinguishable categories? As a starting point, it was assumed that teachers, as active instructors in the sense given by Ausubel and Gagné, use written materials in different ways. For example: while some teachers might be using textbooks only as a source of graded examples, exercises, and be providing the explanations or discussions themselves, others might expect their students to gather explanations from the text as well. Also, it was expected to find teachers who based a large amount of their classroom work on a single series of books, but it was also expected to find others who used several printed materials, including textbooks, and even those who produced their own written materials.

As there is little (if any) previous research on use of written materials for classroom work, the advice from researchers in teaching (Kilpatrick, 1977; Romberg and Carpenter, 1983; Good and Biddle, 1988) must be considered and, in order to categorise these different ways, one must start by identifying actual ways in which teachers use written materials in their classroom work - and this was the main object of the data collection.

A survey-based methodology would cover a larger number of teachers but, on the other hand, would probably give information on teachers' opinions and expectations rather than information on what was really happening in mathematics lessons in secondary schools (Askew et al., 1997). It would be possible to end up with a picture of teachers' opinions and judgements about their own work instead of reproducing some examples of the ways teachers have been using written materials for classroom work.

If a methodology based on observation of classroom work and interviews was chosen, the data collection would have to be restricted to a small number of teachers, and it could not be expected that the sample would be large enough to represent all the possible ways teachers use written materials in classroom work (Good and Biddle, 1988). However, it could be expected that the *identified* ways would be an authentic picture on how those teachers were using written materials for classroom work. It could also be expected to find out differences and similarities in the ways those teachers used written materials - and some of the differences would be sufficiently relevant to allow the categorisation of these ways of using such materials.

Taking into account these considerations, a methodology was chosen that allowed the search for the influence of written materials on as many 'segments' of classroom work as possible for each teacher in the sample (Good, Grouws and Ebmeier, 1983; Leinhardt, 1989). Other aspects highlighted in the review of literature had to be considered if the present study expected to account for the influence of the use of written materials for classroom work: there were indications that considerable differences between experienced and novice teachers'

practices could be expected (Leinhardt, 1989; Livingston and Borko, 1990); there were also indications that different groups of students would have different needs (Brophy and Good, 1983). As data was collected from a small number of teachers, these variables needed to be taken into account when defining the sample. It was a research project decision that experienced teachers would be selected and observed while working with different groups of students.

Based on the arguments presented, the first research question introduced at the beginning of this section needs to be modified from the former (too general) formulation to a more realistic one, as it cannot be expected that identified uses could be considered as representative of all possible ones. A better formulation seems to be: ***Using criteria based on relevant differences, is it possible to group the identified ways of using written materials into distinguishable categories?***

5.2. Second Research Question

The second research question is related to students' performance, and this must be defined (Romberg and Carpenter, 1983; Brophy and Good, 1983). Because of the definition of teacher's sample, one could not expect the students in the sample to be all from the same year group. On the other hand, for each teacher, the sample of students was defined to include a wide range of attainment levels, because it was hypothesised that teachers' decisions about written materials could be different for different students' groups (Brophy and Good, 1983). Due to these factors, it also could not be expected that the student groups would be working on the same mathematical topic.

Nevertheless, it would be necessary to test students' outcomes in some way, if it is expected that this study should comment on the effects of teachers' decisions on how to use written materials for classroom work. Notice also that it was not the case to test for long term aims or to verify these students' performances in some standardised tests. The objective of testing the students was to verify the outcomes related to the observed lessons, more specifically, the objective was to verify students' outcomes related to written materials used during observed lessons. It was decided to define performance as *achievement of teacher's short term aims*. Each observed group of students was tested for its achievement on tasks taken from the written materials used during observed lessons. It was also decided that these tests had to be approved by the teacher before being applied to the students.

As different groups of students were observed working in different ways (in fact, even students from the same group were observed working in different ways, if the instruction was individualised), the sample of students tested had to be defined in such a way that several different uses of materials were represented. Each student in the test sample was tested on a piece of mathematics he/she had been working at during the observed lessons, with emphasis on questions taken from the written materials used.

Because the tests applied were completely different to one to another, one cannot expect to compare groups' performance using these results (Romberg and Carpenter, 1983). Note also that several other variables could influence the results of these tests, such as level of difficulty of the questions, students' previous knowledge, teaching styles, etc. (Good and Brophy, 1997; Koehler and Grouws, 1992; Fennema et al. 1989a). Nevertheless, it was expected to find out if it was possible to establish a link between certain decisions made by the teacher on how to use written materials for classroom work and the performance of students in immediate post-tests directly related to these decisions. Taking these considerations into account, the second research question can be formulated as: ***Do particular identified ways teachers use written materials in their classroom work relate to better achievement of a teacher's short term aims by the pupils?***

5.3. General Design of the Data Collection.

This study is intended to be a qualitative research on how secondary mathematics teachers have been using written materials for classroom work. From the research questions, it became clear that the main objectives were: to identify and categorise actual ways written materials are used and to link these identified uses with students' outcomes in immediate post-tests related to teachers' short term aims. The considerations discussed when proposing the research questions led to decisions that were central when planning the data collection: (1) classroom observation would be the central source of data in identifying actual ways written matters were used and (2) different groups of students taught by the same teacher should be tested for outcomes related to the observed lessons.

Several constraints in the methodology can be regarded as direct consequences of these initial decisions. The use of classroom observation as methodology for data collection implies a small sample of teachers. Taking the size of the sample into account, it is not possible to expect identified uses of written materials to be representative (Good and Biddle, 1988). As a consequence, the categorisation proposed by the first research question would have to take into account only general aspects of the observed strategies (those which could be generalised). Another constraint that has to be considered relates to the proposed tests for student's outcomes. They were tested based upon observed lessons, so no comparison among groups could be made. The tests could only be used as indicators of students' outcomes related to a particular decision on written materials made by their teachers. As Brophy and Good (1983) emphasised, those sorts of results can only be considered as links, no cause-effect conclusions can be drawn. Restrictions also had to be made, because many other variables could have a large influence in students' outcomes (teacher's style and management of classroom , time spent in different segments of the lessons, etc. - Leinhardt, 1989).

Another important decision made at the initial stages of methodology planning was that all teachers in the sample should be *experienced.* This decision was based on previous research (Leinhardt, 1989; Livingston and Borko, 1990) indicating considerable differences in performance amongst experienced teachers as compared to novices. It was decided not to add another variable to existing research problems. On the other hand, one could expect experienced teachers to present richer strategies concerning usage of written materials for classroom work. In this sense, to catalogue their decisions would be a more useful contribution for research on teaching, specially regarding applications to teacher training.

Based on these initial considerations, decisions about teachers and student samples, as well as decisions about other sources of data and data collection methodology were made. They are presented in the following sections.

5.4. Sample of Teachers

The number of teachers in the sample, as well as the number of students tested were strongly dependent on the amount of time estimated to collect and analyse sufficient data to give initial categorisation types. All teachers in the sample volunteered. This means to say that it was necessary to find and interview these teachers. It is also important to note that not all teachers initially interviewed met the required criteria for the sample. The time spent searching for teachers was considerable and, once a teacher agreed to participate, it was necessary to arrange suitable times for about two and a half weeks in the school once. All these considerations taken into account, it can be said that the total time spent searching for teachers was more than a school year (five terms), and some 20 weeks were actually spent in schools.

The sample of teachers in the main data collection is *eight teachers*. It was expected that this number of teachers would be enough to allow the development of a study based on: (a) teacher interviews, (b) observation of classroom work and (c) verification of supplementary sources of data (such as schemes of work, notebooks and records of assessment), (d) a test given to the students based on teacher's short term aims. It was also expected that this number of teachers would be enough to highlight some relevant differences in the ways teachers have been using written materials with their students. The sample included two teachers who did not see themselves as basing their classroom work on a series of textbooks and six teachers who saw themselves as basing their teaching on some of the most popular series of textbooks in this country.

Textbooks series in England are usually presented in different 'tracks', to be used with different groups of students. On the other hand, there were theoretical indications (Ausubel) and research indications in the literature (Cooney, 1988; Brophy and Good, 1983) that it was not possible to be sure that the same teacher used written materials in the same way with all of his/her groups of students. It was decided that it would be necessary to observe each teacher's work with more than one group. Whenever possible, *four* different groups of students for each teacher would be observed. Such a sample should include groups of different perceived attainment levels and at least one of mixed ability. Under such conditions, it was impossible to observe all four groups from the same school year. Nevertheless such group choice enabled the present research to verify if there were differences in the usage of written materials by the same teacher due to year groups, composition of the group or levels of attainment.

To obtain a picture on how a teacher has been using written materials in his/her classroom work with one group of students, it was necessary to observe more than one lesson. To produce enough evidence that written materials were used in a certain way, it was necessary to observe consecutive lessons over a period of time. It was estimated that, for each group of students, all their mathematics lessons during one week would be observed.

Nonetheless, it was also decided that whenever the teacher stated that one certain 'content' or 'topic' was going to be developed, at least the 'complete' series of lessons related to that topic should be observed. It was also estimated that for each teacher, at least one more week at school was needed, in order to collect data from other sources (schemes of work, notebooks, testing the students and interviewing the teacher). So, for each teacher in the sample, a minimum time of a fortnight was required for data collection.

Selecting the Sample of Teachers.

As it might not be possible to recruit eight teachers who would fulfil the desired conditions, selection criteria had to be presented in an *order of importance.* The first, second and third items were considered *essential*, and it was desirable that the teachers should satisfy as many others as possible. The *criteria* of selection follow:

1)All teachers had to be working at a School that includes students aged 11 upwards.
2)All teachers had to have at least five years experience.
3)Two of the eight teachers selected had to be basing their teaching on their (or the school's) own selected materials, while the other six had to base their teaching on one among the most popular series of books in this country.
4)The teacher had to be working with groups of students that the school had classified as high, medium and low attainment groups, as well as one mixed ability group.
5)Among teachers using the same series of books (or using their own material) and fulfilling the same previous requirements, preference was given to teachers that used the material in different ways from others already selected for the sample (evidence of complementation using different materials, changes in the order of the content, different classroom management, etc.).
6)Among teachers using the same series of books (or using their own material) and fulfilling the same previous requirements, preference was given to teachers who had a personal commitment to the written material they used.
7)Among teachers using the same series of books (or using their own material) and fulfilling the same previous requirements, preference was given to the best qualified teachers (P.G.C.E. course in Mathematics, undergraduate course in Mathematics, in service courses in Mathematics Teaching, etc.)

5.5. Sample of Students.

As mentioned before, it was expected that for each of the teachers selected, all his/her mathematics classes for four different groups of children during a week would be observed. So, it was estimated that 32 groups of students were observed. The ideal composition for the set of groups working with the same teacher was:

1)One mixed ability group
2)One group considered by the school as being of high attainment level.
3)One group considered by the school as being of medium attainment level.
4)One group considered by the school as being of low attainment level.

On selecting teachers, the criteria took into account this ideal composition, and also what was possible to concede without changing the main objectives of this research. If it were the case that for a selected teacher it would still be possible to choose between two equivalent groups (for example: two seven year mixed ability groups), the choice would be made by chance. It's important to note that such a set of groups implied that the groups which were observed were from different school years, so the students in the sample were from different age ranges.

In fact, all teachers selected but one matched the student group criteria completely, so the total number of groups observed was 30. From each of these four groups of students working with the same teacher, some students were chosen to be tested (at least 10, but whenever possible, the whole group). The test applied varied depending on how the teacher organised the classroom (all class working at the same task, group work, individualised work, etc.). Nevertheless, it was important to make sure that students who participated in different activities in the classroom were tested and also that test results could evaluate the students' immediate learning from the written materials used.

5.6. Defining the Sources of Data

Many different sources of information were used during the data collection. For each one of those sources of data, a series of questions was developed to be used as guidelines. These questions reflected the underlying theoretical perspective of this research project as well as results from previous research into teaching. Nevertheless, the main tool for data collection was the observation schedule, as results were expected to reflect the real classroom situation.

The observation schedule used also had to fit the general objectives of this research. It was designed to provide an answer to the questions presented in this chapter. It was necessary to form (if possible adapting from existing ones) and to test an observation schedule. The pilot study for this research project was dedicated to test the developed observation schedule.

For each teacher, information was gathered from the following sources:

1) First interview with the teacher.
2) Complementary sources of data on planning: school scheme of work and teacher scheme of work.
3) Written materials given to the students.
4) Classroom observation.
5) Complementary sources of data on classroom work: students' notebooks, teacher's records of previous assessment and informal interviews with teachers after lessons.
6) Test with some of his/her students from each group.
7) Final interview with the teacher.

Except for source (6), which clearly related only to the second research question, all other sources initially related to the first research question, and provided information on how written materials were actually used by the teachers.

For each source, it was necessary to generate a series of general questions on which the data collection was based. These questions enabled the researcher to gather information about the influence of written materials in several activities in teacher's work. It was not possible to ask exactly the same questions for teachers using commercially produced textbooks and for those who produced their own written materials or were adapting them from different sources of printed materials. These questions are presented arranged according to information source and are organised into two main groups:

[GI]- Questions for teachers using textbook series, and

[GII]- Questions for teachers using their own (or school's) selected written materials.

1. Questions for the First Interview with the Teacher

This interview was necessary as part of the selection process described in section 5.4. More than eight teachers were interviewed, but part of the information gathered here is essential for the research, such as: (a) teacher's previous courses and experience; (b) teacher's general beliefs about the use of written materials; (c) groups of students and their perceived

attainment levels; (d) written materials he/she has been using; and (e) criteria on grouping his/her students into different attainment levels.

The questions reflect the views of Ausubel and Gagné that teachers have an important role as direct instructors. They also reflect the intentions of the researcher to verify: (question 1) experience and previous qualification of the teacher, as there were indications in the literature that these factors could influence teacher's performance (Koehler and Grouws, 1992, pg. 121; Leinhardt, 1989 and Livingston and Borko, 1990); (questions 2 and 3) teacher's personal beliefs about the choice of materials adopted in the school, as there were indications in the literature that teacher's views are reflected in their practice (Fennema et al., 1989 a and b) but, on the other hand, they could be affected by other factors (Hoyles, 1988, pg 155); (questions 4, 5, 6 and 7) teacher's personal decisions on how to use the chosen material and how they affect their plans and actions (Fennema et al., 1989 a , pg. 176); and (questions 8 and 9) influence of other factors in teacher's decisions. These questions were the guidelines for the initial interview:

[GI]- Questions for teachers using textbook series:

1) Tell me something about: (1) How long (in time) have you been teaching? And where? (2) Your P.G.C.E. course, (3) Your undergraduate course, (4) Your in-service courses.
2) Was the decision of using "X" as a textbook a personal one? (if it is a school decision: Have you participated in the decision process? How? Do you agree with such a decision?)
3) Do you like your textbook? What are the good points about it? Do you think that to use a textbook is better for your students then to use non-commercially produced written material? Why?
4) Do you have your own written scheme of work for each of your students' groups? (if yes: is it matched to (1) school scheme? (2) textbook or teacher's guide to the textbook (3) students' attainment levels ? / if not: what have you been using instead ?)
5) Do you complement your textbook using other written materials? What sort of materials?
6) Do you use the same textbook with students from different attainment levels? (if yes: Why? Do they all use it in the same way? / if not: What are the different materials being used? Are they all used in the same way?)
7) Is the assessment of the students matched to the textbook? (if yes: is it the suggested assessment from the textbook? Complemented? How? / if not: What is the base of the assessment of the students?)
8) How were the students classified into different attainment levels? How has the textbook influenced the classification?
9) Is the progression of the content based on the textbook?

[GII]- Questions for teachers using their own (or school's) selected written materials:

These questions are almost the same as the previous ones, interchanging *textbook* and *own produced materials*, except for:

4) - (2) must be changed to: While writing the materials, do you use any printed materials as help? Which ones? How are they used?

5) must be changed to: Who produces the written materials? (if the school: Did you participate in the production process? How?) Has any printed material been used as help? How? Do you complement your written materials using any printed materials? (if yes: what are these materials?)

9) must be changed to: how do you obtain the progression of the content? Is it based on any printed material?

Questions for the Complementary Sources of Data on Planning: School Scheme of Work and Teacher Scheme of Work

Planning is an important part of a teacher's work (Cooney, 1988, pg. 273) , and the school scheme of work must be one of the bases upon which teacher's plans are built (Cockcroft, 1980). Even with the National Curriculum offering guidelines (Askew et al. 1993), each school is supposed to produce its own scheme, and the influence written materials exercise in this scheme will probably be reflected in classroom work. So, during the data collection it was necessary to look for references to written materials in the school scheme of work.

Secondary mathematics teachers do not usually write their own work scheme, so it was not expected that this source would be as relevant as the school scheme. Nevertheless, if a teacher had his/her own written schemes, they would probably be more detailed about his/her classroom work, and they would be a useful complementary source of data, because the teacher's personal views on how to use materials would be reflected on them (Fennema et al, 1989 a and b).

The following questions related to the school scheme must be answered :

[GI]- Questions for teachers using textbook series:

1) Is the school scheme based on (or matched to) any commercially produced printed material? Is the title of the chosen series of books written in the school scheme of work? Is the usage of such material obligatory?

2) Does the school scheme of work include any recommendation about complementary written materials? What sort? Are there suggested titles?

3) Is it clear from the school scheme whether or not all the teachers in the school must be using the same textbook? Are they supposed to use the teacher's guide as well?

4) Is it clear from the school scheme of work whether or not students from different attainment levels must be using the same textbook?

5) Are the recommended progression and assessment of the content in the school scheme of work based on (or matched to) any textbook?

[GII]- Questions for teachers using their own (or school's) selected written materials:

1) What are the recommendations in the school scheme of work about using written materials? Does it explain why?
2) Does the school scheme of work include any recommendation about printed materials? Is there any evidence that it is matched to any printed materials?
3) Is it clear from the school scheme whether or not all the teachers in the school must be using the same selection of written materials?
4) Is it clear from the school scheme of work whether or not students from different attainment levels must use the same written materials?
5) Is the recommended progression of the content in the school scheme of work based on (or matched to) any commercially produced printed material? If not, is there any evidence from where the progression of the content was taken? What are the recommendations in the school scheme of work about assessment?

The questions for the teacher schemes of work are quite similar to the questions for the school scheme, except for questions (3) in both groups, that must be replaced by:

[GI]- Questions for teachers using textbook series:

3) Is it clear from the teacher's schemes of work whether or not he/she has been using the same textbook with all his/her students' groups?

[GII]- Questions for teachers using their own (or school's) selected written materials:

3) Is it clear from the teacher's schemes of work whether or not he/she has been using his/her own selection of written materials with all his/her students' groups?

Questions for the Written Materials Given to the Students

Previous knowledge of the written materials themselves was necessary, in order to understand the ways these materials were used by the teachers (Love and Pimm, 1996, pg. 385). It was not intended to carry out a complete analysis of the written materials, but it was necessary to spend some time becoming familiar with them, in order to understand teacher's interpretations and adaptations' (Robitaille et al., 1993, pg. 28) of these materials as important influences in the observed differences in use of such material .

[GI]- Questions for teachers using textbook series:

1) What are the printed materials used by the teacher? Is there one that might be considered as the *textbook*? Is there any supplementary written material being used? If yes, is it developed by the teacher (or by the school) ?

2) Are there different materials for different attainment level students? Do they come from different series of textbooks?
3) Is the instruction in the textbook based on students self-instruction?
4) Are there tests or any other kind of assessment provided by the textbook? (If yes, it is necessary to verify if they have been used by the teacher). How close is the assessment done by the teacher to the examples and exercises in the text?
5) Is there a Teacher's Guide? (if yes, it is necessary to verify if it is used by the teacher)

[GII]- Questions for teachers using their own (or school's) selected written materials:
1) Has the written material been developed by the teacher or by the school? Is there any printed material being used as well? If yes, what kind of activities has been provided by these materials?
2) Is there any evidence that different materials have been used for students of different attainment levels?
3) Are the written materials based on self-instruction?
4) Are the tests and other forms of assessment developed by the teacher or by the school? Is there any printed test or other form of printed assessment being used?
5) If the material has been developed (or selected) by the school, is there any kind of teacher's guide for it? (It is necessary to verify if the teacher has participated in the development of the material and if he/she has been following the recommended usage).

Questions for Classroom Observation

The classroom observations constituted the core of the data collection. The main objective of such observations was to verify how written materials were actually used in classroom work (Askew et al. 1997 pg. 2-25; Brophy and Good, 1983, pg. 329; Good and Biddle, 1988, pg. 131). The classroom environment provided several sorts of information, and it was necessary to ensure that researcher attention during the observation was directed towards the use of texts. So, forming and piloting an observation schedule preceded the main data collection, and these were the subjects of the pilot study.

Nevertheless, it is important to present here the questions to which it was expected the classroom observations could provide answers. They reflect views in teaching and learning expressed in the theoretical framework as well as views presented in literature on differentiation (e.g. Cockcroft, 1980) and about teacher's decision making during lessons (Cooney, 1988). They also reflect the views that a lesson can be divided into several segments (Leinhardt, 1989; Good, Grouws and Ebmeier, 1983):

[GI]- Questions for teachers using textbook series:
1) Are there different printed materials being used with different students?
2) Are decisions about the use of the material taken during the lesson?

3) Are the introductory explanations of the lesson (or a new topic) done by the teacher or are the students supposed to gather information from the text? If the teacher does the introductions, are they based on the text or on the teacher's guide? Is there any evidence that such explanations are complemented by other written materials?
4) Is there any evidence that the teacher is introducing other written materials motivated by student's questions? What kind of materials?
5) Is there any evidence that the teacher is using the printed material in a different way than that suggested by the text or teacher's guide?
6) Are these exercises taken from the book? Is there any evidence of complementary work done using other texts?
7) How does the teacher introduce the exercises? How does the teacher correct such exercises?
8) Does the teacher give homework? Do the students take copies of the written material home? Do they keep a copy of it all the time? Do they use the textbook for homework?
9) (If it happens that during the observed lessons any kind of assessment takes place.) How close is the students' assessment to the text? Is it the suggested assessment in the text? Is there any difference among the assessment of students from perceived different attainment levels?
10) Are general conclusions taken from the activities developed during lessons? How are the written materials used in this segment of the lesson?

[GII]- Questions for teachers using their own (or school's) selected written materials:
These questions are almost the same as [GI] questions, where *textbooks* will be replaced by *written materials*.

Questions Relating to Complementary Sources of Data on Classroom Work

Students' notebooks provided an overview on whether the work developed in other lessons during the school year followed the observed pattern. The notebooks were useful in determining how often the textbook was used for classroom work and the frequency of changes in texts. It was expected to complement the information gathered from the classroom observation using the notebooks.

The informal interviews with teachers after observed lessons as well as the records of previous assessment made by the teacher were used to complete the picture on whether or not teacher's decisions were based on the text. They were also used to complement this research with a picture on teacher's expectations for different attainment level students (Good and Brophy, pp. 104-105).

Test with Some of the Teacher's Students from each Group.

The objective of testing some students was to provide an answer to the second research question: *Do particular identified ways teachers use written materials in their classroom work relate to better achievement of teacher's short term aims by the pupils?* Although the testing of students on short term objectives may say little about their overall mathematical knowledge, a high failure rate, for example, can be associated with a teacher decision that was not successful in judging the appropriateness of the content developed during observed lessons, in the sense proposed by Piaget and Vygotsky.

The tests were developed by the researcher based on the use of written materials during the observed lessons. The test's contents varied according to the work done by the students during the observed classes. Each student was asked to repeat some of the exercises taken from the written materials he/she had been working on during the lessons. These tests were discussed with the teacher, who had the final approval on them, before they were applied. As many students in a group as possible were tested. If all students in the class were working at the same tasks during the observed lessons, all of them performed the test, otherwise a sample of at least 10 students was selected. Such a sample was chosen by discussion with the teacher, and was considered representative of the diversified work done by the students based on written materials, whenever it was the case.

Questions Relating to Final Interview with the Teacher

The objective of the final interview with the teacher was to complete the information gathered during the data collection about teacher's decisions on how to use written materials with each group of students. The specific questions varied from teacher to teacher, depending on the observed work, but were about practical matters observed during lessons. The general guidelines were:

1) Good points about the usage of chosen written materials.

2) Bad points about the usage of such materials.

3) What were the relationships between his/her plans for lessons and the chosen written material, including teacher's guides?

4) How dependent was development of the content on the written materials?

5) How dependent was the classroom organisation from the written materials?

6) How important was it to provide different activities for different attainment level students? Did the written materials help him/her in this matter?

7) How important was the use of written materials other than the *textbook*? What sort of mathematical experiences was provided by these materials that was not provided by the usual textbook?

5.7. The Pilot Study.

An observation schedule that could provide a record for mathematics lessons and in which the role of written materials used by the teacher could be recorded was needed. Several schedules used in the U.S.A. and Australia were reviewed, but none of them seemed to fit the purposes of this research. Two observation schedules used in England were also reviewed: (1) Beeby, Burkhardt and Fraser (1979), who developed an observation schedule for mathematics lessons called SCAN - Systematic Classroom Analysis Notation and (2) Eggleston, Galton and Jones (1975, 1976) who developed an observation schedule to be used in an evaluation of science teaching methods. Neither was suitable, as they did not allow the researcher to record several of the central points related to the present research project (such as actual exercises done by students during the lessons or the notes made by the teacher on the blackboard). So, a draft schedule was prepared, in which codification was kept to a minimum (the few codes adopted were based on SCAN (1979)). Eggleston et al.'s (1975) observation schedule also influenced its design due to its emphasis on the importance of matching the instrument with its objectives. The researcher decided to write notes rather than codes whilst sitting in a class, so the observation schedule should be dedicated to the activities developed by teacher and students when using written materials.

The pilot study for the present research aimed at testing the formed observation schedule in order to verify whether or not it suited the researcher's needs as an instrument to be used for the main data collection. It took place in a secondary comprehensive school in Nottinghamshire, U.K. During one day, four lessons with three different teachers were observed. Those teachers were working with different groups of students: two seventh year mixed- ability groups, one eighth year top ability group and one of ninth year low attainers. The researcher focused its observation on the use of the chosen written materials for classroom work. A copy of the written material chosen by the teachers (textbooks) was also added to the data collected.

The completed observation schedules were used to prepare reports of the lessons. From the analysis of these reports, it transpired that there were a few inadequacies in collecting the necessary data. Some reports were too descriptive. Also, some of the information recorded had no bearing on the use of written materials. On the other hand, some important pieces of information, that could be relevant to the research work were missing: 1) a better description of the exercises was necessary; 2) the researcher did not take notes on the nature of the questions posed by the teacher during lessons; and 3) for some questions, a complete answer was not apparent (for example: were the exercises given for practice or for development?)

A useful device was to add a classroom organisation diagram to the observation schedule. This strategy was used during the first observed lesson, because there was the need to refer to one specific student (who was facing difficulties with the proposed tasks) and this

was a possible way to identify the student. However, the diagram provided much more information than initially expected: it was useful when recalling information from the observation schedule to produce the corresponding report and it was also an easy way of recording interactions between students themselves during the lesson (for example: students who worked in pairs were linked with a line in the diagram). The other modifications done to the observation schedule were fairly small: a column dedicated to 'observations' was eliminated, mostly because these could be recorded in the 'teacher activity' or the 'student activity' columns, which had their width enlarged.

School:.....Teacher:.....

Class:.....

Material used:.....

Diagram of the classroom during the lesson:

class. organi sation	teacher activities	teacher material	close ?	students activities	students material

. . . . . .
. . . . . .
. . . . . .

Figure 5.7.1: A fragment of the observation schedule.

Figure 5.7.1 shows the format of the observation schedule used in the main data collection. The first column was used to record classroom organisation. Data such as 'the teacher was working with the whole group at the same time', or 'the students were working through differentiated tasks', etc.. were recorded there. The second column was dedicated to record teacher activities during the lesson. Data such as the introduction of the lesson, the marking of notebooks, the conclusions offered by the teacher, his/her comments on each student's work, etc. were recorded there. The column 'teacher material' was dedicated to record the material used by the teacher during the lesson, such as textbooks, worksheets, answer

books, etc. The column 'close?' was introduced to allow the researcher to comment during the lesson how close a developed activity was to the one proposed in the teacher guide for the chosen material. The fifth column was dedicated to recording the students' activities. The exercises they were doing, the use of answer books, the discussion with their colleagues are examples of data recorded there. Finally, the column 'students' material' is self-explanatory.

CHAPTER 6

METHODOLOGY FOR ANALYSIS OF DATA RELATED TO THE FIRST RESEARCH QUESTION

> ... the fundamental task of conducting classroom research is to generate and test plausible theory concerning teaching. When relatively primitive, such theories consist of terms that represent classes of observable events and propositions that summarize observed relationships among them.
>
> Good and Biddle, 1988, pg. 137.

The aim of this chapter is to present a description of the methodology used to analyse data generated during the main collection relating to the first research question: *Using criteria based on relevant differences, is it possible to group the identified ways of using written materials into distinguishable categories?*

A description of the methodology used to identify the classroom activities and the teacher's decisions that seem to be influenced by written materials is given in section 6.1. The set of identified decisions and activities are called headings. Section 6.2 presents a description of the codification of the observed teacher's decisions and behaviours under each of the headings defined in section 6.1. In this section, some examples are also given, in order to emphasise the need for a strong set of general codes that could be used throughout the whole set of data related to the first research question. The general description of the set of codes used is given in section 6.3. These codes are called meta categories.

To conclude this introduction, it is important to acknowledge that the methodology of analysis suggested here has been based on suggestions of qualitative data analysis made by Miles and Huberman (1994).

6.1. First Steps in Analysis: Identifying Classroom Activities and Teacher's Decisions Influenced by the use of Written Materials (Headings).

The first step in the analysis of the first research question was the organisation of collected data into Reports of the Lessons. For each group of students observed, a report containing the description of the complete set of observed lessons was prepared. These reports were based mainly on the completed observation schedules, although other sources of data, such as informal interviews with the teacher and analysis of students' notebooks were used to complement them. Remarks and comments were added to these reports, identifying the sources of data used. An example of one of these complete reports is presented in appendix 2b.

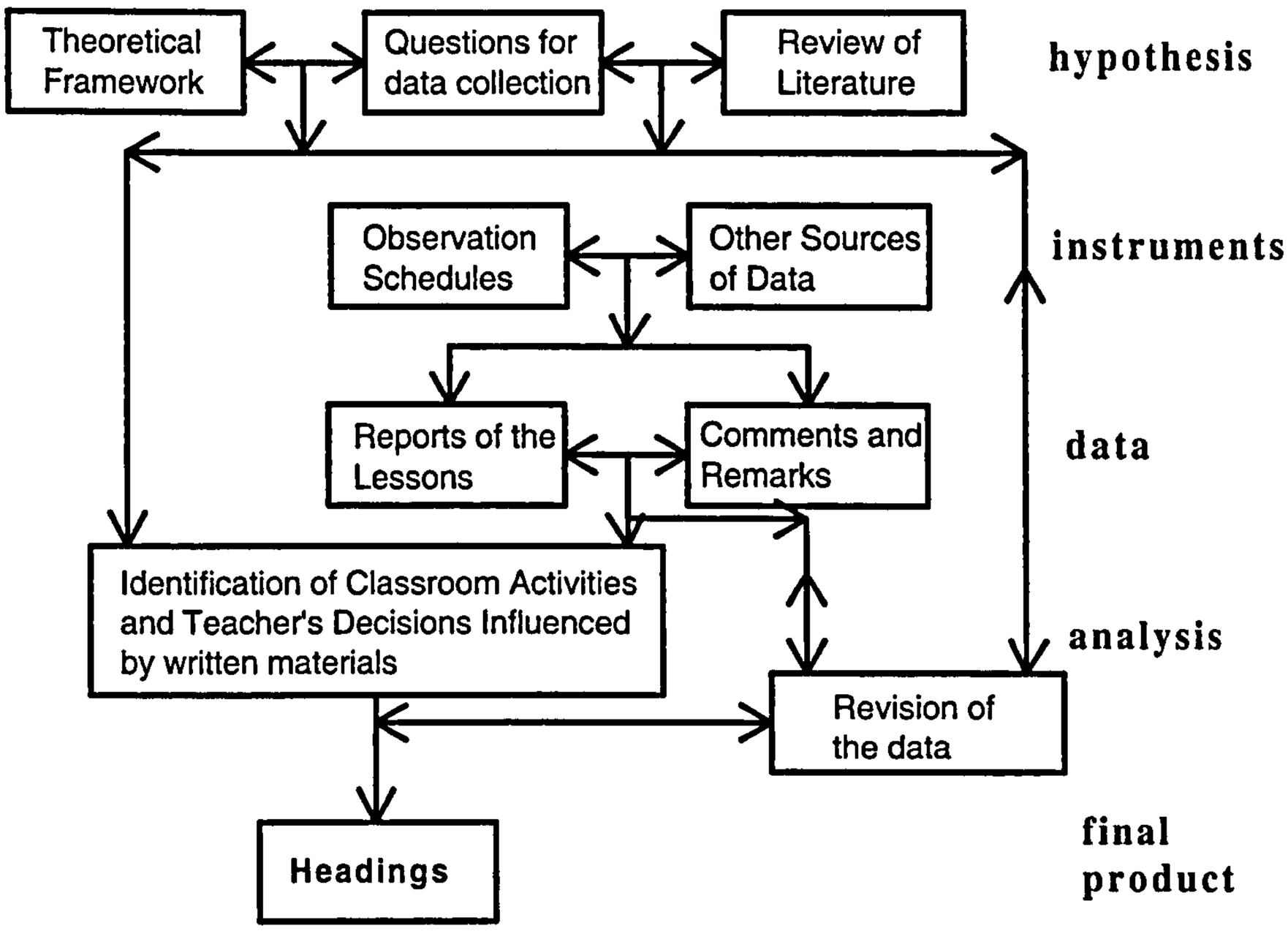


Figure 6.1.1: Flow chart of the process of analysis used to identify classroom activities and teacher's decisions influenced by the use of written materials.

The following step was the identification of classroom activities and the decisions made by the teacher that were influenced by the use of written materials. Figure 6.1.1 presents a flow chart of this step in the process of analysis. The identification was done not only by using the reports of the lessons but also by reviewing the theoretical framework, previous research results discussed in chapter 3 and the questions defined for data collection in chapter 5.

The first group of behaviours and decisions influenced by the use of written materials was then verified against the data (revision) with the objective of trying to recover data lost in

the process. The final product of this analysis is going to be referred as Headings from now on.

.Definition of Headings and Their Organisation into Three Sets

A Heading is defined in this study as an observed teacher's behaviour or decision related to written materials to be used for classroom work. In order to group these headings into sets, previous research results were examined. Initially, there were indications that teachers plan for their classroom activities (Fennema et al., 1989a, pg. 176), and it was considered in the present study that decisions on written materials are a crucial part of the planning process. On the other hand, researchers have shown concerns that school factors could affect teachers' decisions and behaviours. These constraints are likely to influence teachers' plans for the year, and affect their decisions when choosing written materials. Taken these considerations into account, a first set of headings was defined.

- **Set I :** Headings that are related to general decisions made by the teacher concerning materials to be used during the whole school year.

On the other hand, there were also indications in the literature that teachers' decision making ideas do not end during the planning of the activities for the year. Teachers made decisions before each lesson, and also during lessons (Cooney, 1988, pg. 273). If the concept of 'lesson segments' (Leinhardt, 1989, pg. 56) is considered, a lesson can be divided into several moments, including the developmental part of the lesson (Good, Grouws and Ebmeier, 1983) and the 'seat work' moment (Brophy and Good, 1983). Teachers need to make decisions not only when they choose which written materials are to be used in each segment but also when they determine how the chosen materials are to be used during each of these moments. Taking this consideration into account, two other sets of headings were defined:

- **Set II :** Headings that are related to decisions made by the teacher about written materials in each lesson segment. The material to be used seemed to be a decision that could be modified from one lesson to another within the same group of students, or even during a lesson, if in order to solve a 'crisis situation'.

- **Set III:** Headings that are related to the way a chosen material is actually being used for classroom work (or homework). This set differs from the others in the sense that the choice of material is not considered anymore. The question now is 'how is it being used?'.

Because teachers used different methodologies during the observed lessons, the division of the lessons into the two basic segments (developmental and seatwork) proposed in the literature (Leinhardt, 1989, Good et al., 1983) was not enough to characterise the observed use of written materials. The basic segments were subdivided into several headings, mainly supported by the theoretical framework presented in chapter two.

In the first place, it is considered that any lesson starts with an 'introduction' provided by the teacher. Although this lesson segment could be used by the teachers as a simple

description of the subsequent activities, some teachers used the introduction segment of the lesson to provide links with previous knowledge and/or used particular strategies in helping the students to get started. These are important issues discussed by Vygotsky, Ausubel and Gagné, as seen in Chapter Two. To Ausubel, meaningful learning could not take place, unless links with previous knowledge were promoted. Gagné was concerned with pre-requisites and Vygotsky asserted the importance of 'what children can do with ...: assistance' (1978, pg. 85).

Examples also can be used to help children to get started, so this is another heading to be considered. Giving pupils appropriate material with which to work is also supported by the theoretical framework. In fact, the choice of materials for class activities is quite an important consideration when analysing teacher's decisions about the choice and use of written materials.

'Conclusion of a topic' is another important lesson segment according to the theoretical framework. Concluding a topic, by summing up the main results, is one of the most effective ways teachers may help the students' knowledge to become operational, as defined by Piaget (1964, 1969). He stated that once a new piece of knowledge is assimilated, it is necessary process of accommodation to take place, otherwise the knowledge does not become operational (that is: it cannot be applied in new situations). Complementing this argument, Ausubel stated that the knowledge of the content is as important as the development of the capacity for solving problems when formal education is concerned. Assimilation and accommodation as processes of learning also support the importance of providing children with reference material, which will give them a chance to look back to the introductory activities and exercises, either to revise or to improve their previous understanding.

Finally, 'links between related topics or topics in progression' are directly related with Ausubel's idea of meaningful learning and with Gagné's idea of pre-requisites. In establishing connections with previous knowledge during the presentation of a topic, the teacher (or the written material, or both) is helping the students to internalise the knowledge acquired.

Yet other headings were introduced to verify other aspects considered in the literature review. For example, the advice offered to teachers on how to use texts for class work was taken into account when defining headings as 'differentiation'. In the list bel ow, each heading is followed by a brief discussion, in order to complement the arguments already outlined.

. Headings in Set I: (General Decisions)

- **I - 1 :** Main Source of Materials: This set of decisions is related not only to questions of planning (asked during the interviews with the teacher), but also to analysis of the school scheme itself. Schools that do not have a written scheme used one textbook series as the main source of material. This heading provides an overview of the sources of materials being used.
- **I - 2 :** Progression of Content: This heading is related not only to answers to questions about progression of content (interviews with the teacher) but also to analysis of the

school scheme itself. In schools that do not have a written scheme, the teacher was asked some questions about modification and complementation of the suggested progression in the textbook series.

- **I - 3 :** Materials given to the Students for the purpose of reference at home: Reference materials can be defined as those that the students can use in order to look back for explanations, examples, suggestions of work and/or definitions. This set of behaviours provides information about the materials the students usually use to revise and study at home. Teacher's interviews were the main source of data for this heading.

. Headings in Set II and Set III: (Day -by-Day Decisions and Use of Written Materials)

The reasons for these two sets of headings being presented together are: (1) both sets of headings are related to day-by-day teacher's activities and (2) the headings in set II can be matched with the headings in set III. The same features are considered from two different points of view: while set II emphasises teachers' decisions on choice of material, set III changes the focus to teacher's decisions on how to use the chosen materials. The main source of data for these sets of headings was the reports of observed lessons, although there were occasions when data were complemented by an informal interview with the teacher, aiming to clarify his/her decisions on choices and/or usage of the written material.

Another important issue to be considered when describing headings is that of 'matching questions'. If it is considered that the core of a lesson is the set of activities proposed to the students during the lesson, it is important to analyse whether the remaining features in the lesson match it.

- **II - 1 :** Choice of materials to be used in class as source of activities :

  III - 1 : Use of the chosen materials in class as source of activities:

  These headings deal with information about materials used for activities in the classroom during the observed lessons for each particular group of students. They are presented first because students' classwork is considered the kernel of the lesson and used as reference to the 'matching questions'. Any assessments taking place during the observed lessons are also included under this pair of headings. A practical reason for this inclusion is that very few assessment situations were observed during data collection, and a special pair of headings dedicated to it would not be worthwhile.
- **II - 2 :** Choice of materials used to introduce a new topic:

  III - 2 : Use of the chosen material to introduce a new topic:

  While analysing these headings, the focus will be on how the teacher decided to introduce new content, and how close the introduction was to the activities proposed for the lesson.
- **II - 3 :** Choice of materials used to introduce a lesson:

  III - 3 : Use of the chosen material to introduce a lesson:

These headings discuss the links between what is going to be done and what has already been done. Introducing a lesson provides useful links with previous knowledge. Another aspect analysed in these headings is whether the introduction matches the activities proposed for the lesson.

- **II - 4 :** Choice of materials used as reference in class work:
  III - 4 : Use of the chosen material as reference material during the lesson:
  These headings complement the information given in heading **I - 3,** by describing the materials used as a source of reference during the observed lessons for each particular group of students. Whether these materials (if existent) match the activities are also considered during the analysis of these headings.
- **II - 5 :** Choice of materials used to produce exemplification:
  III - 5 : Use of the chosen material as a source of examples:
  Examples can be defined as illustrations of a topic (definition, concept, rules, etc.) through its applications in exercises and/or in problem situations. During the observed lessons, examples were the most common way used by teachers to clarify methods and/or rules, to illustrate the application of definitions. These headings provide a view on how materials were used as sources of examples during the observed lessons. Once more, 'matching' with the activities developed was considered.
- **II - 6 :** Choice of materials used to conclude a topic:
  III - 6 : Use of the chosen material to conclude a topic:
  The main feature to be analysed under these headings is whether the conclusion summarises and organises the content developed during the lesson(s).
- **II -7 :** Choice of materials to establish links between related topics or topics in progression:
  III - 7 : Use of the chosen material to establish links between related topics or topics in progression:
  Again the justifications of the inclusion of these headings are given by the theoretical approach taken in this research. Whenever different sources of materials are used to cover related topics, it should be analysed how the teacher provided the links, as it cannot be expected that the materials would provide such links. On the other hand, if the same source of materials is used to cover both topics, it is expected that some links would be provided by the material.

The following two pairs of headings were included to deal with the issue 'differentiation'. Differentiation has been highlighted in advice material for teachers and it is considered in two different aspects: (1) the teacher decides to promote 'planned differentiation' to address students' individual differences, (usually, this kind of differentiation is based on teacher's previous experience with the students) and (2) the teacher promotes differentiation motivated by a 'crisis' in the classroom (the most common example

of such 'crisis' is: some students are having difficulties in solving the proposed exercises). The present research addresses these two situations separately.

- **II - 8 :** Choice of materials used to promote differentiation within class:
  III - 8 : Use of the chosen material to promote differentiation within class:
  One of the main reasons for criticism of textbook series is that their contents are not necessarily suitable for all the children in the class. Differentiation is defined in this research as the planned introduction by the teacher of different activities simultaneously, in order to respond to students' different needs.
- **II - 9 :** Choice of remedial material used to provide help for students facing a crisis:
  III - 9 : Use of the chosen remedial material:
  Contrasting with differentiation, this pair of headings considers whether the teacher changes the chosen activities (with possible change of material) when realising that some students are facing a crisis. For this research, remedial activities (materials) are defined as the activities (materials) proposed by the teacher motivated by unplanned students' needs.
- **II - 10 :** Choice of materials used as sources of homework.
  III - 10 : Use of the chosen material as a source of homework:
  When analysing these headings, not only choice and use of materials will be considered, but also whether homework matches with the class work observed (or to be developed in the following lessons, whenever homework is considered as a preparatory activity).
- **II - 11 :** Choice of materials used to provide feedback on exercises:
  III - 11 : Use of the chosen material to provide feedback on exercises:
  For the present research, feedback is defined as the set of responses given by the teacher to students' answers to proposed activities. Although in general feedback can be considered in both directions: (1) students → teacher, and (2) teacher → students, only the second one will be analysed here. Feedback on exercises (including homework and tests) was very frequent in the observed lessons.

Although the headings above could be presented in any order, three aspects were considered when deciding their sequence:

1. The first pair of headings examines the material(s) a teacher decides to use as source of activities.
2. The following pairs of headings (from pair II.2, III.2 to pair II.7, III.7), consider whether other lesson segments (such as introduction and conclusion) were based on the same material as those used as main source.
3. Finally, the last four pairs of headings (from pair II.8, III.8 to pair II.11, III.11) are the ones dedicated to appreciate features related to the lesson development in which it is more likely that other materials were introduced to respond to students' specific needs.

In the next paragraph, the role played by the set of Headings in the analysis will become clear.

6.2. Codifying observed behaviours for each heading.

Figure 6.2.1 presents a flow chart for the generation of codes for the different behaviours in each heading. The reports of the lessons and information obtained from other sources of data were matched with the headings. Possible different teachers' behaviours under each heading were identified.

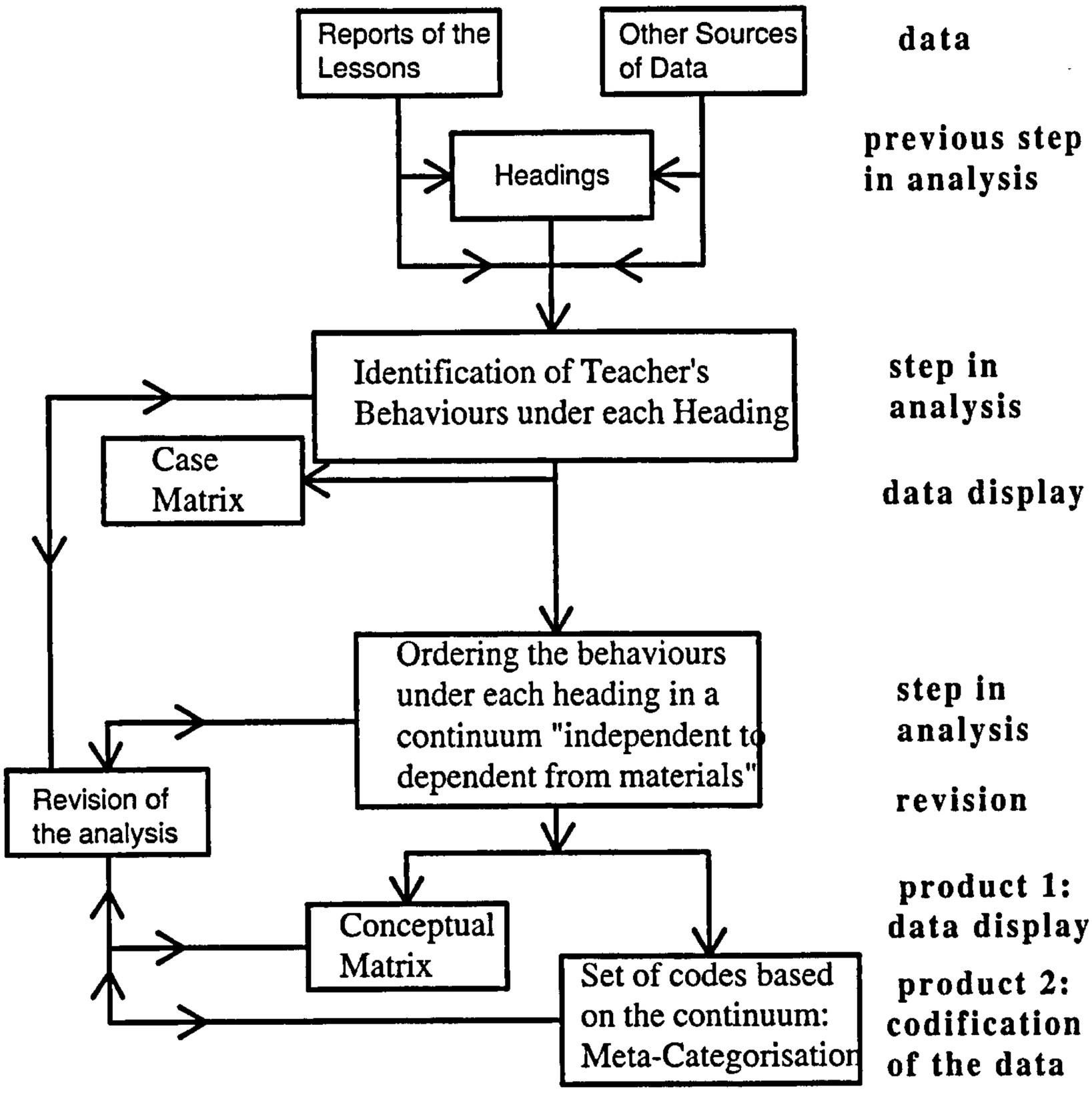


Figure 6.2.1: Flow chart of the identification and codification of the different behaviours presented by the teachers in the sample for each heading.

After identifying the possible behaviours under each heading, it was possible to create an initial display using the matching of the data with the headings. This display is the Case Ordered Meta Matrix, 'which is a matrix that contains first level description data from all cases' (Miles and Huberman, 1994, pg. 189) with the format presented in figure 6.2.2. The schematic example provided in figure 6.2.2 also shows that it is possible for the same teacher to present different behaviours under the same heading with different groups of students, or even within the same group of students. It also shows that it is possible that different teachers present similar behaviour under the same heading. From now on, matrix displays matching headings with student groups are called Case Matrices.

Teacher / Group→ Headings	Teacher A Group A1	Teacher A Group A2	....	Teacher B Group B1	...
Heading 1 (H1)	Behaviour X for H1	Behaviour Y for H1	....	Behaviour X for H1	....
Heading 2 (H2)	Behaviour X' for H2	Behaviour X' for H2	....	Behaviours Y' and Z' for H2	...
Heading 3 (H3)	Behaviours X" and Y" for H3	Behaviour Y" for H3	...	Behaviour Z" for H3	
....	....	....	....	....	....

Figure 6.2.2: The format of the Case Ordered Meta Matrix.

However, the Case Matrix was not an easy display from which to retrieve data. The first disadvantage was its size, the second was that, although it could be used to give an idea of one teacher's behaviour with one group of students (by following the column), it was not so simple to compare behaviours in different groups (comparison of two different columns). Another disadvantage was the fact that behaviours under each heading were presented by case, making it difficult to compare behaviours of a teacher in two different groups even under the same heading (comparisons within the same row). Comparisons between different rows were almost impossible. All these disadvantages pointed to two different types of needs: The first one was to create a set of codes that could summarise the information contained in the Case Matrix. The second one was to create another display, where observed behaviours under each heading were not ordered by case anymore, but presented in some kind of conceptual order (meaning it was generated by some analysis criteria) that would facilitate comparison in between behaviours. A display such as that described above is called a Conceptual Matrix

Observed behaviours under a heading were then organised in a continuum stretching from 'independent from written materials' to 'dependent on printed materials' at the other extreme. Behaviours were coded so that the numerical code 1 was attributed to behaviours that show great independence from any sort of written materials, code 2 to behaviours that show great independence from printed materials, and so on, to '8', this last being attributed to behaviours that show great dependence on printed materials and teacher's guides. These numbers provided a meta-categorisation that could be applied to the whole set of answers to questions related to the first research question.

	independent from written materials ←							→ dependent on printed materials
Meta-Categories. Headings:	1	2	3	4	5	6	7	8
Heading 1	-	A1, A2, B1	A3, B2, C1, C2, C3, C4	A4	B4	-	B3	-
Heading 2	-	C1, C2	A1, A2, A4, B1, B2	-	C3	C4	B3	B4
....	....	....	....	....	....	....	....	....

Figure 6.2.3: Format of the Conceptual Matrix. (In this schematic example, A1 is the code name for the first group of students of teacher A; A2 for the second, etc.). Not real data.

By using the meta-categories, it was possible not only to summarise the Case Matrix, but also to create a Conceptual Matrix, formatted as shown in figure 6.2.3. For each heading, we could identify the teacher's observed behaviours in the continuum independent/dependent from printed materials. In the 'invented' example provided in figure 6.2.3, it is possible to describe teacher A as a teacher who is not very attached to printed materials (behaviours meta-classified as 2 or 3, and none is classified higher than 4). Teacher B seems to use printed materials frequently with groups 3 and 4, although with groups 1 and 2 his/her teaching seems to be more independent from printed materials. Teacher C seems to work independently from printed materials, but for some activities the printed materials are used with some of his/her groups, etc.

An important point to be analysed was whether behaviours being considered under the headings (such as introduction of a lesson, examples, etc.) matched with the main activity developed during the lesson (meaning the set of activities the students were asked to do). Using the meta-categories above, it is clear that no answers to the 'matching' questions are going to be obtained when classifying data into this set of codes. Nevertheless, as only two possibilities can be considered: 'match' or 'not match', a simple device (*) put beside the meta-category attributed to a certain behaviour, can give simple visual access to the summarised answer to this question. As it can be expected that 'non match' cases will be less frequent than 'match' ones, the code (*) will be put beside 'not matching' behaviours.

6.3. The Meta-Categorisation.

Summarising the ideas presented at the end of the previous section, Meta-Categorisation can be defined as a set of numerical codes that orgnised the teachers' observed behaviours under each heading in a continuum stretching from 'independent of any sort of written materials' to 'dependent on a series of textbooks and teacher's guides' at the other extreme. Meta-categorisation can be applied to the whole set of answers for questions related to the first research question. Another advantage of meta-categorisation is that it allows data to be codified in a simple way, as one teacher behaviour could be summarised by a numerical code, transforming the data matrix displays described in the previous section into concise tables. Finally, meta-categorisation established a link between codes used under different headings: the same numerical code used under different headings would have a common 'degree' of independence/dependence from printed materials.

In order to include all possible situations in the meta-categorisation exercise, it was necessary to include the code @. The '@' was used to indicate that the teacher did not present the behaviour being observed. For example, if the heading considered was 'use of written material for the introduction of a lesson', '@' was used to indicate that the lesson has been introduced neither by the teacher nor by the material.

Another point to be considered is that, during the analysis to generate meta-categories, it was observed that sometimes two or even three different behaviours were obtained under the same heading and could be considered as being equally dependent on printed materials, even though these behaviours were quite different among themselves. Analysing these behaviours, it was clear that it was necessary to introduce sub-categories in order to differentiate them. The sub-categories can be described as:

sub - category (a) - Teacher's observed behaviour shows that the written material(s) is(are) being used in a way that is incompatible with the guidelines suggested by teacher's guides.

sub - category (b) - Teacher's observed behaviour shows that some decisions about the use of written material(s) are given to the students (usually choice among different options).

(ab) - is associated with a behaviour frequently observed, that will be called 'keep going'. For this research, 'keep going' is defined as the teacher's behaviour characterised by giving the material to the students with no further introduction nor illustration, and asking the students to follow the activities proposed there.

Meta-categorisation is the central tool in analysing the data connected with the first research question, and a summary of its general interpretations presented below can also be found in Appendix one: description of codes.

. Meta-Categorisation for Headings Concerning Choice of Material.

As discussed, in section 6.1, the headings in sets I and II previously described are bàsically discussing choice of materials. Meta-categories on the continuum independent from written materials to dependent from printed materials can generally be described as:

@ - Teacher does not present the behaviour being observed.

1 - Teacher's observed behaviour shows independence from any written material..

2 - Teacher's observed behaviour shows independence from printed materials.

3 - Teacher's observed behaviour shows that several printed materials are being adapted.

4 - Teacher's observed behaviour shows that printed materials from different sources are being used, without adaptations.

5 - Teacher's observed behaviour shows that the main source of printed material is being used as a resource for free adaptations.

6 - Teacher's observed behaviour shows that main source of printed material is being used as a base for instruction, with evidence of complementation.

7 - Teacher's observed behaviour shows that the main source of printed material is being used, but the teacher is not considering suggestions in the teacher's guides.

8 - Teacher's observed behaviour shows that the main source of printed materials is being used, and the teacher is considering the suggestions in the teacher's guides.

. Meta-Categorisation of the Use of Chosen Materials.

On the other hand, the headings in group III do not take choice of material into consideration, but discuss the ways chosen materials are being used. For these headings, the meta-categories generated by the continuum 'independent'/'dependent on printed materials' can be described as:

@ - Teacher does not present the behaviour being observed.

1 - Teacher does not use written materials,(even when there is a chosen one)

2 -Teacher is adding completely new features to the chosen material(s), modifying it(them).

3 - Teacher is making several modifications to the chosen material(s), without adding new features to it(them).

4 - Teacher is basing the teaching on the chosen material(s), but he/she neither refers to it(them) nor gives a copy to the students.

5 -Teacher is basing the teaching on the chosen material(s), complementing it(them) without modifying its(their) basic structure. A copy of the material(s) is given to the students.

6 -Teacher is basing the teaching on the chosen material(s),doing small modifications, which do not alter its(their) basic structure. A copy of the material(s) is given to the students.

7 -Teacher is actually following the chosen material(s) (and nothing else), without considering the suggestions in the guide(s). A copy of the material(s) is given to the students.

8 -Teacher is actually following the chosen material(s) and considering the suggestions in the guide(s). A copy of the material(s) is given to the students.

In the following section, examples of tables applying the meta-categories described above to the headings of each group are presented. For each meta-category, an interpretation and/or an example of a correspondent behaviour will be presented, in order to clarify its meaning to each particular heading. It is important to notice here that some of the behaviours described below were not observed, although they could be expected. The complete set of tables can be found in appendix three.

6.4. Examples of Tables for the Meta-Categories applied to the Headings:

This section presents some examples of the particular interpretation that can be given to meta-categories when applied to each heading. A brief reminder of the meaning of each set of headings is given, followed by an example.

To be concise, the information is presented in tables: The first column is used to show the number of the meta-category associated with the event described, the second column is used to provide an example and/or a description of the event, and the third column is used to indicate from which group of students the example was taken (this does not mean that the same behaviour was not observed in other groups of students).

The code names for the groups of students used in these tables are the same as those used throughout the analysis, and are also described in Appendix 1B.6. The format of names given to student group is: **(letter)(letter)(number)(letter)**.
The first letter represents school's code name, the second represents teacher's code name, the number represents the school year and the third letter is taken from the set: {T, M, L, X}. T is used if the group is considered a 'top-group'; M, if the group is considered as being of medium ability level, L, if the group is formed by students considered low attainers and finally X, if the group is considered mixed ability.

It is important to note that sometimes the same group of students can be used as an example for more than one event in the same table. This means that the teacher changed his/her behaviour from one lesson to another within this group. Finally, some of the behaviours described in the second column were not observed in any of the students' groups in the sample. Nevertheless, they were considered possible ones, and were expected to occur during the data collection. The complete set of tables presenting particular meta-categorisation interpretations under each heading is presented in appendix three. The interaction of these tables with the reports of the lessons can be considered the main tool in generating the meta-codified data displays, which will be used in the latter stages of data analysis related to the first research question.

Table 6.4.1 shows an example of the meta-categorisation applied to the set of headings concerning the decisions taken by the teacher throughout the whole school year. As we said in section one, headings in this set are constants within the same group of students. Therefore, one single code could be attributed to each group of students. For the headings in this group, a number close to 1 used to codify an observed behaviour means that no printed material was directly or indirectly used, a 'medium' number means that a great variety of sources of written materials is being used, and finally 7 and 8 are used to say that one single source is being predominantly used: a textbook series in most cases.

. Example of the particular interpretation of the meta-categorisation for Set I:

Meta-Category	Descriptions of Observed Behaviours Under the Heading Considered	Observed in
1	School Scheme of Work that does not determine which material should be used.	FF7L
2	School Scheme of Work based on written materials created by the teachers	
3	School Scheme of Work based on written materials adapted by the teachers from printed materials.	AA8X, AA9L
4	School Scheme of Work that determines which printed material should be used, selecting from a large range of such materials	AA10M, AA10T,CC8L, EE7X, EE10L, EE9M
5	School Scheme of Work that proposes adaptation mainly from one series of textbooks.	FF7X, HH7X
6	School Scheme of Work based on one series of textbooks, but suggesting regular complementation.	CC11T, DD10L, FF8M, GG7M, GG8T
7	Textbook Series, with modifications from the suggestions in the teacher's guides	BB10X, BB9L, BB11M, BB9T, CC8X, CC10M, DD7X, DD9M, DD9T, EE11T, FF8T, HH10L, HH9M, HH10T
8	Textbook Series and Teachers Guides	

table 6.4.1: The meta-categories applied to the choice of main source of materials.

. Example of the particular interpretation of the meta-categorisation for Set II:

The headings in this set are also related to the teacher's choice of materials, although in this case they could vary from one lesson to another within the same group of students.

Meta-Category	Descriptions of Observed Behaviours Under the Heading Considered	Observed in
1	Teacher gives no such written materials during the lesson.	AA8X, AA10M, BB9L, FF7X, FF7L, GG7M, GG8T, HH7X
2	Teacher is using own produced written materials, not based in existing printed materials.	HH9M
3	Teacher is using own produced written materials, adapted from several printed materials.	AA8X, AA10T, BB9T, DD7X, EE9M
3ab	(ab) Teacher gives the students a series of worksheets adapted from printed materials and asks them to 'keep going'.	DD7X
4	Teacher gives the students printed material to be used during the lesson.	AA9L, AA10M, AA10T,EE7X, EE10L, EE9M,
4a	(a) Teacher gives the students more than one printed material, asking them to use these materials for different activities than proposed by the material.	CC8L, FF8M
4b	(b) Teacher gives the students the choice from different printed materials.	EE9M, GG7M
4ab	(ab) Teacher gives the students the printed material, and ask them to 'keep going'.	AA10T, EE10L HH7X
5	Teacher gives the student own produced material, adapted from the textbook.	CC11T
6	Teacher gives the student the textbook, complemented with other written materials or with activities proposed by the teacher.	BB9T,FF8M, HH7X
6ab	(ab) Teacher gives the students the material, and ask them to 'keep going'.	DD10L
7	Teacher gives the student the textbook, but uses it in a different way than suggested by the guides.	EE11T, FF8T, HH10T
7ab	(ab) Teacher gives the students the textbook, and ask them to 'keep going'.	BB10X, BB9L, BB11M, CC8X, CC10M, DD7X, DD9M, DD9T, EE11T, GG7M, GG8T, HH10L
8	Teacher gives the student the textbook, and follows the suggestions of use in the guides.	GG7M, GG8T,

table 6.4.2: The meta-categories applied to the choice of materials as source of activities

. Example of the particular interpretation of the meta-categorisation for Set III:

These headings are describing how the previously chosen materials are being used by teachers on a day-by-day basis. In this group, a 'small' number used to codify an observed behaviour means that the teacher is doing several modifications and adding personal comments to the chosen material, while a 'large' number is used to say that the chosen material is being used in close adherence. Table 6.4.3 provides an example of the particular meaning of the meta-codes under the heading 'Use of the Chosen Materials to Introduce a New Topic'.

Meta-Category	Descriptions of Observed Behaviours Under the Heading Considered	Observed in
@	Neither the teacher nor the material introduce a new topic. Students are supposed to work through the exercises straight away.	EE10L
1	Teacher does not use written materials to introduce a topic, doing it in his/her own way.	AA8X, AA10M, BB9L, FF7L, GG7M, GG8T
2	Teacher introduces a new topic adding new aspects to the introduction presented in the material.	BB9T, FF8T, HH7X
3	Teacher introduces a new topic by selecting from the material, modifying its aims.	EE9M, FF7X, FF8M
4	Teacher introduces a new topic based on written materials, but does not give a copy of these materials to the students, doing the introduction him(her)self.	AA9L, EE7X, EE9M
5	Teacher introduces a new topic based on the material, with clear evidence of small complementation.	BB10X, BB9T, GG7M, HH9M, HH10T
6	Teacher introduces a new topic based on the material, with evidence of small modifications.	AA10T
7	Teacher introduces a new topic essentially in the same way done in the material.	FF8T
7ab	(ab) Teacher does not highlight the introduction of a new topic and the students are supposed to read the introduction in the material by themselves.	BB9L, BB11M, CC8X, CC8L, CC10M, CC11T, DD7X, DD10L, DD9M, DD9T, EE11T, HH10L
8	Teacher introduces a new topic close to the material and following the guidelines of the guide.	DD7X, GG7M

table 6.4.3: The meta-categories applied to use of the chosen material to introduce a new topic.

In order to validate the meta-categorisation, an experiment was made with a fellow mathematics educator. After discussing the meanings of meta-categories for each set of headings, exemplified in the tables above, the reports of the lessons from one students' group were given to her. She was asked to codify the behaviours observed under the headings. Without any checking, agreement regarding the use of codes was 82 percent (with 70 percent of disagreement being in 'neighbours' codes). After discussion, this agreement grew to 95 percent, with the researcher feeling the need to change the categorisation once.

Using the tools developed in this chapter, the data collected are analysed. The study of the case of each teacher, aand the comparison of behaviours within different student's groups will be the object of chapter 8.

6.5: Example of Analysis of One Group of Students.

This section is dedicated to the analysis of teacher behaviour with one group of students. This analysis constitutes an example of how the methodology described in this chapter was used to codify observed behaviours. The 'mixed ability' year eight group (AA8X) taught by teacher 'A' is analysed in detail for the second and third headings. The table produced here, summarising the analysis is also presented in Appendix 4,along with others. Appendix 4 presents the complete summary of data for the first research question.

Table 6.5.1 shows the summarised data used when analysing the headings in sets II and III, which are related respectively with day-by-day decisions and usage of materials by the teacher. Group AA8X was one of the groups that had four (instead of three) lessons observed. The reason for this extra lesson was to give the researcher the opportunity to observe the complete development of a topic in the classroom.

The first lesson observed was the last of a series of three based on a series of worksheets. Although the department had produced these worksheets, they were strongly based on a series of textbooks. As the worksheets did not contain examples, the teacher had to provide all necessary support during the lesson. This support was provided individually by the teacher, by reviewing the previous exercises done by the students (or examples given in previous lessons) and reminding them what they had been doing. As the group was a 'mixed ability' one, some students were in need of extra support. For example, all students, except two had mastered during the previous lessons how to use isometric paper to draw three dimensional shapes, while the remaining two still needed the support of the teacher and some plastic cubes. On the other hand, the remaining lessons (second, third, and fourth) observed were completely independent from any source of written materials. The teacher used his own knowledge of the subject to give the lessons. He prepared all the activities developed during the lesson, all the examples, guidance for students' notes in their notebooks and even a small piece of assessment in the form of a mental test, applying the definitions developed during the lessons. To complete this picture, it is worth saying that teacher A seldom 'gave' examples in classroom. Most of his explanations and examples took the form of questions to the students, with the objective of establishing links between new exercises and the previous work.

In principle, all the activities were proposed for all students. Nevertheless, some differentiation has occurred during the first lesson, because some students were able to go further than others in the worksheet series. During the remaining lessons, the teacher treated the class as a whole, and no differentiation occurred.

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of the evidence from the report of the lessons**
Source of Class Activities.	3ab 1 1 1	7ab 1 1 1	**L1** - students are asked to 'keep going' with the exercises they were doing in the previous lessons, using a set of adapted worksheets (strongly based on 'Journey into Maths' - a textbook series) **L2, L3** and **L4** - all the exercises proposed by the teacher. No written material used. **L4** - mental test designed by the teacher given to the students, matched with the other activities developed in Lessons 2, 3 and 4.
Introduce a new topic.	- 1 - -	- 1 - -	**L2** - Teacher introduced the topic by writing its title on the board and asking questions about 'factors'. **L1, L3** and **L4** - No topics were introduced.
Introduce a lesson.	3ab 1 1 1	7ab 1 1 1	**L1**- the teacher asked the students to continue the work from the previous lesson. No other introduction was offered. **L2** - see above - new topic. **L3** and **L4** - the teacher introduced these lessons by proposing own designed activities for the whole group.
Reference During the Lesson	1 1 1 1	5 1 1 1	**L1** - worksheets used have neither explanations nor examples. Teacher gives examples by solving some exercises (or helping the students individually to do so). **L2, L3** and **L4** - students have written in their own words the definitions in their notebooks, following instructions given by the teacher. These definitions were not yet reviewed by the teacher.
Exemplification.	3 1 @ @	5 1 @ @	**L1** - Examples given individually by solving proposed exercises, giving concrete materials (cubes) or recalling previous exercises. **L2**- teacher wrote down examples on the blackboard. **L3** and **L4** - no examples given.
Conclude a topic.	@ - - 1	@ - - 1	**L1** - the topic was not concluded by the teacher or by the material. **L2** and **L3** - no conclusion of topic. **L4** - conclusion made by the teacher, using exercises and poster activity.
Promote links between related topics.	1 1 1 1	2 1 1 1	**L1** -The series of worksheets provided no links. Teacher used students' previous work and support from plastic cubes in order to create links between the different sheets. **L2, L3** and **L4** - Some links were provided by the teacher, who did not use materials to promote them.
Promote differentiation within class.	3ab @ @ @	7ab @ @ @	**L1** - teacher is using a sequenced series of worksheets. At least 30 % of the students did not finish the last one. **L2, L3** and **L4** - no differentiation noticed.
Remedial Material.	1 - - -	1 - - -	**L1** - teacher used previous worksheets exercises and concrete material (cubes) to help students. **L2, L3** and **L4** - teacher followed his plans for the lessons and no students ask for extra support.
Source of Homework.	3(*) @ @ @	8(*) @ @ @	**L1** - worksheet given as homework produced by the school, based on printed material. No connection with classroom work. **L2, L3** and **L4** - no homework given.
Provide Feedback on Exercises.	1 1 1 1	1 1 1 1	**L1** - feedback given by the teacher by using concrete materials (cubes) and answers to the previous exercises. No written materials used. **L2, L3** and **L4** - feedback given by discussing orally the students' answers, with the whole group at the same time.

table 6.5.1: Group AA8X - analysis of the second and third set of headings over four lessons.
In the table above, L1 stands for the first lesson, L2 for the second, and so on.

No homework was given during three of the lessons observed, and homework given during the first one had no connection with classroom activities developed in this lesson nor with the next series of lessons. During the first lesson, the teacher used exercises from previous worksheets as examples to help students facing difficulties. In this sense, some links between the worksheets were constructed, as they were not provided by the worksheets themselves. The teacher did not present a conclusion to the topic, so different students finished at completely different points. As for the remaining lessons, some links were emphasised, and others not. For example: during the second lesson, the teacher did not discuss the links between ideas such as multiple and factors, but did establish the links between prime numbers and factors.

In this series of lessons, the teacher has concluded the topic not only by giving the students a mental test during the last lesson, applying the definitions and concepts developed during the previous ones but also by asking them to develop a poster to hang in the classroom and be used as reminder of the definitions presented and their applications.

CHAPTER 7

THE METHODOLOGY FOR ANALYSIS OF THE DATA RELATED TO THE SECOND RESEARCH QUESTION

> 'the fundamental task of conducting classroom research is to generate and test plausible theory concerning teaching. When relatively primitive, such theories consist of terms that represent classes of observable events and propositions that summarize observed relationships among them.' (Good and Biddle, 1988, pg. 137)

The aim of this chapter is to present a description of the methodology used to analyse the data related with the second research question: *Do particular identified ways teachers use written materials in their classroom work relate to better achievement by the pupils of teacher's short term aims?*

In chapter five it was argued that, due to the nature of the student sample, it was neither possible to use the same test for all the students nor to test each group for the teacher's long term aims. It was decided that a test would be given to the students just after the end of the week of observed lessons. The test would be developed to measure the *achievement of teacher's short term aims for the observed week*, with emphasis on those which were based on written materials.

As already argued, each test was previously discussed with the teacher and it was only applied to the group after the researcher and the teacher had agreed that it was measuring what had been observed and intended. Although high marks on these tests do not necessarily mean that good teaching was achieved, poor performance at least shows that teacher's short term aims were not achieved. It has also to be considered that the achievement of teacher's long term aims are made upof short term aims building on each other.

Testing students' outcomes has been one of the major problems in research on teaching, which seems to remain unsolved, as argued in chapter three (see section 3.5). In order to develop the analysis of the test results, it was considered that: (1) the tests were different from each other (2) the final objective of this analysis was to link students' achievement of their teacher's aims and the decisions on written materials made by the teacher; and (3) finally, the analysis had to take into consideration that this set of tests (as any other) is an incomplete measure.

The first research question categorised the teachers according to the different ways they use written materials for classroom work. As for the second research question, it was expected that the strategies concerned with written materials adopted by the teachers in the sample for different groups of students would vary. About 32 (= 8 teachers × 4 groups each) different observed situations would provide a sample of different ways written materials were used for classroom work by experienced teachers. In fact, there were about 40 different observed situations, due to the fact that some teachers used more than one strategy within the

same group of student during the observed lesons, while others used different strategies with different sub-groups of the same group of students.

In order to verify whether there were links among these strategies and the results of the tests, it was necessary to group these strategies according to similar decisions on written materials made by the teachers in the sample. The way the strategies were grouped was a research decision based on the analysis of data collected not only during the observed lessons but also during the informal interviews with the teachers. The interviews played an important role in complementing the necessary information to describe these strategies, as teachers were asked questions about the choice they made.

In this chapter, the methodology for the analysis is presented. In section 7.1 the methodology used to analyse the results of the tests applied to each group of students is described. Section 7.2 introduces the methodology used to complement the analysis of the strategies on choice and use of written materials, mainly based on the works of Fennema et al. (1989 b) and Askew et al. (1997), as well as the methodology of confronting teachers' identified strategies and students' outcomes.

7.1. Methodology of Analysis of the Test Results

This paragraph describes the initial steps in recording and analysing the results of the tests. Some of the tests applied are presented in Appendix 2,C. During the test, the students were asked to repeat some of the activities they had developed during the observed lessons, and these were taken from the material used (in the cases where no material was used, the tests were designed to conform as closely as possible with the guidance offered by the teachers during the lessons).

The first step in the analysis was to prepare a report for each test. Even if the tests were to be used only as indicators of performance, it was important that the first register of the results of the tests was made as complete as possible. These reports contain a description of the questions, the criteria used to mark the questions, and the table containing the marks given for the test, as schematically represented in figure 7.1.1. An example of a complete report can be found in Appendix 2d, and the tables for all groups can be found in Appendix 5.

Number of the Question→ / Student's Number	1a*	1b*	2	...	La**	Lb**	% grade
1	1	1	0	...	0		(total of '1'
2	1	0		...		1	x 100)/M
...	...	...	...	...	...	...	in each
N	0	1	1	...	1	1	row
Number of Students who answered correctly	total number of '1' in each column						
Number of Students who tried the question	total number of '1' + '0' in each column						
% correct →	(total correct x 100)/N in each column						
% try →	(total try x 100)/N in each column						
National Curriculum Levels →	level of the question in the N.C.						
N.C. Attainment Targets →	Attainment Target (N.C.) of each question						

in the table above, **N** is the total number of students who performed the test, **L** is the number of the final question (La and Lb are its items) and **M** is the total of items in the test.
Key: (1) correct answer, (0) wrong answer and ( ) question not tried
(*) initial questions of a topic (**) final questions of a topic
figure 7.1.1: Schematic example of the table of test results for one group of students.

The special features in figure 7.1.1 were introduced in order to allow the results of the tests to be presented in a common form. They can be described as:

- Marks were put next to the number of the questions: (*) if the question was considered as starting points of one activity, and (**) for final questions.
- Six rows were added to the bottom of the table: the first with the total number of students who received full marks on the item, the second with the total number of students that tried the item, the third with the percentage number of students who had correct answers on the item and the fourth with the percentage number of students who tried the item. The

fifth row stated the level of the item following the National Curriculum (1991 version) and finally the sixth row stated to which Attainment Target of the N. C. the item belonged.

- One column was added to the right of the table presenting the percentage grade of each student in the test. If two different strategies in choice/use of written materials were observed within the same group of students, two extra columns were added, each one of them presenting the percentage grade of each student in the correspondent questions of the test. Whenever a group was divided into sub-groups by the teacher, the results were presented in different tables.
- In almost all cases, these percentage grades were obtained by the usual formula: percentage (%) grade is (100 x number of correct items)/total number of items. In those few cases where this formula was not used, an explanatory note was given, together with the formula used for the case considered.
- Two codes were used for wrong questions: wrong item (0) and a blank ( ) item. This feature was introduced to differentiate between those items which the students felt confident to try and those the students decided not to try.
- The marks beside the initial and final questions were introduced to verify whether the performance of the students varies from one of these sets of questions to the other. It was expected that the performance of the students in each set of questions could be eventually used to improve the understanding of their overall result.
- The rows concerning group results were introduced to allow an overview of the group in each one of the items on the test. Percentages were used to allow a common register of the results from different groups. These rows consider not only the total number of students who obtained a correct answer to the item, but also the total number of students who tried the item.

Some situations occurred during the observations that led the researcher to feel the need to include some kind of measurement of the level of the work developed during the observed lesson (for example, one teacher, working with a year nine 'low attainer' group was asking them to perform, during an investigation, tasks starting from levels 3/4 in the N.C. and going up to levels 5/6, while another teacher, also working with a year nine 'low attainer' group was asking then to solve money problems that can be considered as levels 2/3 in the N.C.). It can be expected that decisions about the levels of mathematical content made by the teachers would have an influence on the test results.

On the other hand, it was never the aim of this research to discuss the teacher's aims, or to evaluate if the 'contents' of one school were, in any sense, 'better' than the 'contents' of another. Nevertheless, this research analyses the achievement of each teacher's aims with his/her students, and this achievement is likely to be influenced by the levels of the questions. Thus, these differences should be registered here, and the N.C. levels were a natural choice,

specially since this is close to the way teachers in the sample did it themselves. The 1991 version was chosen because it was the one used in the review of literature and the one that has been used in this research since its beginning. It is important to stress once more that the levels of the questions asked in the tests reflected the level of the questions asked by the teacher during the observed lessons.

It is also important to register that all teachers declared that new mathematics content was being developed during the observed week with all groups of students. In fact, it was clear from the observed lessons that the students were not acquainted with the material used (whenever that was the case) or were discussing a new content with the teacher (if it was the case that no material was used). The tables with the results of the tests used in this chapter and in chapter nine are in appendix five.

An Overview of the Results of the Tests.

As the tests were to be used only as indicators of performance, it was necessary to have an overview of the results of the students tested, in order to verify whether it was possible to define levels of performance based on group achievement of teacher's short term aims. As it was decided that they should reflect the results of the present sample, these levels were not previously determined.

In 27 out of the 30 groups of the sample, all students completed the test . The other three groups (CC8X, CC11T and DD9M) had students selected for the test, and in those few cases, the researcher and the teachers agreed that the sample of students was representative of the diversified work developed by the children during the observed lessons. In this sense, the percentage of students who were able to perform in the test within certain levels can bring some light on students' achievement of teacher's short term aims.

Type of	Teacher→	A	B	C	D	E	F	G	H
Group ↓	Range↓	Percentage of students within considered ranges of performance:							
X	50% or more	Part 1: 64 Part 2: 77	Reds: 88 Yells: 92	75	Part 1: 83 Part 2: 78	Part 1: 94 Part 2: 90	84	-	Part 1: 97 Part 2: 66
(mixed)	70% or more	Part 1: 52 Part 2: 32	Reds: 35 Yells: 50	42	Part 1: 65 Part 2: 48	Part 1: 35 Part 2: 90	37	-	Part 1: 55 Part 2: 28
L	50% or more	55	Part 1: 100 Part 2: 55	Part 1: 93 Part 2: 74	83	74	100	-	78
(low)	70% or more	22	Part 1: 55 Part 2: 18	Part 1: 47 Part 2: 47	25	42	100	-	56
M	50% or more	33	Blue: 44 Red: 100	100	84	79	60	Text: 62 / 46 Logo: 66 / 83	74
medium	70% or more	0	Blue: 0 Red: 75	89	42	39	20	Text: 0 / 31 Logo: 0 / 83	37
T	50% or more	17	89	100	93	96	57	100	96
(top)	70% or more	3	86	92	83	52	30	100	81

Table 7.1.1: Percentage of students in each group within selected ranges of performance.

In order to decide how to classify the groups according to the tests' results, table 7.1.1. was constructed. In this table, the percentage of students in each group that achieved more than 70% of the test is presented (this result was usually considered by the teachers in the sample as highly satisfactory in immediate post-tests) as well as the percentage of students in each group that achieved more than 50% of the test. Whenever more than one percentage is presented in the same cell of the table for the same group, either the group was split into sub-groups by the teacher or the test had two parts, each one related to a different strategy in choice/use of materials. Group GG7M had each sub-group tested in two different strategies of use of materials.

When looking at these percentages as a whole, four levels of performance could be seen, with clear 'gaps' between any two of them. These levels allow display of the results of the tests in more concise tables. They are, in decreasing order of achievement:

- **L1:** At least three fourths of the students (75%) were able to achieve a 70% grade or more in the test (in all those cases, at least 83% of the students tested were able to correctly complete more than half of the test items).
- **L2:** At least three fourths (74% or more) of the students were able to correctly complete more than half of the test items (excluding those groups who achieved L1 level)
- **L3:** At least half (55% up to 66%) of the students were able to correctly complete more than half of the test items.
- **L4:** Less than half (46% or less) of the students were able to correctly complete more than half of the test items.

Table 7.1.2 presents the distribution into the four levels described above, of the results associated with all different strategies in choice/use of materials observed in the sample. As it was expected that different strategies would be used for different 'types' of groups, the results of the tests are distributed in columns using this criteria. To be concise, the group was identified only by the teacher's code.

'type' of group → level of result ↓	**mixed (X)**	**low (L)**	**medium (M)**	**top (T)**
L1	E (2)	F	B(r), C, G(L2)	B, C, D, G, H
L2	A(2), B(r), B(y), C, D(1, 2), E(1), F, H(1)	B(1); C(1, 2), D, E, H	D, E, H	E
L3	A(1); H(2)	A, B(2)	F, G(t1), G(L1)	F
L4	-	-	A, B(b), G(t2)	A

table 7.1.2: Levels of Achievement of the Results of the Tests, distributed by 'Types' of Groups

7.2. Methodology of Analysis for the Second Research Question

The methodology of analysis for the data related to the second research question took into consideration the model proposed by Fennema et al. (1989 b), already presented in chapter three (see figure 3.4.1). In this model, teachers' knowledge and beliefs, as well as their knowledge about their students' behaviour, influenced their decisions. On a second level, these decisions play a central role in classroom instruction. The model also considered students' outcomes as the final product of classroom interactions. Although these outcomes were related to classroom instruction, there were also several other factors that could affect them. Taking this model into account, there was no sense in proposing a methodology of analysis for this research that would establish cause-effect relationship between teachers' decisions on written materials and students' outcomes in an immediate post-test, especially due to the constraints discussed in chapter five.

The research decision of choosing experienced teachers for the sample was made in order to consider the variable 'teacher's knowledge' under control. This means to say that the present research made no attempt to measure teachers' mathematical or pedagogical knowledge. It was hypothesised that all teachers in the sample were confident when teaching secondary school mathematics to the observed groups of students. This hypothesis was confirmed by data collected from classroom observation and interviews: all teachers in the sample were generally confident with the content and the methodology adopted. They also declared (and demonstrated during the observed lessons) that they know well the students they taught.

On the other hand, it was hypothesised that teachers in the sample would be dealing with real constraints related to school policies and the implementation of the National Curriculum (Millett and Johnson, 1996). Advice given to teachers was also considered as a possible influence. As Thompson (1992) argued, teachers' knowledge and experiences influence teachers' beliefs, so the factors mentioned above would probably influence teachers' decisions about how to use written materials.

Questions about their personal system of beliefs were not directly put by the researcher to the teachers in the sample. On the other hand, each one of them, at some point, declared his/her decisions on how written materials should be used for classwork by different groups of students. These statements were usually made during the informal interviews after the lessons, when teachers were questioned about a particular choice or usage of written materials, so they also reflected their classroom practice. These decisions were clearly influencing teachers' pratices, although, during the interviews, teachers also demonstrated that there were other features relevant to their decisions. The main ones seemed to be their previous knowledge and experience with the chosen materials and the constraints imposed by the school or by implementation of the National Curriculum. The data from the interviews

were important throughout the analysis of both research questions, and a summary of these data can be found at the introduction of each section in the next chapter, and in Appendix 4.

Another aspect considered in the analysis was teacher's expectations of each group of students. Although they should be included in the set of teacher's relevant beliefs (Good ad Brophy, 1997), they are not necessarily related to written materials, so they had to be placed separately. As the objective of the second research question was to verify if there were links between the identified strategies on use of written materials and students' outcomes, the influences suggested above had to be taken into account when describing these decisions. The adapted model for this work, which considered only experienced teachers, is presented in figure 7.2.1.

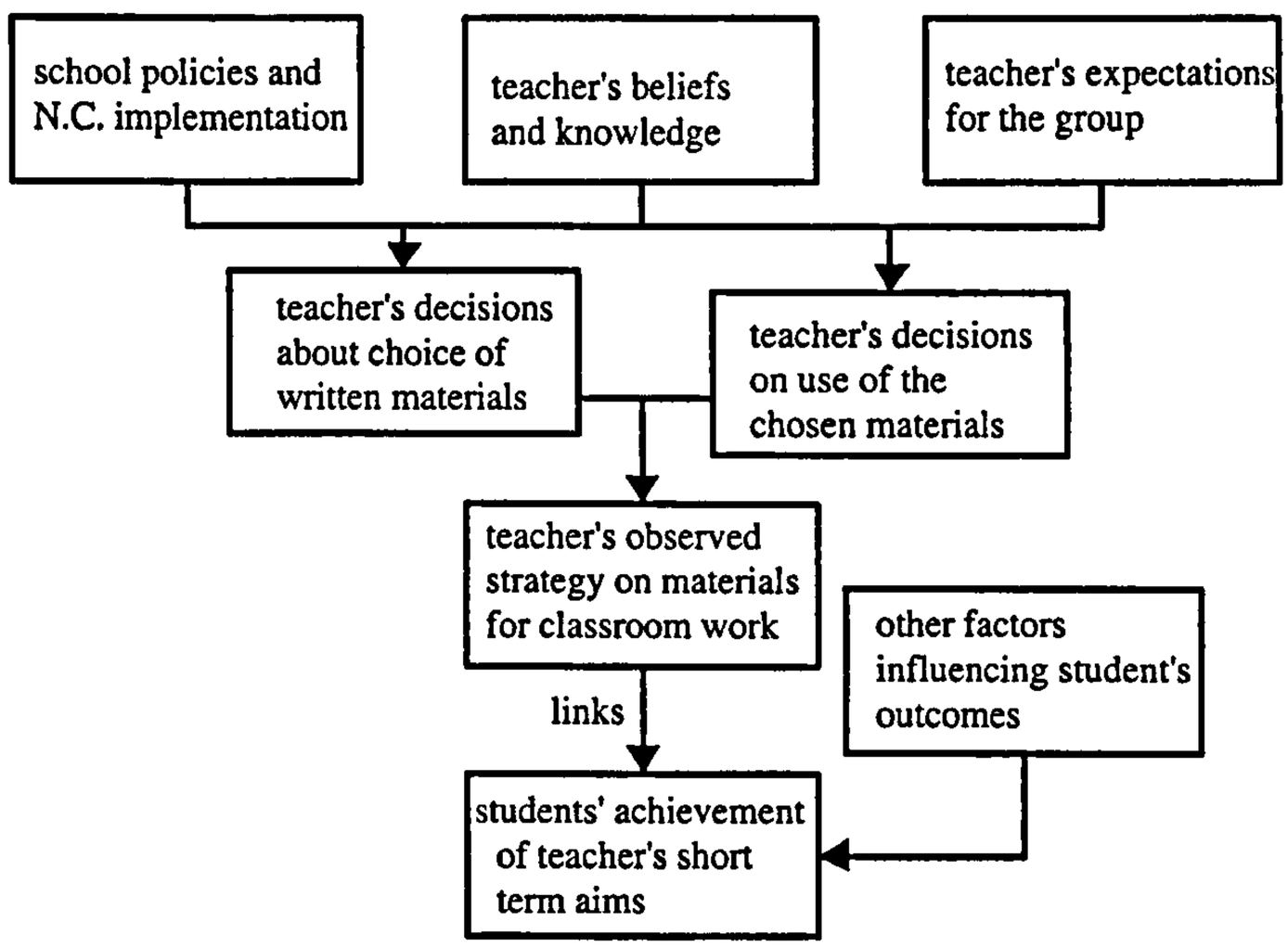


Figure 7.2.1 Model for the analysis of the links between experienced mathematics teachers' decisions on written materials and students' achievement .

The influences of school policies on written materials and the description of strategies in choice and use of written materials were the object of the study described in chapter eight. The data were already codified, using the methodology described in chapter six, and the same set of codes could be used once more to analyse the links of the observed strategies with students' outcomes. So, at this point of the analysis, it was necessary to construct a framework in which the sets of teachers' expectations and decisions related to use of written materials could be analysed and classified.

Teachers' Decisions about Written Materials

The practical way Askew et al. (1997) analysed teachers' belief orientations were used as a starting point. They compared the beliefs of the teachers in their sample with three basic models: discovery oriented, transmission oriented and connection oriented. The definitions of

these models were summarised in chapter three (see also Askew et al., 1997, pg. 2-27). As these models were built for primary teachers when teaching numeracy, it was not possible to apply the same model to the teachers in the present sample, as far as use of written materials for classroom work was concerned. Also, the data collected during the interviews and observed lessons did not allow the researcher to draw conclusions about teachers' beliefs, but only about teachers' decisions. So, it was necessary to define new models to classify these declared decisions about written materials made by the teachers.

It is significant to stress once more that this set of categories for classifying teachers' decisions on written materials were based not only on teachers' interviews, but also on what the teachers actually did during the observed lessons. Note also that, as far as written materials were concerned, it was a possibility that the same teacher declared different decisions for different groups of students, for different sub-groups in the same group or even for different sets of materials being used within the same group of students.

The first aspect considered in defining teachers' decisions about written materials concerns choice: whether or not the teacher considered it important that the written material chosen included mathematical statements besides the proposed activities to be developed by the students. This feature was emphasised in the literature as one of the main advantages of adopting a textbook series (e.g. Love and Pimm, 1996). Those teachers who were only concerned with whether the material could provide suitable classroom work material, without regard for the mathematical connections, were considered as *worksheet oriented* (**W**), while the others were considered as *textbook oriented* (**T**).

Once the orientation in choice was decided, two other aspects concerning use of the chosen material were taken into account when defining teachers' sets of decisions: (1) whether or not it was important to give the same material to the whole group, and (2) whether or not it was important that the students completed the proposed task before starting another mathematical activity.

Based on these approaches, the main orientations in teacher's decisions presented by the teachers in the sample for their different groups are summarised in table 7.2.1. As far as use of chosen materials are concerned, they can be described as:

SaCo: The same material is given to the whole group, and all students should finish the proposed task before starting a new one.

SaIn: The same material is given to the whole group and each student works through it individually (or in small groups), so they can finish the task at different points.

DiCo: Different materials are given to different students and each one of them is supposed to work through the material at his/her own pace. A student changes tasks only if he/she has already finished the previous one.

DiIn: Different materials are given to different students, each one of them is supposed to work through the material at his/her own pace. A student may change tasks with no regard as to whether the previous one is already finished.

	Worksheet oriented		Textbook oriented	
	all students completed the task (Co)	not all students completed the task (In)	all students completed the task (Co)	not all students completed the task (In)
same material for all students(Sa)	**WSaCo**	**WSaIn**	**TSaCo**	**TSaIn**
different material for different students(Di)	**WDiCo**	**WDiIn**	**TDiCo**	**TDiIn**

table 7.2.1. Teacher's decision orientations about written materials

It is interesting to note that, for the teachers in the sample, these orientations appeared to have strong connections with teachers' observed roles during the lessons:

- All of the teachers who presented decision orientations **WSaCo** or **TSaCo** towards a certain group of students also demonstrated during the correspondent lessons that they acted primarily as 'active instructors' with these groups. They were in charge of promoting whole group discussions, and were generally concerned with emphasising the main mathematical ideas and connections. Class teaching was frequently observed during these lessons.
- On the other hand, all of the teachers who demonstrated during the observed lessons that they acted primarily as facilitators, helping the students to carry on with the proposed tasks, presented decision orientations **WSaIn** or **TSaIn**. Within these groups, these teachers usually introduced the task for the whole group, and from this moment on, allowed them to work at their own pace. Appart from the introduction of the task, these teachers never promoted group discussions and were not concerned whether the students shared their results or conclusions.
- Finally, those teachers in the sample who demonstrated during the observed lessons that they acted primarily as managers, being in charge of verifying students' progress through the given materials were predominantly associated with decision orientations **TDiCo** or **TDiIn** (there was no case of **WDiCo**, nor **WDiIn**). Within these groups. the teachers never promoted group discussions and were not concerned whether the students shared their results or conclusions. These teachers usually tested students in these groups on an individual basis. The only exception to this pattern was teacher C, who kept the students working through the same tasks (even if on an individual basis), managing this situation by proposing regular homework and, periodically, common assessment tasks.

Teacher's Expectations on Student's Performance

Good and Brophy (1997) emphasised teachers' performance expectations as another important factor that could affect students' performance. During the interviews, the teachers in the sample made statements about the observed groups of students that showed their expectations. For the same reasons discussed above, it was possible that the same teacher expressed different expectations for different groups of students, or even for sub-groups of the same group. From the analysis of the interviews, there were four main orientations for teachers' performance expectations for a group of students:

(H) - High: The teacher showed his/her belief that all (or almost all) students in the group were able to achieve the teacher's aims, and these aims should be somehow ambitious. These expectations were expressed by the teachers in different ways, specially when different ability level groups were considered. Some examples extracted from the interviews: 'I give my students common tests... those students who are more 'slow' during the lessons would have to make some extra effort at home... and they do... with... good results' (teacher C); 'Once the students are comfortable with the textbook track they are using they usually perform well, and can be stretched a bit' (teacher C); 'these students benefit from a more structured and organised way of work...' (teacher E); 'if we carefully help them ... building their confidence, these students can perform better' (teacher F). There were also situations when the teacher demonstrated high expectations for a sub-group of the group considered: 'Some students in this group could follow a higher track of the book' (teacher B, who actually divided the group into sub-groups).

(R) - Regular: The teacher showed his/her belief that all (or almost all) students in the group were able to achieve the teacher's aims, whenever these aims were not too ambitious. Some examples: 'they can perform these tasks well ... asking them to go further could affect their confidence' (teacher C); 'we sometimes use SMP suggested assessments, and the students usually perform well in them' (teacher D); 'these students are comfortable using the book...but sometimes I do not stretch them as much as I should' (teacher F)

(L) - Low: The teacher showed his/her belief that the group of students as a whole usually had problems in achieving teacher's aims, even if these aims were not too ambitious. Some examples: 'These students need special attention, and ... [sometimes] it is difficult to give this kind of attention to them.' (teacher B); 'we cannot expect these students to grasp general mathematics concepts ... we give them tasks based on everyday maths...' (teacher B), 'they must feel comfortable with their task' (teacher B, about the same group of students); 'these students cannot concentrate for long periods on the same task...' (teacher D); 'we keep these students working at the best of their ability' (teacher D, about the same group of students); 'it is difficult to ask them to work two consecutive lessons using the same material' (teacher E); '[one topic in the school scheme] can have different interpretations for different groups of students... for my 'bottom group' ...I will keep it simple' (teacher E); 'we give these students

the chance to work at their own pace...' (teacher H). There were also occasions when the teacher divided the group of students into sub-groups, and demonstrated a low 'sub-group' performance expectation: 'I sometimes wonder if they would not be better using the green book [a lower track]' (teacher B).

(I) - Individualised: The teacher showed his/her belief that some students in the group were able to go further than others, when achievement of teacher's aims was considered. Some examples: 'The activities are the same for all students. The difference is the outcome' (teacher A); 'We have some specially bright students... and I do believe they can go for an A^+ grade' (teacher C); 'extension material is always offered for the brighter students' (teacher E); 'the whole group starts at the same point ... different is the final point: some students can go further than others... ' (teacher G).

Although in the present analysis the set of teachers' expectations on students' performance was placed separately from their decisions on choice/use of materials, the influence of those expectations on teachers' decisions about written materials cannot be denied. Just to give one example: all teachers who frequently changed materials for a group of students declared that those students could not concentrate for a long period of time using the same material, and this decision was always associated with low expectations about students' performance.

Notice also that such expressions as 'feel comfortable with their task' or 'working at the best of their ability', used by the teachers associated with different levels of expectations, were never associated with high expectations. In fact, all teachers who showed high performance expectations for a determined group (or sub-group) also demonstrated their belief that it was part of their role to encourage these students to go further. Expressions such as 'stretch', 'challenge' or 'build their confidence' were used, depending on the ability level of the group. These attitudes seemed similar to the different ways 'effective junior high U.S.' teachers motivated students from different social backgrounds, as described by Brophy and Good (1983, pg. 342)

Confronting teachers' decisions with the results of the tests.

Once the framework for analysis of teachers' decisions about written materials and teachers' expectations was built, the next step in the analysis was to build a case matrix display where the main features of an observed strategy could be summarised. The data display should also present some information about the level of the questions of the tests and the level of achievement of each group, as defined in section 7.1. As the analysis of data concerning the first research question had already shown that the 'type' of group being taught was quite influential in teachers' decisions, the analysis of the strategies developed for the second research question was first divided according to this criterion.

So, for each 'type' of group considered (mixed ability, top, medium and low groups), a case matrix (Miles and Huberman, 1994) was developed, in which a summary of the data presented for different observations on use/choice of materials (case matrices described in chapter six and eight) was integrated with the results of the analysis of teachers' beliefs and expectations and with the levels of performance of the students, presented in this chapter.

mixed- ability	**B10**		**C8**	**E7**		**F7**
feature →	Red	Yellow	Samp.	Part 1	Part2	-
Decision Orientation	**TDiCo**	**TDiCo**	**TDiCo**	**WSaln**	**WSaln**	**WSaln**
Expectations	**I**	**H**	**I**	**I**	**H**	**R**
Choice of Materials						
Introduction	7ab	7ab	7ab	1	4	5
Classwork	7ab	7ab	7ab	1	4	1
Conclusion	7ab	7ab	7ab	@	@	@
Differentiation	7ab	7ab	7ab	@	@	@
Mathematical Links	7ab	7ab	7ab	1	1	1
Use of Materials						
Introduction	7ab	7ab	7ab	1	4	3
Classwork	7ab	7ab	7ab	1	4 / 7	1 / 2
Conclusion	7ab	7ab	7ab	@	@	@
Differentiation	7ab	7ab	7ab	@	@	@
Mathematical Links	7ab	7ab	7ab	1	1	2
Levels of Questions	**6 / 8**	**7 / 8**	**4 / 5**	**4 / 5**	**4**	**5**
Results of the tests - Percentage of students in considered ranges of performance						
50 % or more of test items correct	**88**	**92**	**75**	**94**	**90**	**84**
70% or more of test items correct	**35**	**50**	**42**	**35**	**90**	**37**
Test Performance	**L2**	**L2**	**L2**	**L2**	**L1**	**L2**

Figure 7.2.2 . Example of case matrix for the second research question: Extract from the Case Matrix for Mixed Ability Groups

An example is presented in figure 7.2.2, containing part of the data for the mixed ability groups. The complete set of data is presented and analised in chapter nine. At this moment, the interest is in describing the features of the data display. As the groups were all of the same 'type', their identification was made (first row) by the letter correspondent to the teacher and the number correspondent to the year group. If the group was observed while the teacher adopted two different strategies, or if the group was divided by the teacher into sub-groups, or additionally if the group was tested by sampling, these data were registered in the second row (particular features of the group). The third and fourth rows address respectively teachers' decision orientations towards written materials and teachers' performance expectations, registered according to the analysis presented earlier in this section.

Use and choice of materials were the issues considered in the next two sets of rows. The codes used were defined in chapter six, and the data are summarised from the case matrices presented in chapter eight. The rows in these sets summarised the main aspects of

use and choice of materials during different segments of the lessons, according to the review of previous research on teaching.

The last two sets of rows were dedicated to the test. The row 'levels of questions' was included in order to inform about the levels of questions that were actually developed by the students during the observed lessons. As discussed before, the levels of national Curriculum (1991 version) were used in this codification. The final set of rows presented the percentage number of students who performed within determined ranges, and the last row presented the levels of performance as defined in section 7.1 of this chapter.

The actual case matrices, presented in chapter nine, were used as the basis for the final analysis of data related to the second research question, aiming at establishing links between observed strategies and students' performance. In chapter nine, they were also used to generate the conceptual matrices, in which the data is reorganised according to the decision orientations.

Constraints of the Methodology.

It was argued in chapters three and five that researchers have been considering verification of students' outcomes as a major constraint in research on teaching, specially because they can be influenced by several variables (Good and Brophy, 1997; Koehler and Grouws, 1992; Romberg and Carpenter, 1983; Fennema et al. 1989a). On the other hand, the use of classroom observation as methodology for data collection implied a small sample of teachers, so it cannot be expected to be representative (Good and Biddle, 1988).

It also has to be considered that students were tested based on the observed lessons, so no comparison among groups could be made. It was also not possible to draw conclusions on students' overall mathematical knowledge, as they were tested on teachers' short term aims. Taking these considerations into account, the results of the tests could only be used as indicators of students' outcomes related to a particular decision on written materials made by their teachers. As Brophy and Good (1983) emphasised, these sorts of results can only be considered as links, no cause-effect conclusions can be drawn.

Nevertheless, as argued by several researchers (Kilpatrick, 1977; Romberg and Carpenter, 1983; Good and Biddle, 1988), research based on qualitative data related to classroom observation is necessary in order to illuminate the actual practices being developed, and students outcomes are 'the ultimate dependent variable' (Romberg and Carpenter, 1983). As Thompson (1992) argued, research related to teachers' decisions can have implications for teacher educators and staff developers, as they bring to light 'how teachers learn from their experiences' (pg. 143).

CHAPTER 8

CATEGORISING THE WAYS TEACHERS USE WRITTEN MATERIALS

> ...Studies concerning the *what* and *why* of mathematics are quite common. You can still find studies on *who* (who teachers? who is being taught?), or *with what* (what equipment, what textbooks, etc.) but as to *how* ... As you have realised, we mean *how* in the descriptive sense, how teaching is done, not how it ought to be done... [authors' italics].
>
> Bodin and Capponi (1996, pp. 566-567)

The aim of this chapter is to categorise the ways in which written materials are used by teachers. In order to do so, data concerning each one of the teachers in the sample is analysed separately in each of the following eight sections. Each teacher is introduced by presenting aspects of his/her training and experience. This introduction also includes the analysis of his/her answers to part of the questions in the interviews as well as the analysis of data collected from the mathematics school scheme of work and from the teacher's scheme of work. The data are analysed against the three sets of headings, creating concise tables to summarise results by using the meta-categorisation defined in section 6.3.

The categorisation presented in section 8.9 is based on a comparative study among different groups of students taught by the same teacher, so in each of the previous sections a teacher is introduced and the comparative study among his/her groups of students is developed from the displays described in chapter six (Case Matrix and Conceptual Matrix), containing the summarised set of data applying the meta-categorisation. Throughout this chapter and the next, each student group will be called by the code name presented in chapter 6 (section 6.4 - see also appendix 1: description of codes).

8.1. Analysis of Data Concerning Teacher A.

Teacher A is a particularly enthusiastic teacher working in school A, a comprehensive secondary school in the Midlands (UK.). His degree is in Science and he completed his PGCE in Mathematics 14 years ago. Since then, he has been teaching secondary mathematics. He has been working for six years in school A. Teacher A is not the head of the department, but has actively participated in the organisation of the school scheme of work. He strongly defends the school's position of not using textbooks saying that 'it is impossible to meet all students' needs using them'. He has also participated in several in-service courses.

School A has three groups of students in each year. All groups in year seven and eight are 'mixed ability'. From year nine upwards, the students are classified into three groups according to their level of attainment. The mathematics school scheme consists mainly of a list of printed materials to cover topics on the curriculum. This is organised by half-terms. It is left to the teacher to decide the sequence in which the topics should be presented and whether some complementation is necessary. Some of these topics have no suggested material. They are listed in the school scheme only by a title such as 'Vocabulary'. Therefore, it is left to the teacher to decide how they should be presented to the students. For example, teacher A decided to present the lessons on multiples, factors, prime numbers, square numbers, square roots, etc. without using any written material to support his lesson.

Despite admitting that the scheme did not work well for teachers who have joined the school after it was implemented, teacher A is very fond of it. He says that the scheme is 'alive' and 'new additions and changes of materials are welcome'. He also says that the head of department encourages all teachers to experiment with new materials and to discuss the outcomes within the department. Nevertheless, on being asked what could be considered as 'recent changes' in the school scheme, teacher A recognised that not many have occurred during the past few years. He thinks this is due to the fact that 'teachers got used to the materials, and they know how to make the most of them'. He also says that some of the materials listed in the school scheme are not the original printed ones anymore: 'they are adaptations we made from the original published materials'.

When asked why he thinks that the teachers who have joined the school after the implementation of the scheme of work have been facing problems, teacher A comments:

> This is a real problem. At the beginning, we had guides to all suggested materials, and we had a lot of discussions on how to use them. As time went by, we stopped using the guides and we did not write down the suggestions discussed in our meetings. By now, nobody knows where the majority of the guides are and the new comers have a lot of problems trying to use the materials. I suppose most of the time they just ask around to find out what we have been doing, but I'm not sure we give them enough support.

The school scheme does not differentiate students by attainment level. When being asked about his work within different groups of students, teacher A explains:

> We think it is better like that. In principle, all students should have a go in all the proposed activities planned for their school year. It actually happens in this way in year seven and eight. The activities are the same for all students. The difference is the outcome. One bright student produces a lot more work than a student that is facing difficulties. When it comes to year nine upwards, it is not always possible to give all the planned activities for the low attainers... I mean... they all have a go in the investigations and computer work, you know... but some of the most difficult topics ... it is better not to discuss them with the low-attainers... I will certainly not present the Pythagoras' theorem for my students in year nine.

Teacher A has no personal scheme of work. Nonetheless, he uses his diary to plan ahead (usually for a week or so). Usually only small reminders are written there, but teacher A is capable of explaining in detail his plans for the following lessons by reading these brief notes. In addition, teacher A has a personal file with printed materials, but he says that he is so well acquainted with the materials that he seldom consults it. This file also contains some of his previous own written materials. However he never uses them a second time. He justifies this:

> I only produce my own materials when I feel that something has gone wrong during the lesson using the suggested materials. In this case, it is always better to produce new material to try to solve the problem. I also cannot remember all the materials I have produced.... It is easier to produce a new one than to find an old one ... specially because I cannot be sure whether the old will be useful.

Set I - Headings: General Decisions	Students' Group→	AA8X	AA9L	AA10M	AA10T
Main Source of Materials for Class Work		3	3	4	4
Progression of the Content		3	2	4	4
Materials given to students for reference		2	1	3	4

table 8.1.1 - Case Matrix for the first set of headings - Teacher A.

Table 8.1.1, the Case Matrix for the headings in general decisions (set I) taken by teacher A, summarises the introduction above, and is based on table 4a.1 in Appendix 4a. In this table, the meta-categorisation defined in section 6.3 is used (see also appendix 1b). During one week, the three lessons given by teacher A to four different groups of students were observed. The groups were: one year eight 'mixed ability' group (AA8X), one year nine, classified as 'low ability' (AA9L), and two year ten groups: one classified as 'medium ability' (AA10M) and the other as 'high ability' (AA10T). In two cases (AA8X and AA10M), one extra lesson was observed, in order to complete the minimum set of data required.

. Comparative Study of Headings in the Second and Third Sets for Teacher A's Four Groups of Students:

As shown in table 8.1.1 above, the meta-categorisation allows the data to be displayed in concise tables. These tables will be especially useful when applied to these sets of headings (II and III, concerning day-by-day decisions and uses of materials). They will be the first main tool in developing a comparative analysis among teacher A's behaviours with his four groups of students. Tables 8.1.2 and 8.1.3 are the summarised versions of the Case Matrices for teacher A, each one concerns one set of headings, and the data used to produce them are summarised in Appendix 4a. Based on these tables, an overview of the choice and usage of written materials by teacher A with each group of students will be presented. As there were lessons when two behaviours under the same headings were observed, two codes can appear in the same cell of these tables.

Student's Group →	AA8X				AA9L			AA10M				AA10T		
Lesson → / Set II: Choice of Material to:	1	2	3	4	1	2	3	1	2	3	4	1	2	3
Source of Activities	3ab	1	1	1	4	4	4	4	4	4	1	4	4b	4b/3
Introduce a new topic.	-	1	-	-	-	-	4	-	-	-	1	4	-	-
Introduce a lesson.	3ab	1	1	1	4	4	4	4	4	4	1	4	4	3
Reference in classroom.	1	1	1	1	1	1	1	1	2	2	@	1	4	4/3
Exemplification.	3	1	@	@	4	4	4	4	4	4	@	4	4ab	4ab
Conclude a topic.	@	-	-	1	-	@	-	-	-	@	-	-	-	3ab
Promote links between topics.	1	1	1	1	-	-	@	-	-	-	-	4	-	-
Promote differentiation.	3ab	@	@	@	4b/1	4b/1	1	4b	4b	4b	@	@	4b	4b/3
Remedial material.	1	-	-	-	1	1	1	1	1	1	1	-	@	4b/3/@
Source of homework.	3*	@	@	@	4	4	4	4	4	4/1	4	4*	@	@
Provide Feedback on Exercises.	1	1	1	1	1	1	1	1	1	1	1	1	1	1

table 8.1.2: Set II Headings - Case Matrix for Teacher A

Group AA8X: Code '1' occurs often with this group because three out of four lessons with 8X were without the aid of written materials. During these three lessons, teacher A worked through the concepts of factors, multiples, prime numbers, square and cubic numbers and square roots using a personal approach and asking the students to write their own definitions in their notebooks. All the activities were also proposed by the teacher (including a mental test, applying the definitions and the design of posters, assigned as group work). This approach was a complete change from the first lesson, when the students were asked to 'keep going' with an activity using a series of adapted worksheets. While in the first lesson the

students were using written materials and working individually (the pupils worked through the worksheets in their own time), during the other lessons students worked as a group, being asked to do the same work, with no differentiation noticed. Homework was only assigned to these students during the first observed lesson. It was based on material different from that used during the lessons and did not match class activities in any of the lessons observed.

Student's Group →	AA8X				AA9L			AA10M				AA10T		
Lesson →	1	2	3	4	1	2	3	1	2	3	4	1	2	3
Set III: Use of Material to:														
Source of Activities	7ab	1	1	1	4	4	6	4	2	2	1	4	7	5
Introduce a new topic.	-	1	-	-	-	-	4	-	-	-	1	6	-	-
Introduce a lesson.	7ab	1	1	1	4	4	4	4	4	4	1	4	7ab	5
Reference in classroom.	5	1	1	1	1	1	1	1	8/1	8/1	@	1	7ab	3
Exemplification.	5	1	@	@	2	2	2	5	5	5	@	4	7ab	7ab
Conclude a topic.	@	-	-	1	-	@	-	-	-	@	-	-	-	7ab
Promote links between topics.	2	1	1	1	-	-	@	-	-	-	-	5	-	-
Promote differentiation.	7ab	@	@	@	2	2	1	4b	4b	4b	@	@	7	6
Remedial material.	1	-	-	-	1	1	1	1	1	1	-	-	@	8ab/@
Source of homework.	8*	@	@	@	6	6	6	6	6	6/1	6	7*	@	@
Provide Feedback on Exercises.	1	1	1	1	2	2	1	2	2	2	1	1	1	1

table 8.1.3: Set III Headings - Case Matrix for Teacher A

Group AA9L: Teacher A used printed materials (code '4' in the first row, table 8.1.2) in all lessons observed with this group. During the first two lessons and the beginning of the third, the strategy used by teacher A was to base his teaching on the materials without giving a copy to the students (code '4' in the initials rows, table 8.1.3). Although the same material was used, teacher A promoted differentiation among the students during the first two lessons by asking them to achieve different targets while developing the investigation. During the third lesson, teacher A differentiated the students by allowing them to work at their own pace. During this lesson, teacher A only gave the material to those who had already finished the initial task proposed. Homework was given in all lessons, based on class work, with a target given by the teacher.

Group AA10M: Teacher A had also proposed an investigation for these students. The strategy used during these lessons (the first three observed) was very similar to the one adopted in group 9L, although this time the teacher had supplied the students with an extra advice sheet, giving guidance on how to develop and report an MA1 activity. During the fourth lesson observed, the teacher introduced a new topic (the relationship between

circumference and diameter of a circle - introduction of number π), without using any written material but basing his lesson on measurements of round objects and tables constructed by the students showing these results. The students had homework assigned during all the lessons observed. It was to develop their report of the investigation up to a target determined by the teacher. A second piece of homework was also given in the third lesson, when the teacher asked the students to do a preparatory homework for the next lesson.

Group AA10T: Teacher A demonstrated several different behaviours when using printed materials with this group of students. All the observed lessons were based on a series of worksheets, designed to revise regular polygons and to use the results to generate a procedure in LOGO to draw any regular polygon, given the number of sides. Teacher A used the first lesson (which took place in the usual classroom) to develop the ideas associated with the task, without giving a copy of the material to the students. This lesson was to revise the pre-requisites for the task. The other lessons took place in the computer room. The students, working in pairs, could choose one of the worksheets to start with, and were asked to generate the procedure. Extension was also planned: a procedure to draw 'stars'. During the first lesson in the computer room, the teacher noticed several pairs of pupils facing difficulties, even when working through the introductory worksheet. Teacher A tried to help these students, but they still had problems by the end of the lesson. On the other hand, three other students had finished the task and were already working through the extension. Teacher A tried to solve the 'crisis' by producing an adaptation of the worksheets to guide the students' work during the second lesson at the computer. This strategy seemed to present good results and the pairs facing difficulties were working better using the new sheet. On the other hand, the students who were ahead finished their task at the beginning of the lesson. Teacher A admitted that he was not prepared for the situation and had no other task to assign to these students.

To conclude the analysis of tables 8.1.2 and 8.1.3, two more features are analysed here: (1) The answers to the 'match' questions; and (2) The sub-categorisation. As described in Chapter Six, the 'match' questions are the ones created to analyse how close other features in the lesson are from the main activity developed. Concerning teacher A, it is possible to say that almost all features developed in the classroom (such as introduction, illustrations, etc.) were matched with the main activity, the only exceptions being the homework given to groups 8X and 10T, which were completely different from any other activity developed in the lesson, and which also did not connect with the following lesson.

The sub-categories were introduced to differentiate among behaviours with the same 'degree' of independence from printed materials, as discussed in Chapter Six (see also appendix 1.b.4). Tables 8.1.2 and 8.1.3 show that teacher A gave choice to the students

during the development of their investigations in both groups 9L and 10M. Choice was also given to the students in group 10T concerning their starting points on the series of worksheets in lesson two and they could also choose whether to change materials for a new one in lesson three. Nevertheless, in group 10T the use of another sub-categorisation was necessary because the teacher adopted the behaviour defined as 'keep going' during the second and third lessons.

To complete the picture on how teacher A chooses and uses written materials, two other data displays are used: the graphs of the conceptual matrix presented for teacher A in figure 8.1.1 for choice and 8.1.2 for use of materials. The graph is defined as a display of data where the rows of the conceptual matrix (headings x meta-categorisation) are presented in the form of a bar chart, and the groups are differentiated. This display facilitates the visualisation and analysis of the characteristics of the teacher when choosing and using written materials and whether these decisions change with the student groups considered, which is the main objective of this research.

Analysis of the Ways Teacher A Chooses the Materials.

An overview of figure 8.1.1 shows that considering choice of materials, teacher A can be classified as a non-textbook user, who bases his teaching mainly on several sources of printed materials. It also shows that teacher A occasionally produces his own material, although usually adapting from printed materials. Another possible attitude of teacher A is to use no written materials. The figure also shows that these different choices do not seem to depend on the group of students considered, and that the same choice can be observed in different groups, although the column '1' is specially noticeable in group AA8X, due to the fact that three out of four lessons with this group used no written materials. Starting the analysis by 'rows', a look into the 'choice of materials' row shows that the general impression of the graph reflects closely the way teacher A chooses the main material.

Other characteristics of teacher A when choosing written materials can be observed looking into other 'rows': (1)'Reference in classroom' shows that teacher A seldom gives any reference material to his students, and usually they have to rely on their notes in the notebooks; and (2) 'links between topics' shows that teacher A usually has to promote these links himself, which is not surprising, given that different sources are used in different topics, even if they are related.

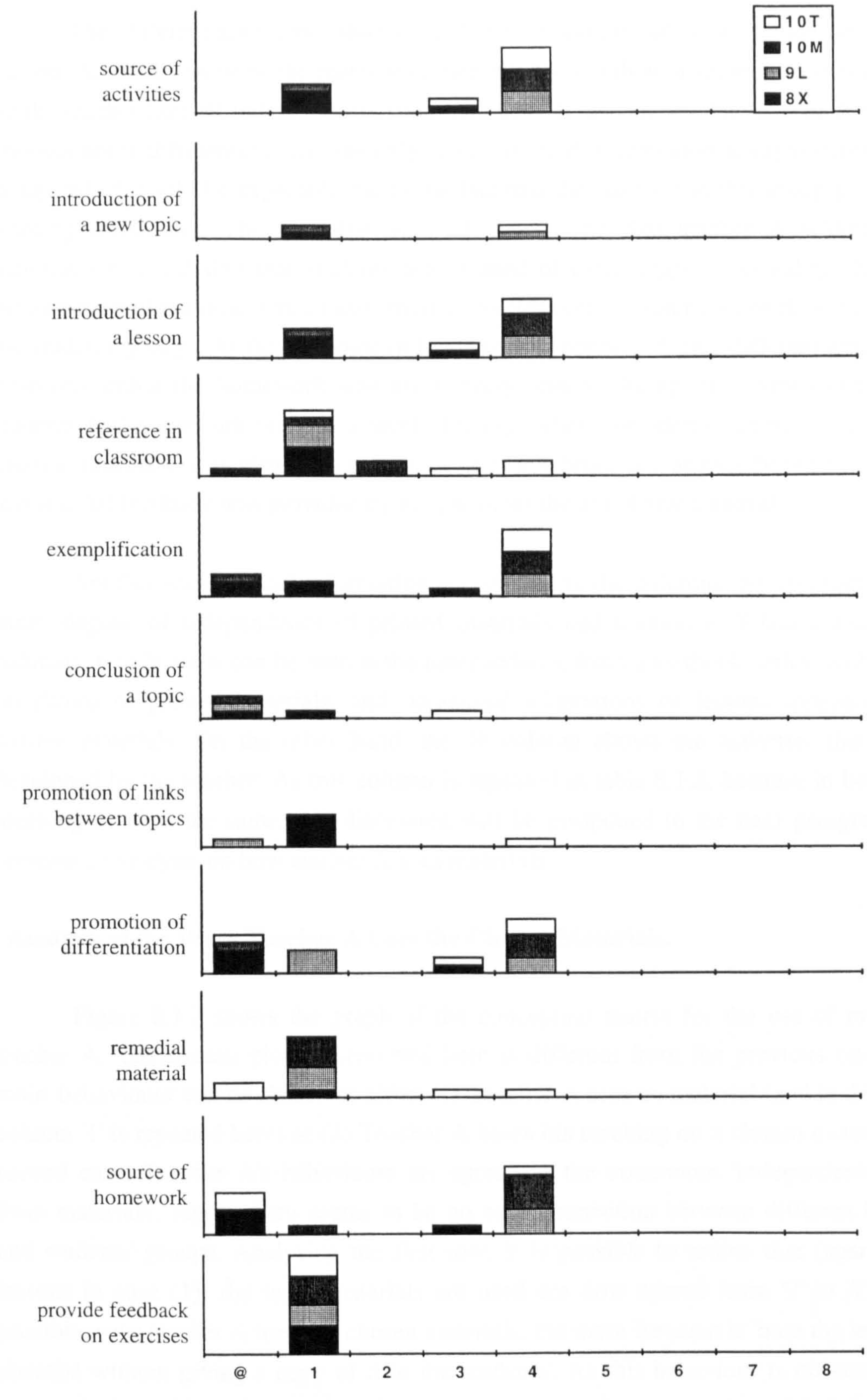


figure 8.1.1: graph of the conceptual matrix for choice of materials - teacher A

The 'differentiation row' shows that differentiation occurs in all groups working with teacher A, usually because the material chosen allows it, although sometimes it is introduced by the teacher himself independently from materials. It also shows that teacher A sometimes chooses not to differentiate, and the only group where differentiation always occurs is the 9L group, which could be expected, due to the fact that the students in this group present more learning difficulties. The 'remedial material' row shows that teacher A seldom changes material when realising that students are in need of extra support, providing this support himself most of the time. Once more, there is no clear connection between these attitudes and the students' groups. As far as choice of homework is concerned, two different attitudes were observed: either the homework was given every lesson (during the investigations), or the students had homework once in a week. Finally, when considering choice of materials to provide feedback, it is clear that teacher A does not bring any answer books for any of his lessons. All feedback was provided by him, without the aid of any material.

Another way to 'read' information is considering the columns, which characterise the same 'degree' of independence of printed materials and textbooks. When considering the columns '1' to '8' what can be seen is the independence from a textbook series, with emphasis on choice of printed materials, and occasional adaptations or lessons independent from written materials. On the other hand, the @ column shows the activities that were not developed by the teacher. As this column is repeated in table 8.1.2, because in both sets the meaning of @ is the same, this discussion will be postponed to the next paragraph, which presents an analysis on how teacher A uses materials.

. Analysis of the Ways Teacher A Uses the Chosen Materials.

Figure 8.1.2 shows the graph of the conceptual matrix for the use of materials by teacher A. The overall picture presented here is different from the previous one, and two main 'behaviours' can be observed: either (1) Teacher A uses no material (and in this case, the column '1' is repeated here) or (2) Teacher A bases his teaching on a chosen material. In this second case, Teacher A's behaviours are spread in the continuum 'independent/dependent from materials'. Again there seems to be no easy connection between different behaviours and students' groups. Analysing the first row, it is possible to notice that (apart from the lessons in case (1)) the ways materials are used are now spread from '2' to '7'. From all possible ways teacher A uses the chosen materials, the more frequent is 'base the lesson in the material without giving a copy of it to the students'. As this behaviour is observed in three groups, it does not seem group related. Notice also that the group where this behaviour was not observed had three lessons observed with no materials used.

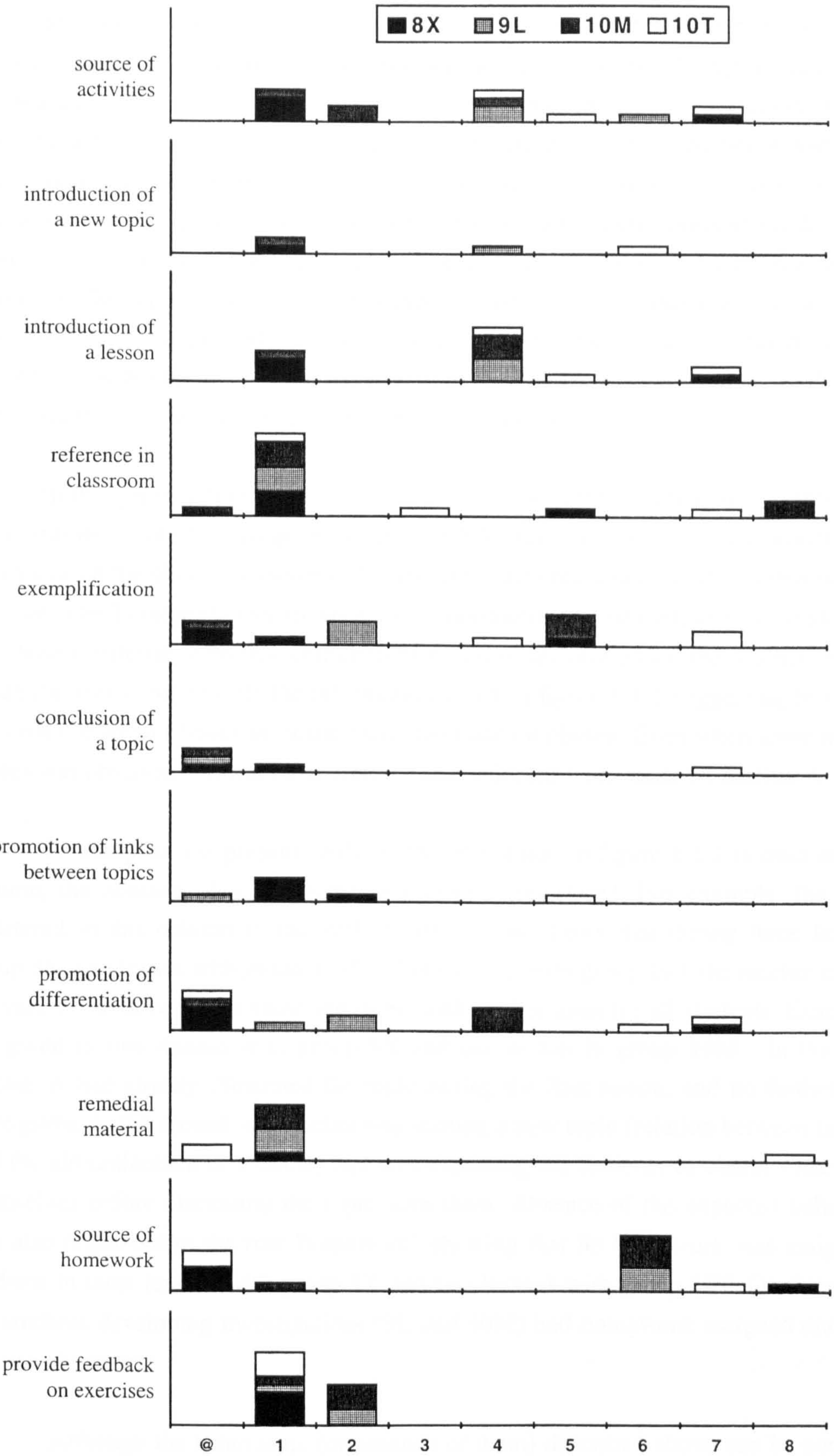


figure 8.1.2: graph of the conceptual matrix for use of materials - teacher A

The use of material without giving a copy to the students was even more frequent when considering introductions to lessons, which means that usually the teacher introduced the lesson by himself, even when materials were given to the students during the lesson. The main characteristic shown in the row concerning reference is that teacher A had a position independent from written materials. For the few cases observed when written materials were used as reference, they were used in a variety of ways: some were complemented, others were given to the students without guidance and others were emphasised by the teacher in class. Examples also show how teacher A varied his behaviours: sometimes the students were supposed to read the examples by themselves, sometimes the teacher presented the examples himself (either basing them on the material or expanding from it), sometimes all the examples were 'invented' by the teacher and sometimes examples were not given.

In this paragraph other 'rows' are analysed. The 'differentiation' row seems to receive some influence of the group considered: while for three groups differentiation seems dependent on the choice of material, for group 9L differentiation is more independent of the material. The 'homework' row shows a more concentrated behaviour, as it is usually based on the chosen material, with few complementary exercises offered by the teacher. Finally, the 'feedback' row complements the information given in figure 8.1.1 suggesting that teacher A does not use answer books associated with the material chosen. Even when some influence of guides was observed, several modifications and additions were made by teacher A.

To complete the present analysis, the @ column in figure 8.1.2 is discussed. In this column, the absence of an expected behaviour is registered. For example, the five cases registered in this column in the 'differentiation' row shows that during three lessons with group 8X, one lesson with group 10M and one lesson with group 10T the teacher expected all the students to develop the same activities, with similar aims for all students. Examples were not given in two lessons with group 8X and one lesson in group 10M. In the first case, teacher A had already illustrated the topic during the first lesson, and no further examples were given. In the second, the teacher was starting a new topic (relation between the diameter and the circumference of a circle) and was expecting the students to obtain some results by themselves before discussing the topic with them. Absence of the expected behaviour (@) was also registered in the row 'homework', showing that no homework was assigned to the students in three lessons with group 8X and two lessons with group 10T. On the other hand, the students developing investigations (9L and 10M) had homework assigned during all the lessons observed.

Although the behaviours (or absence of them) described above can be considered as part of teacher A's teaching strategy, other absences (@) were noticed in figure 8.1.2:

'conclusion', 'links' and 'remedial material'. Out of five lessons, when topics were finalised, three had no teacher's 'conclusions', and the students were expected to reach them by themselves. Although 'links between topics' were already discussed when presenting teacher A's choice of materials (due to the fact that he chooses 'non-linked-materials'), one more case is discussed here. In one lesson with group 9L, the teacher reported to the researcher that he decided to change the order proposed by the school scheme because he perceived that no student had applied the idea of factors in the previous investigation. Nevertheless, no links were established during the lesson and as the teacher was not planning to go back to the investigation, it is difficult to believe that the students would perceive the links by themselves. Finally, the absence of remedial situations was observed twice within group 10T.

To conclude the discussion of figure 8.1.2, some percentages are used. From the 33 occurrences of lessons in categories from 5 to 8, that characterise materials being given to the students and used more closely, 13 (39.4%) occur with group 10T. This means that this was the group where the lessons were kept closer to the material chosen. On the other hand, from 48 occurrences in categories 1 and 2, that characterises great independence from materials, 20 (42%) occur with group 8X and 13 (27%) occur with group 9L. Considering that three lessons with group 8X were developed without the aid of written materials, this feature for this group is not so surprising as the one observed with group 9L, which had a chosen material for all three observed lessons. This percentage of 'independent' behaviour reveals that teacher A makes several adaptations to the chosen material when using it with the 'low attainers' group.

Summary of the Analysis

Summarising the discussion of the behaviours of teacher A concerning materials, it is possible to say that teacher A bases most of his teaching on a wide range of selected printed materials, eventually including own developed lessons, without using any material. Students use the notes and examples in their notebooks as main sources of reference, and these notes are not always reviewed by the teacher. Once materials have been selected, teacher A uses each one of them in a particular way, varying from great independence to close adherence. The way one material is used does not seem completely dependent on the group of students considered, although the data suggest that with group 10T teacher A uses the material closer to its objectives than with the other groups. At the other extreme, the data indicate that with group 9L Teacher A adapts and changes features of the material more than with any other group.

The data also suggest that teacher A usually introduces his lessons, matching this introduction to the lesson's activities, with the exception of those that provide continuity to previous work, with copies of the material given to the students. In these cases teacher A expects the students to remember what they have been doing by looking at the material and asks them to 'keep going'. The data also suggest that teacher A provides feedback for students' activities by himself, adding his own explanations and comments to suggestions offered by the guide (that never is in the classroom, so the students do not access it directly). Based on this feedback, teacher A introduces differentiated tasks and/or remedial situations without changing materials most of the time. On the other hand, teacher A rarely provides conclusions for the topics developed and, even when recognising the existence of links between topics, rarely develops these links in class.

8.2. Analysis of the Data Concerning Teacher B.

Teacher B is a teacher with particularly diversified experience working in school B, a comprehensive secondary school in the Midlands, UK. Teacher B completed her mathematics undergraduate course more than twenty years ago, and she did not become a teacher straight away. She had experience working as a mathematician in industry and only when personal reasons led her to give up work did she decide to complete her PGCE course. Teacher B re-started her career as a part-time teacher working within adult education, and for the past nine years has been working in secondary schools as a Mathematics teacher. At the time of the observations, teacher B was engaged in completing her part-time Masters Degree and had been working in school B for approximately seven years.

The School Scheme of Work in School B is 'in development', meaning it is already complete for years seven and eight, and for years nine upwards it only contains suggestions for investigations and course work. The school used the SMP 11-16 booklets for years seven and eight but has changed to another series of textbooks: 'Journey into Maths'. For years nine upwards the adopted textbook is the SMP 11-16 series, with its four tracks: (yellow, the highest; red; blue; and green, the lowest). Teacher B says:

> Because in years seven and eight we use 'Journey into Maths', complemented by the SMP booklets, it was important to have the scheme written quickly... with the other students we have been using the SMP series for a long period of time... we are quite used to it... a written school scheme was not that urgent... I cannot say that we base our teaching on the school scheme... it is really based on the books and in our personal experiences... the head of the department encourage us to exchange those experiences... and usually a particularly successful worksheet or some material based on the exercises of previous exams prepared by one of us become available in the department files for the other teachers.

School B has five groups of students per school year. When asked how these students are set in groups, teacher B says:

> In theory, the students are 'mixed ability' in years seven and eight and set by ability in the other years... In practice, we have a group for 'low ability' students from year seven... they need special attention and in a 'mixed ability' group it is very difficult to give this kind of attention to them... so, the school has this group with about ten to fifteen students in each year group... and it is better this way... to attend to their individual needs...
> [As for year nine upwards] in practice they are not set by 'levels of ability' in a complete sense. For example, 'my' year 10: I have some students there that can follow the higher course in the SMP books... but they cannot cope with the 'speed' of the 'top group'... they use the yellow book...the other students are using the 'red track' in SMP... but some of them go slowly through the book...I sometimes wonder if they would not be better using the blue book... it is really a mixed ability group... without the students with special needs in it...as in year seven and eight....

Teacher B also believes it is important that students keep the textbook with them all the time. She says:

> These students' courses are based on the SMP... we use the assessment suggested by the book... and even their final GCSE exams are based on it. If they keep the books, they can review their work... also the revision sections of the book are a good source of homework... the exceptions being the 'low attainers'... they always lose their material... so we decided not to allow them to take the book home...and we do not expect these students to work by themselves at home anyway...

Finally, when asked about the decisions on the progression of contents, teacher B says:

> With years seven and eight, because we use two different sources, it was important to decide how the progression would be organised... basically, some topics are based on one source and others on the other source, which means that each topic has its progression taken from one of the sources... for the other school years the question was not even discussed... I guess it is because we all assumed that the book would take care of it... and it does really

Set I - Headings: General Decisions **Students' Group→**	**BB10X**	**BB9L**	**BB11M**	**BB9T**
Main Source of Materials for Class Work	7	7	7	7
Progression of the Content	7	7	7	7
Materials given to students for reference	8	@	8	8

table 8.2.1 - Case Matrix for the first set of headings (general decisions) - Teacher B.

The information about the general decisions taken by teacher B discussed above is summarised in table 8.2.1, presenting the meta-codification for the headings in group I. During a week all three lessons given by teacher B to groups BB10X (the year ten 'mixed-ability group mentioned by the teacher in the extract of the interview above), BB9L (the year nine 'special needs' students group), BB11M (the third out of five groups in year 11), and BB9T (the 'top' group in year nine) were observed.

Comparative Study of the Headings in the Second and Third Sets for Teacher B's Four Groups of Students:

Tables 8.2.2 and 8.2.3 are the summarised versions of the Case Matrices for teacher B, each one concerned with one set of headings. They were developed based on the tables of summarised data presented in Appendix 4b. Based on these pairs of tables, an overview of the choice and usage of written materials by teacher B with each group of students is presented. As there were lessons where two behaviours under the same headings were observed, two codes can appear in the same cell of these tables.

Group BB10X: This group of students is organised in such a way that is almost impossible for the teacher to use 'whole group' teaching as a strategy: about 40 percent of the students are following the yellow track of the SMP series (the 'highest') while the other 60 percent are using the red track (second out of possible four). This implies for example that if one of the sub-groups has had the lesson introduced the other has not, and so on ... The columns corresponding to this group in tables 8.2.2 and 8.2.3 show that the teacher essentially adopts the strategy referred to in this research as 'keep going': the students being expected to work from the book, in an individualised way. Nevertheless, there were some opportunities to observe some class teaching: teacher B used the fact that coincidentally both groups should be starting chapters on proportionality to give some general explanations on the topic (it has to be said that this introduction of the topic did not match the class activities for more than half of the 'reds' and it was actually a review for the 'yellows). Teacher B also did some class teaching directed at one of the sub-groups, while the other students were told to 'keep going'. During the observed lessons, the teacher used more than half of the available lesson time to mark students' notebooks individually, with the aid of the answer book. The homework set by the teacher was also taken from the textbook, using the book's revision sections.

Student's Group →	BB10X			BB9L			BB11M			BB9T		
Lesson →	1	2	3	1	2	3	1	2	3	1	2	3
Set II: Choice of Material to:												
Source of Activities	7ab	7ab	7ab	7ab	7ab/1	7ab/1	7ab	7ab	7ab	6	3	6
Introduce a new topic.	6*	7ab	7ab	7ab	7ab/1	7ab/1	7ab	7ab	7ab	6	-	5
Introduce a lesson.	6/7ab	6/7ab	7ab	7ab	7ab	7ab	7ab	7ab/7*	7ab	6	3	5
Reference in classroom.	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	6	6	6
Exemplification.	5/7ab	7ab	7ab	7ab	7ab/1	7ab/1	7ab	7ab	7ab	6	8	3
Conclude a topic.	-	5/7ab	7ab/-	7ab	7ab	7ab	7ab	7ab	7ab	6	-	6
Promote links between topics.	-	-	-	-	-	-	7ab	7ab	7ab	5	5	5
Promote differentiation.	7ab	7ab	7ab	7ab	7ab/1	7ab/1	7ab	7ab	7ab	@	@	@
Remedial material.	@	@	-	1	1	1	-	4	7	-	-	-
Source of homework.	@	@/8	8	@	@	@	@	7ab	7ab/8	7	3	7
Provide Feedback on Exercises.	7	7	7	7	7	7	7	7/1	7/1	6/7	8/7	6/7

table 8.2.2: Set II Headings - Case Matrix for Teacher B.

Group BB9L: This group is small, composed of 12 students who work through the SMP green track on an individualised basis. Teacher B keeps all the students on the same book, by offering extra activities for those students who have completed a certain stage of the book ahead of the others.For example, during the observed lessons one student was working with 'magic squares' at the computer. During the second and third observed lessons, teacher B also developed some work with one student at a time, without using any written material, using

coins and notes related to market bills and change. From tables 8.2.2 and 8.2.3 it can be seen that teacher B did not assign homework for these students during the observed lessons (in fact, the teacher says that she never does).

Student's Group →	BB10X			BB9L			BB11M			BB9T		
Lesson → / Set III: Use of Material to:	1	2	3	1	2	3	1	2	3	1	2	3
Source of Activities	7ab	7ab	7ab	7ab	7ab/1	7ab/1	7ab	7ab	7ab	7	8	7
Introduce a new topic.	6*	7ab	7ab	7ab	7ab/1	7ab/1	7ab	7ab	7ab	5	-	2
Introduce a lesson.	5/7ab	2/7ab	7ab	7ab	7ab	7ab	7ab	7ab/7*	7ab	5	8	2
Reference in classroom.	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab
Exemplification.	5/7ab	7ab	7ab	7ab	7ab/1	7ab/1	7ab	7ab	7ab	5	8	2
Conclude a topic.	-	5/7ab	7ab/-	7ab	7ab	7ab	7ab	7ab	7ab	5	-	2
Promote links between topics.	-	-	-	-	-	-	7ab	7ab	7ab	2	2	2
Promote differentiation.	7ab	7ab	7ab	7ab	7ab/1	7ab/1	7ab	7ab	7ab	@	@	@
Remedial material.	@	@	-	1	1	1	-	7ab	1	-	-	-
Source of homework.	@	@/8	8	@	@	@	@	7ab	7ab/8	6	6	6
Provide Feedback on Exercises.	7	7	7	5	5/1	5/1	7	7	7	6/7	8/7	6/7

table 8.2.3: Set III Headings - Case Matrix for Teacher B.

Group BB11M: Similarly to group 10X, this group also used two different tracks of the SMP series: the majority of the students used the blue track while the others use the red track. The textbook was used in a way similar to group 10M, during the observed lessons there were opportunities to observe a special solution to a crisis situation. The teacher gave one student facing difficulties some extra work in addition of fractions from another book (which is kept by the school as a resource material), asking him to revert to the textbook afterwards, which he did successfully. Another special feature observed during these lessons was an introduction of a lesson offered by the teacher for the 'blues': the teacher highlighted one of the methods developed by the book in the 'value for money' chapter. As the students were working in an individualised way, only two students were actually working through that section during the lesson, indicating that the introduction given by the teacher did not match the class activities.

Group BB9T: Tables 8.2.2 and 8.2.3 show that teacher B works with this group in a completely different way from the pattern described in the previous groups: the textbook was complemented with other materials and students worked as a group (no differentiation observed) on exercises selected by the teacher. Notice also that the teacher introduced the new topics and also all the lessons, providing examples that complemented the textbook. This

was also the only group observed that was assigned homework in all the observed lessons. Differences were noticed even in the way feedback was given to the students: although the general pattern of marking students' notebooks during the lessons with the aid of the answer book was observed, teacher B complemented that feedback in this class by discussing students' answers with the whole group.

Complementing the analysis of tables 8.2.2 and 8.2.3 notice that the sub-category (ab) also had to be used several times, especially with groups 10X, 9L and 11M, showing that with these groups the students are supposed to interact with the textbook directly most of the time, each one at his/her own pace. This individualisation can be one possible explanation for the fact that on two occasions introductions presented by the teacher were classified as 'not-matching' with the main activities developed during the lesson.

Having analysed the main features observed in each group, a comparative study on how written materials were used can be developed, using Conceptual Matrix graphs for choice and use of materials respectively, as presented in figures 8.2.1 and 8.2.2.

.Analysis of the Ways Teacher B Chooses the Materials.

The overall look of figure 8.2.1 shows that teacher B bases her teaching mainly on the textbook series adopted by the school. In almost all 'rows' of the table, the predominance of behaviours '7' shows that the material is closely used, although not as suggested by the guides.

The first 'row' shows that the textbook was used as the main source of activities with all four groups of students during the observed lessons. Nevertheless, with group **9T** the textbook was complemented with other materials and even not used during one lesson, when the teacher was using an adapted material instead. The strategy of working with one student in group **9L** without using textbooks while the others work through the book is also shown.

Introductions of topics and lessons were not made by the teachers. The students were supposed to read from the book. The high frequency of introduction of topics with groups **10X, 9L** and **11M** occurs not because the teacher was doing it, but because each student initiated a new chapter in the book at his own pace. Only group **9T** had their topics and lessons introduced by the teacher.

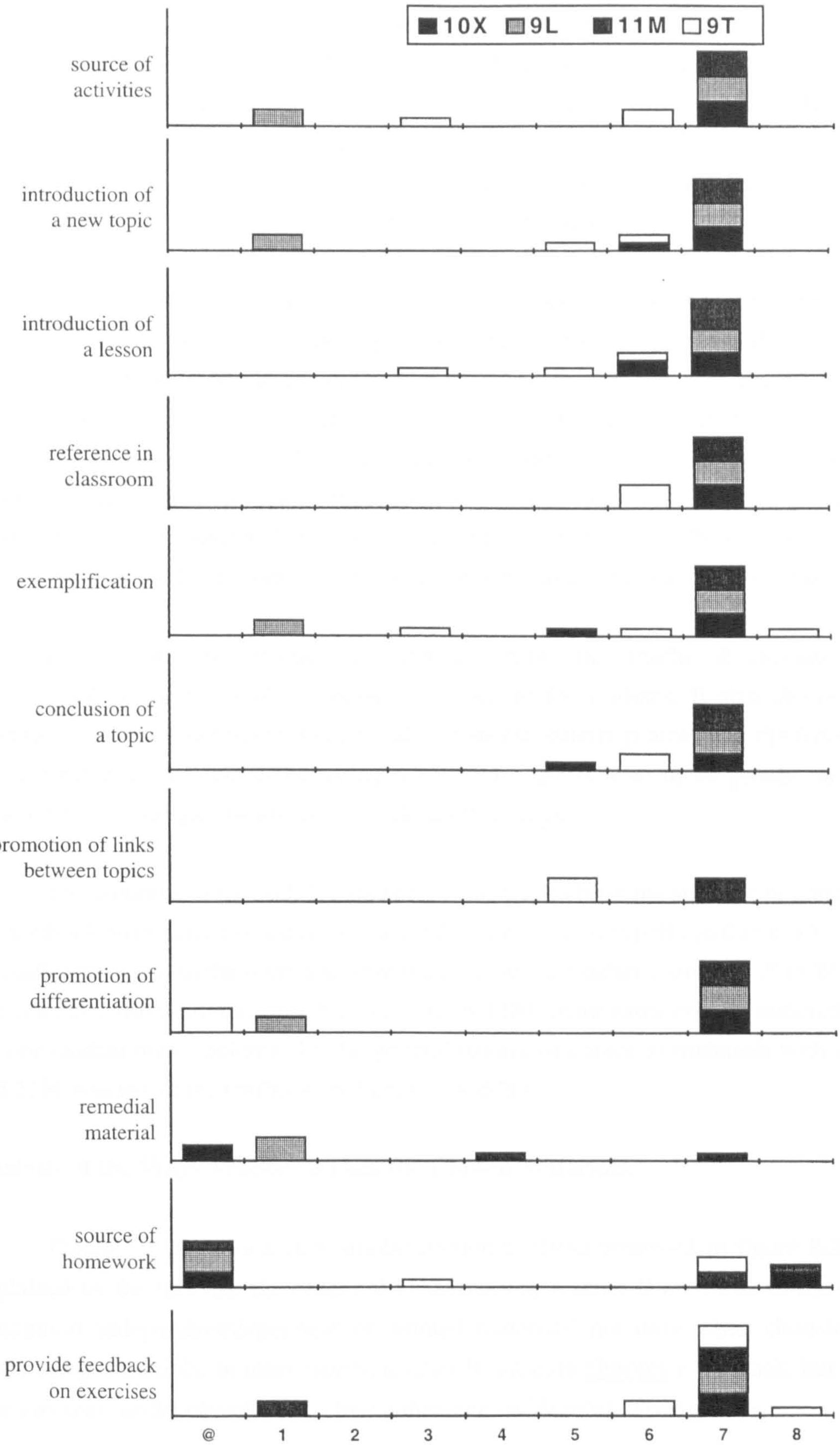


figure 8.2.1: graph of the conceptual matrix for choice of materials - teacher B.

In summary, it can be said that introductions, examples, references and conclusions are all taken from the book in groups **10X** and **11M**. The few exceptions to this general pattern within these groups in the first six 'rows' of the table were described as non-matching situations. Group **9L** is also taught in the same way, but in parallel one student at a time developing a single activity individually with the teacher.All students had the opportunity to develop this activity. The way of using materials changes when group **9T** is considered: in all these activities the textbook was complemented by other materials.

Another feature that reinforces the difference between the groups can be observed in the row 'differentiation'. The textbook promotes differentiation in groups **10X** and **11M**. The same happens in group **9L**, although the teacher complemented the book activity with another differentiated activity using no materials. With group **9T**, teacher B chooses the higher track of the same textbook series, and this time there is no differentiation among the students in the group. The 'homework' row shows that group **9L** had no homework assigned, groups **10X** and **11M** had homework assigned in some lessons, always from the textbook. Group **9T** was the only group that had homework assigned in all lessons, and not always from the book.

Finally, the row 'feedback on exercises' shows that teacher B chooses to use the answer book of the textbook to provide feedback to the students. It also shows that, apart from those activities not based on materials, a general pattern in providing the feedback exists (correct the students' notebooks during the lessons) and is used in all groups including **9T**, although this group also received feedback in other ways.

The columns in figure 8.2.1 show that teacher B chose the strategy of complementing the textbook with activities using no materials, only with group **9L** (column '1'). With group **9T**, different extras to the textbook were planned by the teacher (columns '2' to '6'). Although one remedial situation was introduced in group **11M** using extra printed material (once and for one student only - column '4'), the general pattern of choice of materials with groups **10X** and **11M** was to use the textbook (columns '7' and '8').

Analysis of the Ways Teacher B Uses the Chosen Materials.

Figure 8.2.2 shows a very similar picture to those observed in figure 8.2.1.. This is explained by the fact that the observed behaviours of teacher B are close to the ends of the continuum 'independent/dependent on printed materials' not only when choosing but also when using materials. In other words, teacher B not only chooses a textbook, but also uses it in a way that can be classified as 'close adherence' with most of her groups.

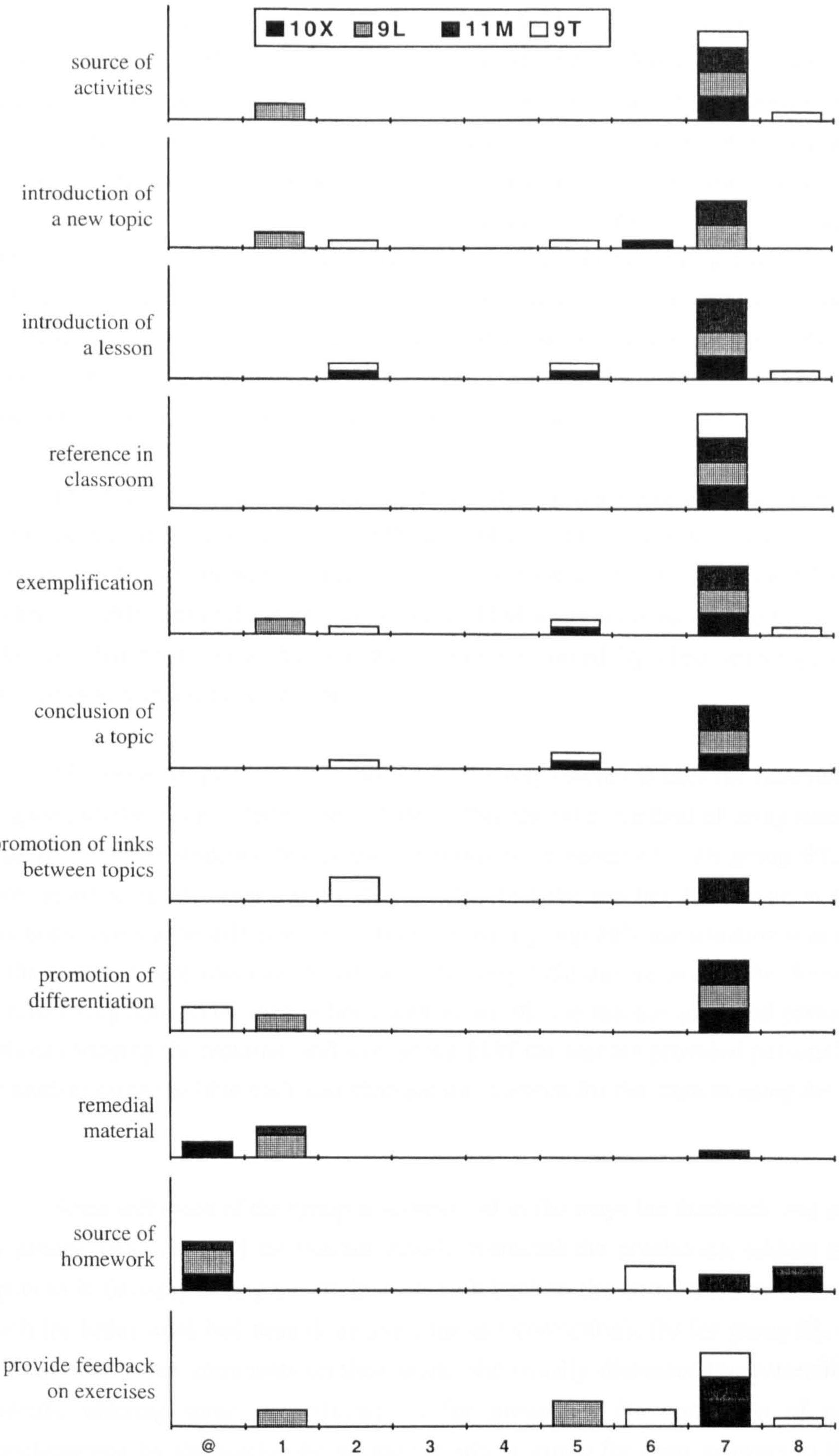


figure 8.2.2: graph of the conceptual matrix for use of materials - teacher B.

As discussed when comparing the groups, the most common behaviour adopted by teacher B when using the textbook in groups **10X, 9L** and **11M** is to ask the students to work individually from the material. In these cases the book promotes the differentiation not only because different tracks are used but also because the students work through it at their own pace. As for group **9T,** the teacher asks the students to work as a group (all developing the same task at the same time with no differentiation noticed). She also adds several complementations to the textbook as well as personal inputs. This pattern when using the books is reflected in the first six 'rows' of the figure and also in the 'differentiation' row, with the exception of the 'reference in class' one. The use of the textbook as reference is not reinforced by the teacher in any of the observed groups, and students are expected to use the book as reference in their own way and at their own time.

The 'links' row shows that the general pattern when using materials is also followed for this particular heading: in groups **11M** and **9T** introductions of topics which included the promotion of links with previous knowledge of the aims were observed. The behaviour of the teacher was different in these situations: group **11M** was supposed to read from the book the links provided while group **9T** had these links reinforced by class teaching, with several personal inputs added by teacher B.

The 'remedial material' row shows that the way teacher B uses the material with group **9T** appears to be more suited to these students than the other method of using materials seems to be to the other students. No 'crisis' situations were observed with group **9T,** while they were observed in all other groups. Notice also that the teacher behaviour in these 'crisis' situations seems to be different for each group: with group **10X** the situation was not resolved by the teacher (there was one student who did very little during two of the observed lessons without being noticed by the teacher); with group **9L** the teacher provided personal support, without changing the material, and with group **11M** the teacher provided personal support for the student using the blue track and changed the material for the student using the red track.

Some influence of the group was observed in the ways the feedback was provided: (a) for groups **10X** and **11M** the teacher mainly corrected the notebooks, adding few personal inputs to it, (usually asking the students to look back to the corrections or commenting how much (or little) work had been done since the last correction); (b) for group **9L** teacher B is more careful in her comments on their work: she usually discusses the corrections with the students, offering some suggestions; (c) for group **9T** the correction of notebooks is complemented by the teacher by asking the whole group for their answers to the exercises and commenting on them on the blackboard.

Summary of the Analysis

Summarising the analysis above, teacher B can be classified as a textbook user who uses the textbook almost all the time during lessons. The way the book is used seems to be very much group related: (a) for the 'mixed ability' and the 'medium' groups the teacher allows the students to 'keep going' most of the time, with occasional interventions to keep each sub-group working within a certain range of the book; (b) for the 'low ability' group the book is used in the same way but teacher B adds some extra work, usually using no printed materials; and finally (c) for the 'top' group the teacher uses the textbook complemented with other materials and in a teacher-led way.

Teacher B also seems to display group related behaviours when 'homework' and 'feedback' headings are considered. She sets homework every lesson for group **9T**, always related to class work; she sets homework sometimes for groups **10X** and **11M**, usually using the revision sections of the book or asking them to complete one chapter, as a way to keep the students in each sub-group not so far apart in the book and finally no homework is assigned to group **9L** (actually these students do not take any material home). Feedback was promoted by notebook correction within all groups, but some 'class correction' was observed with group **9T**, while with group **9L** the teacher seems to be more careful on her comments about students work.

8.3. Analysis of Data Concerning Teacher C.

Teacher C is a highly respected teacher working in School C, a comprehensive secondary school in the Midlands, UK. Teacher C is a deputy head in her school, she is also the substitute head of the Mathematics Department and her fellow teachers comment (and she agrees) that the students are always on their best behaviour when she is around. Teacher C has a Chemistry degree and a Bachelor in Mathematics and Education degree. She also has a Degree in Outdoor Education, and she has been working for twenty years in School C.

The school has adopted the SMP 11-16 series of books and booklets as their textbook, and has no written scheme of work. Teacher C says that they follow the books, apart from 'a couple of topics, that we decided to take out '. She also says that the teachers felt the need to complement the SMP series as far as investigations are concerned. She comments:

> SMP does not provide material enough for investigations... not in the way the N.C. suggests it anyway... on the other hand, the in-service courses for the last few years have been almost completely dedicated to MA1 activities... so I believe we have enough materials for these [activities]... but they are not taken from SMP... they came from other sources...

School C has five groups of students per year and also runs a sixth form course. Because of her degrees, teacher C is considered a highly qualified teacher of Mathematics, and usually only gives lessons to year nine pupils and above. Nevertheless, for this particular year, she is in charge of two year eight groups, because one of the teachers in the department in on leave. Teacher C says:

> It has been a good experience... for more than ten years I have been working only with the year nine upwards and I usually do not work with low attainers... but the teachers who have been doing this work did an excellent job... I mean, by now, everything is highly organised... they have textbooks series chosen to replace SMP for the low attainers in year eight, they have a scheme of weekly homework sheets that take into consideration the different levels of the students... they also have some complementary booklets for those topics in which they felt the SMP did not work very well... I mean, the teachers that are usually in charge of the youngsters and of the low attainers have been working as a team... and they did a good job... it is really easy for someone coming from outside to fit in their scheme of work and to do a proper job.

When asked if the teachers working with year nine upwards also have a similar scheme, she says:

> Well, you can say we also have our organisation... in our own disorganised way... I mean, the scheme for homework exists for those students as well, up to the middle of year eleven... after that the students usually have finished the SMP track they have been working through and we use the final part of year eleven to revise the contents for their GCSE exams... a lot of material has been developed... we have

several series of revision sheets for students who had worked through all the tracks of SMP... these sheets are based on the SMP series and include questions of the previous GCSE exams... I also have been developing some more sheets this year... we have some specially bright students in year eleven and I do believe they can go for an A* grade in their GCSE Maths, so I have been organising some extra sheets for them, including questions of the previous A* exams in Maths... but you cannot say it is a real team effort... it is more like the sum of individual efforts... so I guess it would be more difficult for a new teacher to fit in it... unless you consider the textbook, of course... as every piece of extra material has been based on the textbook, I believe that a teacher who is confident using SMP would have no problem in fitting in...

Teacher C had a reaction that can be considered unique among the teachers in the sample when asked about the 'levels' she would consider her students to be. Some teachers in the sample stated 'levels' for their students even before being asked to, others were more reluctant, but answered the question as well. Teacher C refused to answer the question. She says:

> Call me old fashioned... but I believe 'kids' learn what you teach them... I mean, if you never teach them probability, they would be level zero in AT 5, would they not? ... even if they are level ten in algebra or number... Those very bright students in year eleven are well above all the levels in the N.C.- 'level 14' if you really want me to classify them... but the others... all other students would be in different levels for different ATs... I do not think 'average' can be applied here... no... it makes no sense... sometimes they know things that are 'level seven' and do not know things that are 'level five'... no, I really refuse to classify my students this way!

Finally, when asked about the policy adopted by the school on reference materials, the teacher says that students in year eight do not take books home, but students in year nine and upwards do, except for the 'low attainers'. Teacher C comments:

> Once we let the 'bottom' group take material home... and it was an expensive experience... the books came back destroyed, or did not come back at all... so, these students only take their notebooks and the homework material home... The other four groups we have each school year - one using the 'yellow' track, two using the 'red' and one using the 'blue' - take the textbook home all the time...the eventual loss of one book is a small price to pay for the benefits of taken the material home... I mean... they work in an individualised way in class, but they are going to do the same GCSE exam... so I believe it is a good experience for them to be stretched a bit... I give my students common tests... this means that those students that are more 'slow' during lessons would have to make some extra effort at home... and they do really... with surprisingly good results.

The summary of general decisions about written materials for the school year is presented in table 8.3.1., which is based on the summary data presented in table 4c.1, Appendix 4c. During a week, all three lessons given by teacher C to the groups CC8X, one of the four year eight 'mixed ability' groups (from which the 'low attainers' had been taken out);

CC8L, the only group in year eight that is set by ability level; CC10M, the second out of five year ten groups; and CC11T, the first group in year eleven were observed. In the following section, a comparative study amongst the four groups of students during these lessons will be developed, considering the headings in sets II and III.

Set I - Headings: General Decisions / Students' Group→	CC8X	CC8L	CC10M	CC11T
Main Source of Materials for Class Work	7	4	7	6
Progression of the Content	7	4	7	6
Materials given to students for reference	2	1	8	8

table 8.3.1 - Case Matrix for the first set of headings (general decisions) - Teacher C.

Comparative Study of the Headings in the Second and Third Sets for Teacher 's Four Groups of Students:

Tables 8.3.2 and 8.3.3 are the summarised versions of the Case Matrices for teacher C, each one concerns one set of headings, and the data used to produce them are summarised in Appendix 4c. Based on these tables, an overview of the choice and usage of written materials by teacher C with each group of students is presented. As there were lessons when two behaviours under the same headings were observed, two codes can appear in the same tables' cell.

Group CC8X: This mixed ability group works in a completely individualised way. The students are supposed to 'carry on' the work from the booklet they are using, and at the end of each booklet, they do a test. Depending on the result, the students are asked either to do some revision work or to start a new booklet. The teacher divides the lesson time between two main activities: either marking students work or helping students with their work. Teacher C has a characteristic way of carrying out both activities: she sits at her table and calls one student at a time to have their work corrected. She uses the answer book to mark the work and after finishing it, she writes the solution to selected exercises in the student's notebook, while commenting with the pupil on the mistakes he/she made. If there are no students queuing for help she calls another student, otherwise, she helps the students who have been waiting, always by solving the exercise (or a similar example) in their notebooks. During the observed lessons, most of the students were 'on task' almost all the time. There were two students who did not seem to progress in their booklets, and they did not ask for help. As they were not among the students called by the teacher to have their work marked, teacher C did not notice the 'crisis' situation and no help was offered for these students.

Student's Group →	CC8X			CC8L			CC10M			CC11T		
Lesson →	1	2	3	1	2	3	1	2	3	1	2	3
Set II: Choice of Material to:												
Source of Activities	7ab	7ab	7ab	4	4/1	4	7ab	7ab	7ab	5ab	5ab	5ab
Introduce a new topic.	7ab	7ab	7ab	4	4	4	7ab	7ab	7ab	5ab	5ab	5ab
Introduce a lesson.	7ab	7ab	7ab	4	4(*)	4	7ab	7ab	7ab	5ab	5ab	5ab
Reference in classroom.	7ab	7ab	7ab	4	4	4	7ab	7ab	7ab	7ab	7ab	7ab
Exemplification.	7ab	7ab	7ab	4	4	4	7ab	7ab	7ab	7ab	7ab	7ab
Conclude a topic.	7ab	7ab	7ab	-	@	-	7ab	7ab	7ab	7ab	7ab	7ab
Promote links between topics.	7ab	7ab	7ab	-	-	-	7ab	7ab	7ab	7ab	7ab	7ab
Promote differentiation.	7ab	7ab	7ab	4ab	4ab	4ab	7ab	7ab	7ab	5ab	5ab	5ab
Remedial material.	@	@	@	1	1	1	-	-	-	-	-	-
Source of homework.	@	5(*)	@	@	@	5(*)	@	5(*)	7	5/7	5/7	5/7
Provide Feedback on Exercises.	6	6	6	1	1	1	6	6	6	5	5	5

table 8.3.2: Set II Headings - Case Matrix for Teacher C.

Group CC8L: Teacher C does not use the SMP series with this group. Teacher C says that the SMP booklets and books expect the students to have learned some 'basic Mathematics', which these student had not. Teacher C uses three series of 'remedial textbooks' in alternate lessons to produce some number work (number skills and place value) and some shape and space work with these students. During the observed lessons, two of these materials were being used. Apart from the game activity developed at the end of the second lesson with the whole group, all other activities were individualised and taken from the book being used during the lesson. Although the choice of material was quite different from the other groups observed, teacher C's behaviour was very similar: during the lesson, her main activities were either to mark students previous work or to help students facing difficulties. In contrast to the previous group, there was an opportunity to observe the teacher asking the group if there were students needing help. During the third lesson, when an auxiliary teacher was present, teacher C spent the lesson marking students' exercises, and commenting on them. She did not use the answer books from the adopted material with this group.

Group CC10M: Although this group also work through the SMP series in an individualised way, teacher C gives them common tests regularly. She manages to do this by telling the students in advance when the test is going to be and by assigning extra homework, in order that they can all reach a common chapter by test day. Teacher C uses the homework sheets developed by the school with this group as well. Teacher C does not use 'class teaching' with this group. They are supposed to 'keep going' with their work from the book, and teacher C

uses the lesson time to mark their previous exercises or to help the students requiring support. No crisis situations were observed in this group. The students who face difficulties called for the teacher and were able to carry on with their work after receiving some explanations. During the observed lessons, some students were already ahead of the teacher's proposed chapter for the test and some were behind. The teacher asked the former to revise their previous work at home and the others to do extra homework.

Student's Group →	CC8X			CC8L			CC10M			CC11T		
Lesson → / Set III: Use of Material to:	1	2	3	1	2	3	1	2	3	1	2	3
Source of Activities	7ab	7ab	7ab	7ab	7ab/1	7ab	7ab	7ab	7ab	7ab	7ab	7ab
Introduce a new topic.	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab
Introduce a lesson.	7ab	7ab	7ab	7ab	2(*)	7ab	7ab	7ab	7ab	7ab	7ab	7ab
Reference in classroom.	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab
Exemplification.	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab
Conclude a topic.	7ab	7ab	7ab	-	@	-	7ab	7ab	7ab	7ab	7ab	7ab
Promote links between topics.	7ab	7ab	7ab	-	-	-	7ab	7ab	7ab	7ab	7ab	7ab
Promote differentiation.	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab
Remedial material.	@	@	@	1	1	1	-	-	-	-	-	-
Source of homework.	@	8(*)	@	@	8(*)	@	@	8(*)	7	7	7	7
Provide Feedback on Exercises.	5	5	5	1	1	1	5	5	5	5	5	5

table 8.3.3: Set III Headings - Case Matrix for Teacher C.

Group CC11T: This group has already finished the SMP series they were using and are at the moment working through a series of revision sheets prepared by the school based on the textbook and on previous examinations. There are two series of revision sheets: yellow and red, to be used by students who did the corresponding track of SMP textbooks. The students work through the worksheets at their own pace, and homework is also set from these sheets, with time assigned by the teacher. During the observed lessons the teacher returned a series of exercises done the previous week 'under test conditions' to the students. The teacher gave the feedback on this test in the same fashion as with other feedback: she called one student at a time and commented on the test with him/her, giving the solutions to exercises and using the support of a school produced answer book. She also uses test results to suggest extra revision that each student should do.

Notice also that the (*) sign (characterising non-matched activities) had to be used for a lesson introduction with group **8L** (in fact, the only introduction given by teacher C observed), and for homework with all groups except **11T.** The (ab) sub-categorisation had to be used for all groups, characterising a predominance of the 'keep going' behaviour.

Having analysed the main features observed in each group, a comparative study on how written materials were used can be developed, using Conceptual Matrix graphs for choice and use of materials respectively, as presented in figures 8.3.1 and 8.3.2.

Analysis of the Ways Teacher C Chooses the Materials.

The overall look of figure 8.3.1 shows that there is a clear influence of the type of group of students on the way teacher C chooses the material: She uses the textbook for almost all activities with groups **8X** and **10M**, other printed materials for almost all activities with group **8L**, and written materials adapted from the textbook and the textbook with group **11T.**

The first row shows that teacher C rarely chooses to develop an activity without using materials. It was observed once, at the end of one lesson with group **8L**, when the teacher decided to play games involving table skills with the group. All other activities with this group were based on books designed as remedial materials, which were not related to the textbook series adopted by the school. This row also shows that students' activities during the observed lesson were taken from the textbook with groups **8X** and **10M**, and the activities for group **11T** were taken from adapted materials developed by the school, based mainly on the textbook series.

The second and the third rows show that the introduction of new topics and the introductions of lessons are left to the material chosen, which is also used as reference for classroom work and as a source of examples for all groups except **11T.** This group uses the textbook as reference and as a source of examples, not only in class but also at home, because the students are allowed to take any volume of the book home to use as reference for their revision for final exams.

'Conclusion of a topic' row shows that the material is also responsible for the conclusions and summary of topics within all groups, except **8L.** This group usually has its material changed to a different one, covering a completely different topic every two lessons, without any conclusion of a topic before the change is made. This choice of material reflects the teacher's belief that those students do not work well if kept for long periods of time on the same kind of activities. The 'Links' row also shows that the teacher has a different behaviour with this group: while all other groups use the same source for all activities, so that links are established by the material, group **8L** uses different sources, and no links are established by the teacher or by the materials chosen.

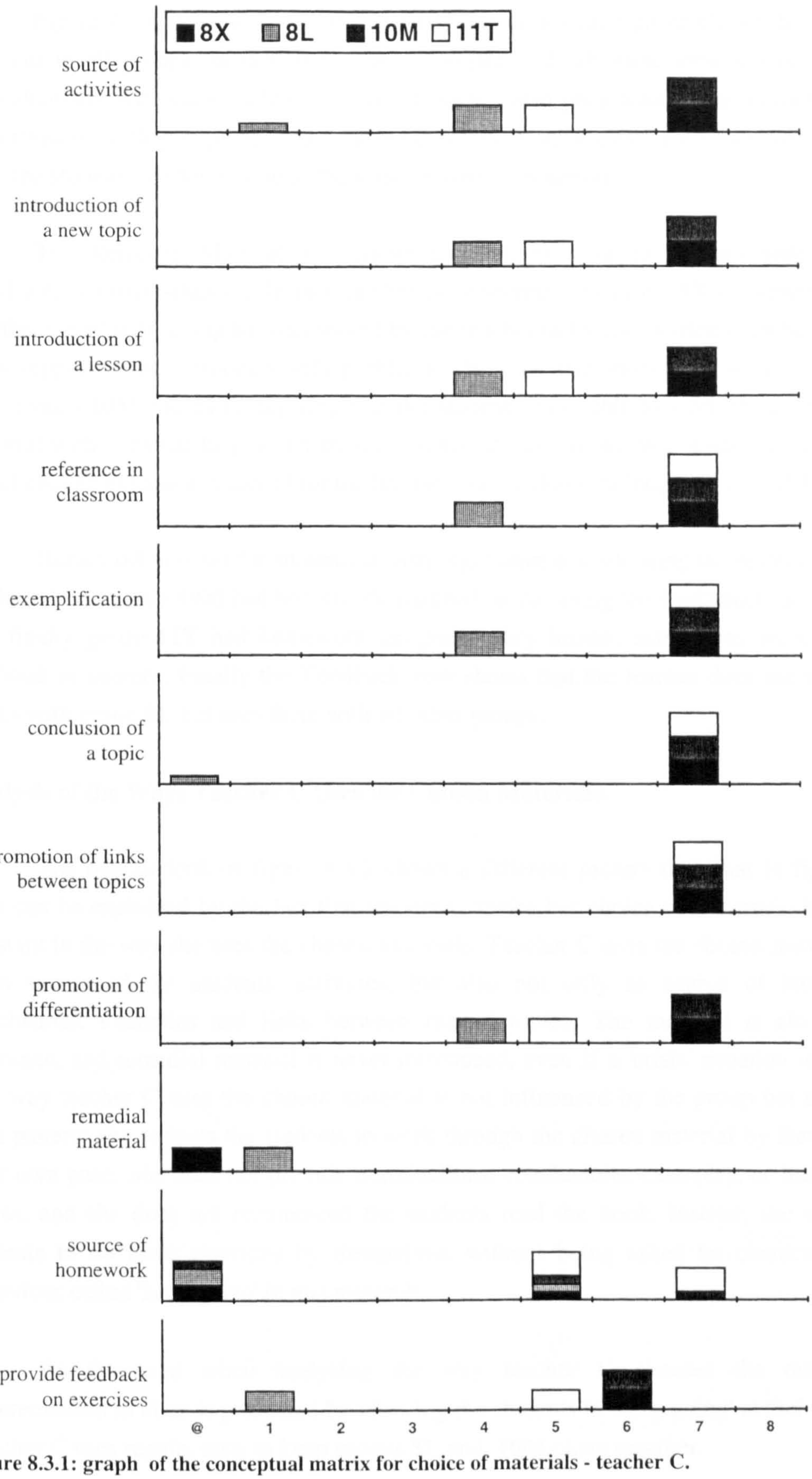


figure 8.3.1: graph of the conceptual matrix for choice of materials - teacher C.

Figure 8.3.1 also shows that the material chosen for each group allows differentiation to occur in all groups. In fact, the work developed with all these groups was completely individualised, with each student working at his/her own pace through the chosen material. Nevertheless, with groups **8L** and **10T** some teaching strategies were observed in order to keep the students working through the same section of material.

The 'Remedial Material' row shows that teacher C never changes materials when faced with a 'crisis' situation. In fact, the 'crisis' observed with group **8X** remained unsolved, and the 'crises' with group **8L** was solved by the teacher or by the auxiliary teacher by giving extra support to those students with problems. This row also shows that no 'crisis' occurred with groups **10M** and **11T**, implying that the students were able to work though the chosen material with eventual help given by the teacher. It also shows that teacher C had planned ahead enough extension material for the brighter (or quicker) students in group **11T**.

Homework was set for students in year eight once a week using the material produced by the school; group **10M** had homework assigned twice: using the worksheets and textbook; and finally group **11T** had homework assigned every lesson, using both worksheets and textbook as sources. Finally the 'Feedback' row shows that the teacher does not use answer books with group **8L** but uses them with all other groups.

Analysis of the Ways Teacher C Uses the Chosen Materials.

The overall look of figure 8.3.2 shows a different picture than that in figure 8.3.1. This can be explained by the fact that teacher C varies her choice of materials, but is more constant in the way she uses the chosen materials. Teacher C uses the chosen material as the main source of the students' activities, but also not only as source of introductions, conclusions, examples and links between related topics. The material is also used for reference, and remedial material is never introduced, even if a 'crisis' situation is observed. The way teacher C uses the chosen material is not influenced by the group but follows her own pattern: she expects the students to work through the chosen material by themselves, at their own pace. She does not provide introductions, conclusions, examples, or links between topics, and she does not recommend the students read the book. Instead, she expects the students to do these activities by themselves, without being asked to, characterising the behaviour called 'keep going' in this research.

As discussed when analysing the way teacher C chooses the material, the differentiation in class is promoted by allowing the students to 'keep going' at their own pace. Teacher C uses regular tests to keep groups **8L** and **10M** close together.

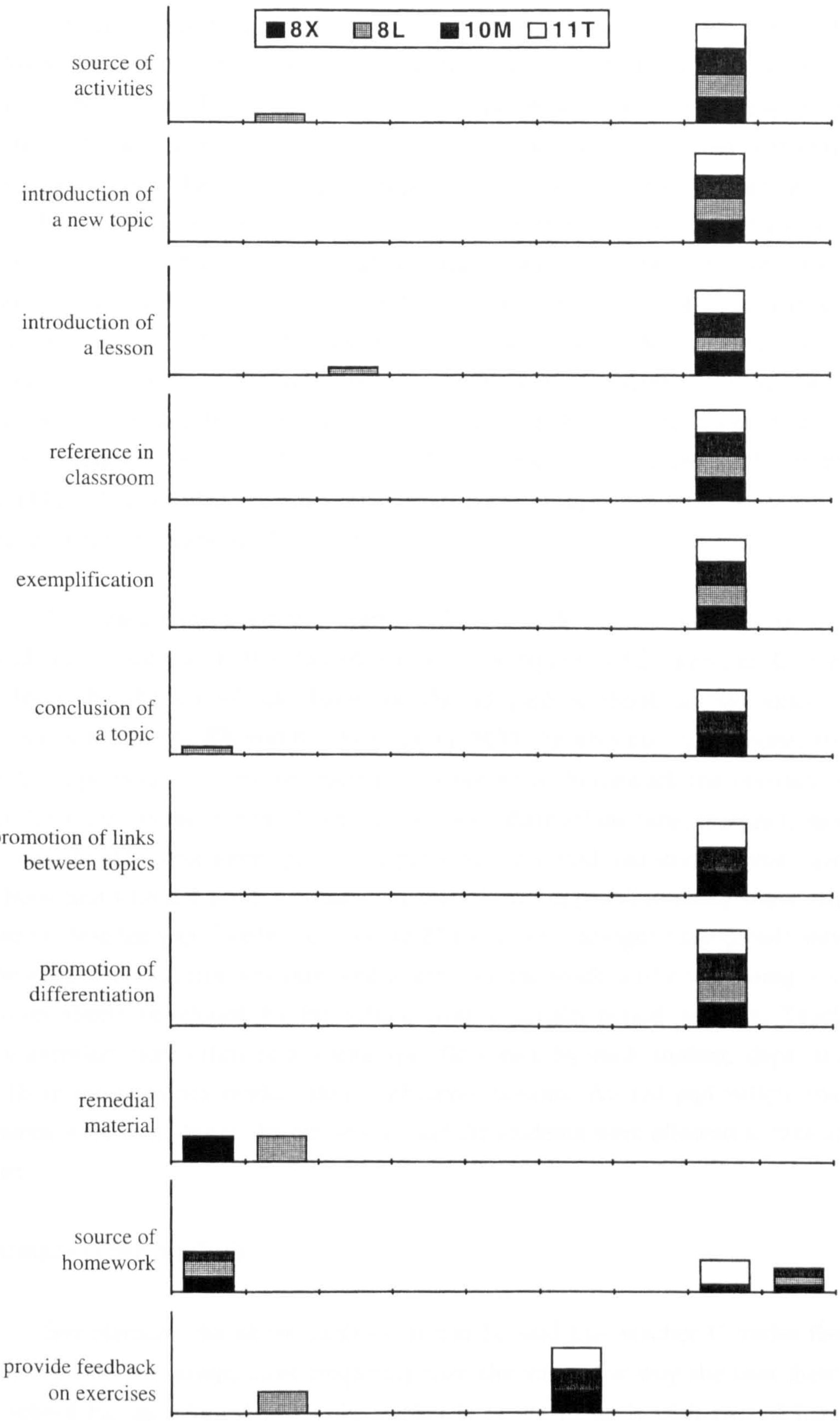


figure 8.3.2: graph of the conceptual matrix for use of materials - teacher C.

The general pattern of behaviour observed also includes the way teacher C provides feedback for students: she uses the time of the lesson to call one student at a time and then marks his/her notebook, usually with the aid of the answer book (except for group **8L**). She interrupts this activity whenever one student asks for her support, coming back to the marking activity afterwards. This way of providing feedback means that some students can stay for more than a week without receiving any support from the teacher. This situation was observed in group **8X** (see 'remedial material' row), where two students who were not progressing through the activities and did not ask for help remained in an undetected 'crisis' for the whole observed week, because they were not called by the teacher to have their work checked. Nevertheless, it has to be said here that these two students were the only exceptions found to a general pattern of students working hard through the material and asking for support whenever they needed it. In fact no 'crisis' situations were observed with groups **10M** and **11T**, indicating that the students in both these groups were able to work through the material in the way expected by the teacher.

The feature observed that seems to be more dependent on the type of the group considered is shown in the 'homework' row in figure 8.3.2. Teacher C uses material developed by the school and based on the adopted textbook as the unique source of homework for groups **8X** and **8L**. With group **10M** she also uses these materials, but there was an opportunity to observe teacher C assigning as homework the revision of previous work for a test. As the students keep the book with them all the time, teacher C recommended that they should revise some specific chapters. She also said that students that were behind in the book and who did not finish these chapters were supposed to make some extra effort at home to close the gap. Finally, with group **11T** teacher C assigned completely individualised homework. Each student was supposed to carry on the work he/she was doing in class, using revision sheets developed by the school over a certain period of time. Teacher C also recommended more effort in revising specific topics by each student, depending on their results in the exercises marked during observed lessons. All red and yellow track of SMP volumes were available in the classroom, and the students were allowed to take any of them home.

Summary of the Analysis:

Summarising the above analysis, it can be said that teacher C varies the source of materials with her groups more frequently than she varies the way she uses them. Although the school has an adopted textbook, teacher C is not using it with two of her groups for different reasons: one group (**11T**) had already finished all the volumes of the textbook, and is revising for final exams and the other group (**8L**) is considered by the teacher to be 'too

weak' to use even the remedial series associated with the adopted textbook. The way teacher C uses the material follows a general pattern: she expects the students to progress individually through the given material ('keep going') whilst providing support by marking and commenting on their exercises and being available throughout the lesson for those students asking for her support. It was also noticed that teacher C does not change the chosen material when faced with a crisis.

The school has a series of material to be used as homework. For each group, the material given for homework is the same, which implies that it does not match with class activities for the majority of the students, because they are working though different activities. Students in group **10M** had this homework complemented by an individualised one: each student was supposed to do his own work in order to prepare for a test. Students in group **11M** do not use these homework sheets anymore. Instead, they use a series of worksheets designed by the school based on the textbook and on previous examinations as a source for class and homework.

8.4. Analysis of Data Concerning Teacher D.

Teacher D is an experienced teacher who has worked for twenty years in school D, a comprehensive secondary school in the Midlands, UK. Teacher D completed her undergraduate course in English about 25 years ago, complemented by a PGCE course. When asked how she became a Mathematics teacher , she says:

> It happens as soon as I got my first job... the school was missing a maths teacher and I had been a good student in maths... A in my A levels... so the asked me if I could give some maths lessons... and here I am... 25 years as a Mathematics teacher...

School D also runs a sixth form, but teacher D says she never teaches these students. She explains:

> The school has teachers that are maths specialists, so they are better qualified than I am to teach the highest levels. I usually teach years seven, eight, and nine... and the last group in year ten.

School D adopts the SMP series as textbook. The school has no written school scheme and uses the textbook as basis for progression of contents. Nevertheless, there are complementary materials available in the school. Teacher D says:

> We use some complementary printed material... mainly for investigations... most of these materials came from in-service courses... We also have a particular system of homework: the department prepared a series of worksheets, based on the SMP series. We give these homework sheets weekly for the students throughout the course... so, since year seven the students get used to it... there are also some supplementary materials for the students to revise for their GCSE exams, but I do not have much contact with this material.

When asked about the school policy on reference material, teacher D explains that students in year seven and eight are not allowed to take their booklets home. She also says that whenever supplementary material is provided (usually as photocopies of printed materials or adapted materials) students are allowed to keep the copies. From year nine upwards students are allowed to keep their copies of the books, except for those students in the 'low attainers' group 'because they usually lose the material'.

Teacher D says that the school is mixed ability in year seven. In year eight, the groups are still mixed ability, but students with learning difficulties form a group that do not use SMP booklets, but use other materials instead. Teacher D says the teacher must decide what to use, with the head of department usually giving her approval. From year nine upwards students are set in ability groups. Teacher D says that usually each of the four groups withineach year uses one of the four SMP tracks.

Teacher D says that the assessment system is also based on the one proposed by SMP, with complementations. She comments:

> Yes, we use the SMP assessment sheets, and the tests provided by the series... we believe that as these students are going to do their GCSE exam based on the book, it is useful that they get used to the system... but we also assess the students using our own tests... especially if we are complementing the material.

The summary of the general decisions on written materials for the school year is presented in table 8.4.1. This table is based on summary data presented in table 4d.1, in appendix 4d. During a week, all three lessons given by teacher D to the groups DD7X, one of the five year seven mixed ability groups; DD10L, the fifth out of five groups in year ten; DD9M, the second out of five year nine group; and DD9T, the first group in year nine were observed. In the following section, a comparative study among the four groups of students during these lessons will be developed, considering the headings in sets II and III.

Set I - Headings: General Decisions **Students' Group→**	**DD7X**	**DD10L**	**DD9M**	**DD9T**
Main Source of Materials for Class Work	7	6	7	7
Progression of the Content	7	6	7	7
Materials given to students for reference	2	1	8	8

table 8.4.1 - Case Matrix for the first set of headings (general decisions) - Teacher D.

Comparative Study of the Headings in the Second and Third Sets for Teacher 's Four Groups of Students:

Tables 8.4.2 and 8.4.3 are summarised versions of the Case Matrices for teacher D, each one concerns one set of headings, and the data used to produce them are summarised in Appendix 4d. Based on these tables, an overview of the choice and usage of written materials by teacher D with each group of students is presented. As there were lessons when two behaviours under the same headings were observed, two codes can appear in the same table's cell.

Group DD7X: Teacher D commented that her previous experience with the SMP booklet on ratio did not work well. She has decided to produce a 'booklet' herself and during the first two observed lessons she was using this material with the students. Teacher D also says that this material is not an invented one, but one adapted from several sources. Teacher D introduced the first lesson carefully, giving examples and explanations about the concept of ratio. After this first introduction, the students were supposed to 'keep going' with their work up to the end of the booklet. Teacher D used class teaching once more at the end of the first lesson, when she asked students for their answers to the first two series of exercises from the 'own produced' material. Another moment of class teaching was observed at the beginning of the

second lesson, when the teacher discussed students' answers on the previous homework assigned. The students who finished the material on ratio were assigned some work taken from SMP booklets. These pieces of work were completely individualised, with each student working through a different booklet (at his/her own pace) and some even working through a piece of assessment taken from the SMP series. Homework was assigned by the teacher once: it was one of the worksheets she described as produced by the school and based on the textbook. As these worksheets follow a sequence and the students work on an individual basis, the homework did not match the class work.

Student's Group →	DD7X			DD10L			DD9M			DD9T		
Lesson → Set II:Choice of Material to:	1	2	3	1	2	3	1	2	3	1	2	3
Source of Activities	3	3ab/7ab	3ab/7ab	6ab	6ab	6ab	7ab	7ab	7ab	7ab	7ab	7ab
Introduce a new topic.	3	-	7ab	6ab	6ab	6ab	7ab	7ab	7ab	7ab	7ab	7ab
Introduce a lesson.	3	3ab	7ab	6ab	6ab	6ab	7ab	7ab	7ab	7ab	7ab	7ab
Reference in classroom.	3	3/7ab	7ab	6ab	6ab	6ab	7ab	7ab	7ab	7ab	7ab	7ab
Exemplification.	3	3ab	7ab	6ab	6ab	6ab	7ab	7ab	7ab	7ab	7ab	7ab
Conclude a topic.	-	7ab	7ab	6ab	6ab	6ab	7ab	7ab	7ab	7ab	7ab	7ab
Links between topics.	-	7ab	7ab	6ab	6ab	6ab	7ab	7ab	7ab	7ab	7ab	7ab
Promote differentiation.	@	@/7ab	7ab	6	6	6	7ab	7ab	7ab	7ab	7ab	7ab
Remedial material.	1	1	1	1	1	1	-	-	-	-	-	-
Source of homework.	@	5(*)	@	@	5(*)	@	@	@	5	7	7/@	@
Feedback on Exercises.	3	3ab/7ab	3ab/7ab	7	7/1	7	7	7	7	7/7a	7/7a	7/5

table 8.4.2: Set II Headings - Case Matrix for Teacher D.

Group DD10L: Teacher D works with these students in an individualised way. Ten students are using the SMP series green track, and at the time of observation were all working through book G2. The remaining four students in the group were considered by the teacher to be at a lower level than the others, and were using a series of books designed to reinforce number skills instead. Teacher D also has the help of an auxiliary teacher with these students. The homework is set for this group in the same way as described above, with group 7X. Teacher D says that she gives these students' worksheets for homework that are usually given for students in year eight. The only opportunity to observe class teaching was at the introduction to the second observed lesson, when the teacher commented on students' answers to the previous homework sheet. All other activities taking place during lessons were individualised, including assessment: there was an opportunity to observe one student reaching the end of book G2 and being asked to start a test on it. Teacher D says that depending on her result, she will be assigned either the next book in the green track or

revision work. Teacher D does not change materials when noticing a student with difficulties, offering personal support instead.

Group DD9M: Although this is considered a 'medium' group, it is the second out of five groups in year nine. They are working through the SMP red track and are presently using book R1. Teacher D says that there are three students in this group who started the school year in the 'top' group, but were facing several difficulties using the yellow track, and were moved to this group. She also says that they are probably the best students in the group. Although these students also work in an individualised way, teacher D establishes a 'minimum' point by setting dates for assessments. During the observed lessons she set an assessment for the following week that included the first six chapters of the book. Some students were still working on chapter five, and she told them that she expected them to do extra work at home, so that they could perform in the test. Homework was assigned using school worksheets, although this time it was matched with revision work the students were expected to do for the test.

Student's Group →	DD7X			DD10L			DD9M			DD9T		
Lesson → / Set III: Use of Material to:	1	2	3	1	2	3	1	2	3	1	2	3
Source of Activities	8	8/7ab	8/7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab
Introduce a new topic.	8	-	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab
Introduce a lesson.	8	8	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab
Reference in classroom.	8	8/7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab
Exemplification.	8	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab
Conclude a topic.	-	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab
Links between topics.	-	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab
Promote differentiation.	@	@/7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab
Remedial material.	1	1	1	1	1	1	-	-	-	-	-	-
Source of homework.	@	8(*)	@	@	8(*)	@	@	@	8	6	6/@	@
Feedback on Exercises.	8	7ab	7ab	5	5/1	5	5	5	5	7/7a	7/7a	7/5

table 8.4.3: Set III Headings - Case Matrix for Teacher D.

Group DD9T: This is the 'top' group in year nine, and the only one using the book's yellow track. Differently from the other groups observed, this group as a whole was working on the same chapter of book Y2, except for two students (working as a pair) who were using book Y1. Teacher D says they both had to stay out of school for a period of time, due to health problems, and since then have been working at a different pace from the other students in the group. As in the other groups observed, teacher D also asked these students to 'keep going' with their work and used the lesson time to check on their work. The behaviours that were

different from the other groups observed were: (a) it was observed several times that the teacher interrupted students work to provide feedback, by reading answers to exercises from the answer book (once this activity was complemented by discussion of the students' answer with the whole group); and (b) instead of using the worksheets developed by the school as source of homework, the teacher assigned the conclusion of the chapter, including the revision section, except for those students working through book Y1, who were assigned 'half hour of maths work'.

Having analysed the main features observed in each group, a comparative study on how written materials were used is developed, using the Conceptual Matrix graphs for choice and use of materials respectively, as presented in figures 8.4.1 and 8.4.2.

Analysis of the Ways Teacher D Chooses the Materials.

The overall look of figure 8.4.1 shows that teacher D bases her teaching mainly on the textbook series adopted by the school. In almost all 'rows' of the table, the predominance of behaviours '7' shows that the material is used in close adherence, although not necessarily as suggested by the guides. Other strategies when choosing materials are observed with group **7X** and group **10L**, when the teacher introduced 'own produced' material and used a different textbook for some of the students in the group

The first 'row' shows that the textbook was used as a main source of activities with three out of four groups during observed lessons. In the case of group **7X,** the textbook was also used, although for two lessons the teacher used another material instead. The strategy of adopting more than one series of books in group **10L** is also shown in this row.

Introductions of topics (and also introductions of lessons) were observed during all lessons with almost all groups not because the teacher was doing it, but because each student initiated a new chapter in the book at his/her own pace. The only group for which the introductions were not left to the book was **7X**, during the first lesson.

By way of summary, it can be said that introductions, examples, references, conclusions, and links are all taken from the book in groups **9M** and **9T.** Groups **7X** and **10L** are also taught in the same way, but another textbook is being used in parallel with some students in the former group while the first had the textbook exchanged for another material for the development of a specific topic (in fact, the teacher said that she had used the SMP booklet before for this topic and was not satisfied with the final results).

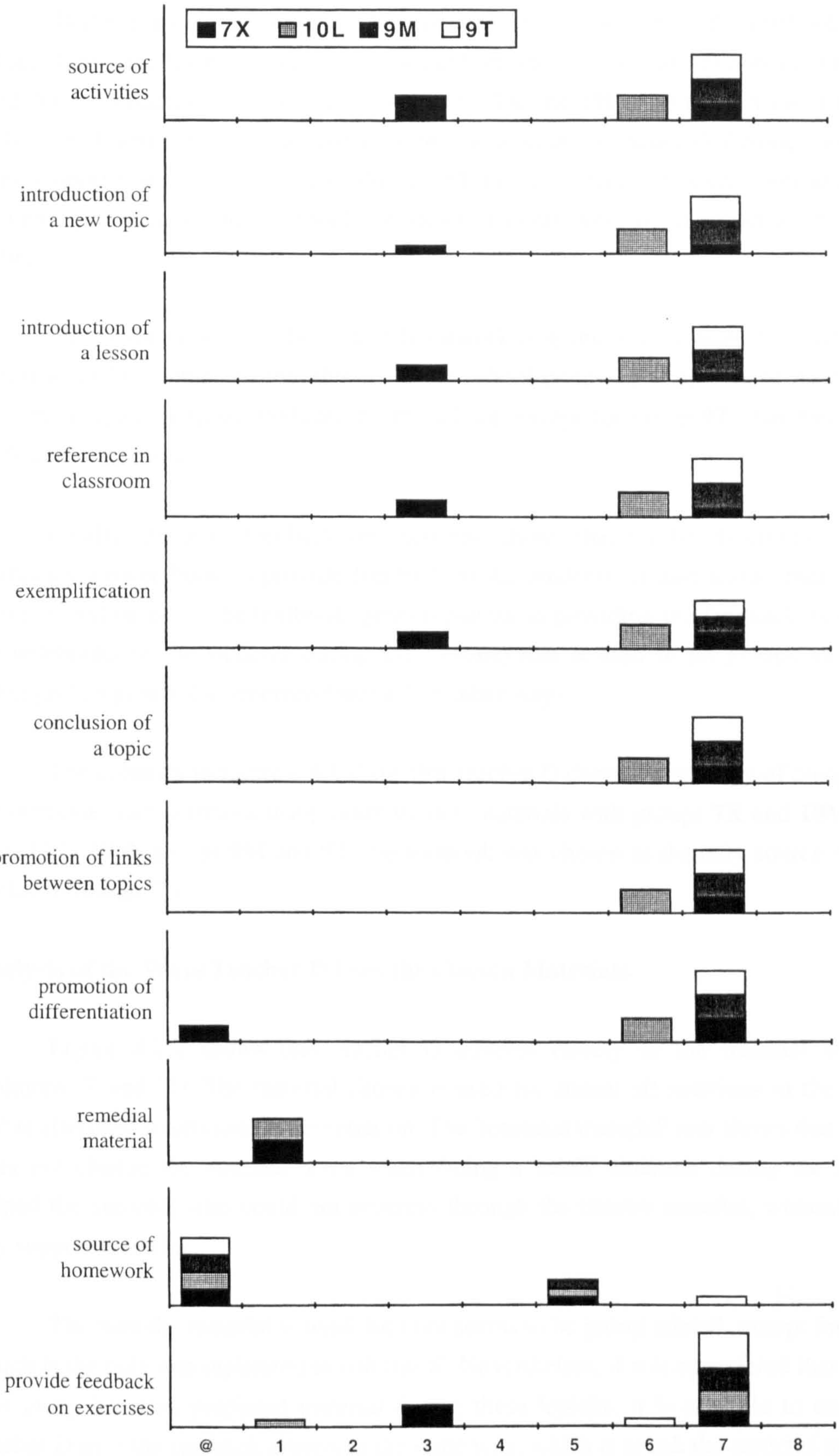


figure 8.4.1: graph of the conceptual matrix for choice of materials - teacher D.

Differentiation also seems to be a feature depending on how the textbook was used. In fact, during the lessons when it was not used, differentiation was not noticed. On the other hand, 'crisis' situations were observed in groups **7X** and **10L**, and in both cases the teacher deals with it without changing materials for those students facing difficulties, but offering extra support instead. With groups **9M** and **9T** no crises were observed, indicating that the activities proposed by the textbook for those students were more suited to their level of ability.

The 'homework' row shows that homework is given once a week for each group (in fact, teacher D commented that this is part of school policy). For all groups homework was set from adapted material produced by the school, except for group **9T**, that had homework set from the textbook.

Finally, the row 'feedback on exercises' shows that teacher D chooses to use the textbook's answer book to provide feedback to the students. It also shows that, apart from activities not based on the textbook, general pattern in providing the feedback exists (correct the notebooks of the students during the lessons) that is used in all groups including **9T**, although this group also received feedback in other ways.

The columns in figure 8.4.1 show that teacher D chose the strategy of complementing the textbook with activities using other written materials with groups **7X** and **10M** (columns '3' and '6'). With groups **9M** and **9T**, the textbook was chosen as the only source of activities in class (column '7').

Analysis of the Ways Teacher D Uses the Chosen Materials.

Figure 8.2.2 shows that teacher D adheres closely to the material she chooses (columns '7' and '8'). The material chosen is used for almost all activities in the classroom, and is also used to promote differentiation. The 'remedial material' row shows that the teacher does not change the material, even when facing a 'crisis' situation during the lesson. She helped the students who could not progress through the chosen material, without the aid of any support material.

The way the material is used does not seems to be group related, except for group **7X**, which is the only one registered in column '8'. Nevertheless, if it is considered that the teacher was using her own produced material during these lessons, it is possible to conclude that teacher D uses the textbook always in the same way, which is to ask the students to follow the book's sequence, at their own pace.

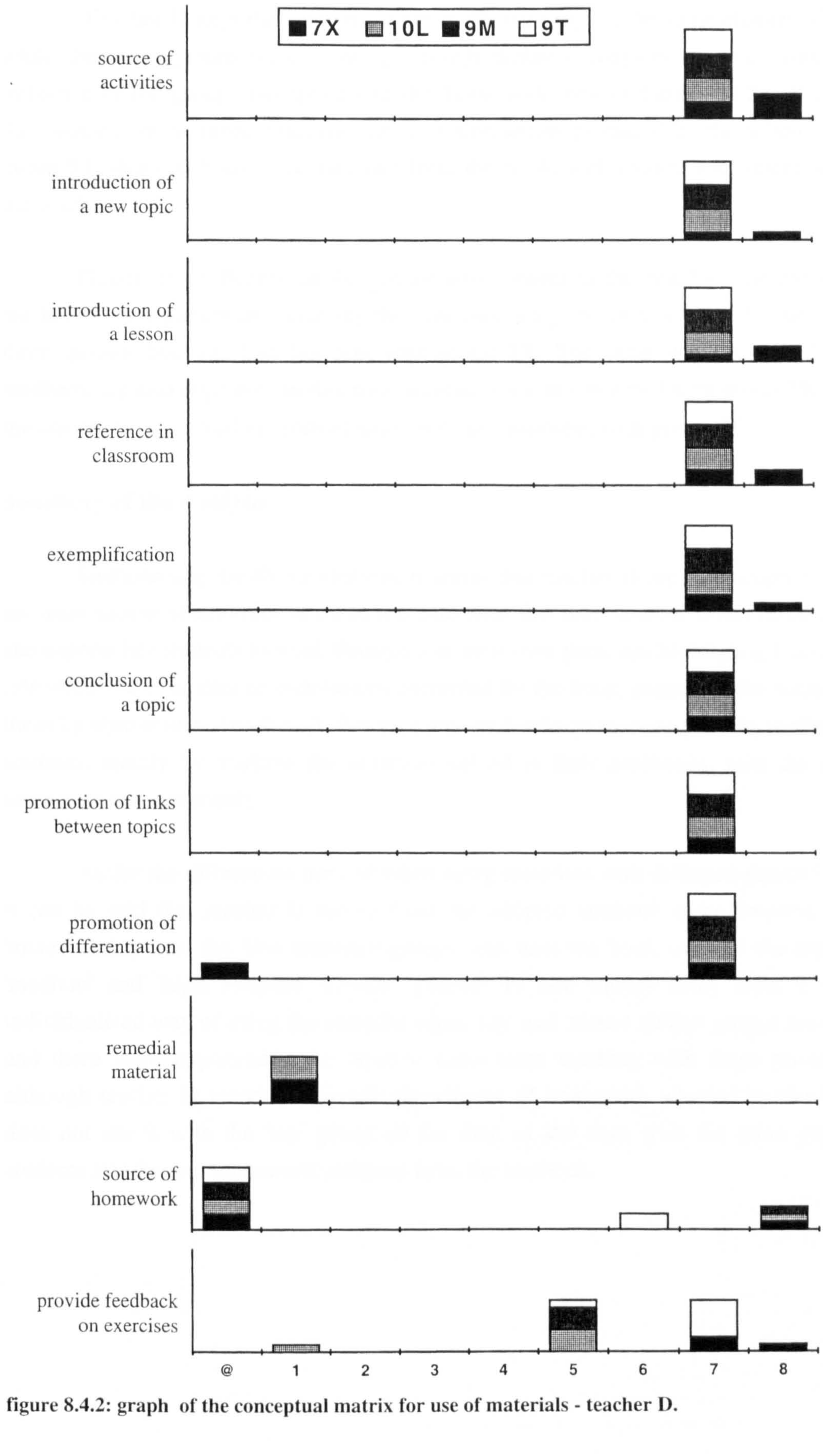


figure 8.4.2: graph of the conceptual matrix for use of materials - teacher D.

Teacher D kept the students in group **9T** working on the same chapter of the book, while the other groups were working through different chapters (or even volumes). The influence of the group also appears in the 'homework' row of figure 8.4.2.: all groups had their homework assigned from the series of worksheets produced by the school, except for group **9T**, that had homework assigned from the book, with evidence of selection made by the teacher.

Finally, the influence of the groups also appears in the 'feedback on exercises' row: teacher D has a pattern of correcting the exercises using the answer book that was noticed in three groups, but was less frequent with group **7X**. The other ways teacher D provided feedback are also registered in this row: without using any material with group **10L** and using the answer book to read the correct answers to the questions with group **9T**.

Summary of the Analysis

Summarising the above analysis, it seems that teacher D uses the adopted textbook as the main source of activities in class. It is also clear that once teacher D has chosen a material she expects her students to work through it at their own pace, not highlightig features such as introductions, examples or conclusions presented by the book, expecting the students to read them by themselves. Teacher D also uses answer books as tools to provide feedback for her students, mainly by marking the exercises solved in their notebooks, with the addition of some personal comments.

As for the differences noticed when using materials with different groups of students, it can be said that teacher D moves from the adopted textbook more frequently with the 'mixed ability' and the 'low attainers' groups, and uses the book most of the time with the 'medium' and 'high attainers' groups. Teacher D also moves away from a completely individualised way of using the material when 'top' and 'mixed ability' groups are concerned, and there were opportunities to observe some class teaching with these groups. Finally, although teacher D seems to approve the scheme of homework adopted by the school, she does not use it with the 'top' group all the time as she does with the other groups, these students also having homework assigned from the textbook.

8.5. Analysis of Data Concerning Teacher E.

Mr. E is a teacher with a particularly extensive knowledge of published materials working in school E, a comprehensive secondary school in the Midlands, UK. Teacher E completed a Science Degree about 14 years ago and has been working as a Mathematics teacher in school E ever since. By now, teacher E is the head of the Mathematics Department, but says that he mostly gave continuity to the policies decided by his predecessor.

Although School E does not have an adopted textbook, teacher E says that for two years they have been using the textbook series 'Tasks Maths' with the 'top' group in years nine, ten and eleven. Other groups do not use textbooks and have their progression based on the School Scheme of Work. The School Scheme has been in use for almost eight years, but teacher E says it has been revised and modified constantly. The School Scheme is organised by school years in a manner unique among the schools in the sample: (a) the four groups in year seven are all mixed ability and spend the first year at the school working only through investigations; (b) to 'compensate' for year seven, the four groups in year eight (which are set as two 'top half' groups and two 'bottom half' groups) have the emphasis of their maths work changed to 'reinforcement of skills and understanding of fundamental areas of mathematics'; (c) from year nine upwards students are divided into four groups according to levels of attainment and the 'top' group' starts to use textbooks, while the others have their progression of contents decided by the school scheme, which does not suggest materials for the activities, except for investigations and course work.

When asked who decides what material should be used for these groups in years eight and upwards, teacher E says:

> Well... theoretically speaking, the teacher should be the one to decide... in practice...well... we have a huge file with materials... most of them printed materials or photocopies of sections of books... some adaptations as well... nothing really original... these materials can be used as bases for teaching the topics listed in the school scheme... by now we know the materials well enough... what we choose depends on the group, of course... but this file is not closed... and it never will be... new materials are always being added to this file... I suppose you can say that there is this set of materials and each teacher can decide which one to use, depending on the group.

When asked why the students are 'mixed ability' for the investigations in year seven and set after that teacher E comments:

> Well, the emphasis on investigations is not ours... it is a N.C.'s. What is really our decision is to do it in mixed ability groups. We decided to do this because the 'low attainers' did not take much from the investigations in a set group... the interaction with the brighter students give them <u>ideas</u> and they can learn a lot by developing these ideas... in a group set by attainment what happens is that the ideas never come

> out... and the 'low attainers' spend the investigation lessons working through very basic mathematics... and it feels like a waste of time really... unless the teacher provides lots of directions... so, we prepare our students for Key Stage Three starting by investigations, and after that we emphasise the skills and the mathematics concepts... in years eight and nine.

Teacher E says that the textbook is kept by students at all times, while the other students keep their notebooks and a copy of the worksheets they work through, except the 'bottom' group that never takes any material home 'because experience has shown they do not bring it back'. He also comments:

> I do not think this policy is fair with the low attainers... I think they would benefit from having homework assigned... but it has to be in a worksheet format, so they do not have to take their notebooks home... This year I meant to try one experience with them: to assign non obligatory homework from worksheets... but I did not... it is one of those things that requires some extra time planning, and I just did not have the time to do it... As you can see, this is one of the situations where the pressures of everyday teaching and the constrains of real situations interferes in what one thinks should be done...

Teacher E positions himself as a 'non-textbook user'. He says that most textbooks 'keep the students busy, but do not enhance reflectiveness'. When asked why textbooks are used with the 'top' group in each school year, teacher E says that the textbook was carefully chosen. He believes that this particular textbook 'reflects the experience of teachers who were trying to create a real learning environment in their classroom. He also comments that the experience with the textbook for the 'top' group has been a very positive one. Teacher E says:

> It is because it helps them in every sense... we started the experience two years ago, and we could feel the difference in the GCSE exams results straight away. I believe that the brighter students benefit from having a reference material, so they can carry on with their work if they feel like it... the way the book is organised also helps... extension material is always offered to the brighter students... I mean, if you do not use a textbook, you sometimes do not have any activities prepared for those students...

Teacher E also admits that although the school scheme is the same for all groups in the same school year, in practice different groups have different courses. He says:

> Apart from the groups in year seven, we use different materials with different groups... for example... the school scheme for year ten includes the topic 'planning'... well, this can have different interpretations for different groups of students... it can mean plan for a holiday or plan your savings to achieve a certain target in a certain period of time... anything really... for my year ten 'bottom group' it will mean plan the hours of your day in order to achieve two or three different targets... I will keep it simple...

Table 8.5.1, the Case Matrix for the headings in general decisions (set I) taken by teacher E, summarised the introduction above, and is based on table 4e.1 in Appendix 4e. During a week, all three mathematics lessons involving the following groups were observed: group EE7X, one of the four mixed ability groups in year seven; group EE10L, the fourth out of four groups in year ten; group EE9M, the third out of four groups in year nine; and group EE11T, the first out of four groups in year 11. In the following section, a comparative study of the four groups of students during these lessons will be developed, considering the headings in sets II and III.

Set I - Headings: General Decisions **Students' Group→**	**EE7X**	**EE10L**	**EE9M**	**EE11T**
Main Source of Materials for Class Work	4	4	4	7
Progression of the Content	4	4	4	7
Materials given to students for reference	4	@	4	8

table 8.5.1 - Case Matrix for the first set of headings (general decisions) - Teacher E.

Comparative Study of the Headings in the Second and Third Sets for Teacher E's Four Groups of Students:

Tables 8.5.2 and 8.5.3 are the summarised versions of the Case Matrices for teacher E, each one concerning one set of headings. Based on these tables, an overview of the choice and usage of written materials by teacher E with each group of students is presented. As there were lessons when two behaviours under the same headings were observed, two codes can appear in the table's cell. They were developed based upon the tables of summarised data presented in Appendix 4e.

Group EE7X: As stated in the introduction, this group of students works through investigations throughout the year. During the observed lessons the students were working through an investigation called 1, 2, 3, 4, for which the final objective is to write all numbers between one and fifty as an expression using the four operations and combinations of 1, 2, 3 and 4 (for example, 22 can be written as (43+1)÷2). The teacher proposed the activity as 'open-ended', and the students were creating expressions without having an aim during the first two lessons. The teacher proposed the aim at the beginning of the third lesson, and the worksheet was given to the students so they could record their answers. The teacher says these results will be used in an investigation using computers. Teacher E introduced all the lessons, and also established links with previous work done in expressions, reminding the students the rules about the order of operations. Teacher E used no material to provide feedback to the students. Homework was assigned once, and was related to class work.

Student's Group →	EE7X			EE10L			EE9M			EE11T		
Lesson →	1	2	3	1	2	3	1	2	3	1	2	3
Set II: Choice of Material to:												
Source of Activities	4	4	4	4	4	4b	4b	3	4	7	7ab	7ab
Introduce a new topic.	4	-	-	@	-	-	3	-	4	7ab	-	-
Introduce a lesson.	4	4	4	4	4	4b	3/4b	3	4	7ab	7ab	7ab
Reference in classroom.	1	1	1	4	4	4	1	3	1	7ab	7ab	7ab
Exemplification.	4	4	4	4	4	4	4	@	3	7ab	7ab	7ab
Conclude a topic.	-	-	@	@	@	@	-	3	-	7ab	7ab	7ab
Promote links between topics.	1	1	1	-	-	-	1	3	-	7ab	7ab	7ab
Promote differentiation.	@	@	@	@	@	4b	@	@	@	7ab	7ab	7ab
Remedial material.	-	-	-	1	1	4b	1	1	-	-	-	-
Source of homework.	@	4	@	@	@	@	3	@	3	7	7	7
Provide Feedback on Exercises.	1	1	1	1	1	1	1	1	1	1/7b	1/7b	1/7b

table 8.5.2: Set II Headings - Case Matrix for Teacher E.

Group EE10L: The main characteristics observed with this group of students was the attention given by the teacher to introducig a new piece of material in each lesson. Teacher E started the topic 'planning time' by developing an activity involving timetables with the whole group, using a 'remedial textbook'. Although the activity was proposed for all students, they were allowed to work at their own pace and the teacher introduced differentiation himself, by verifying what students were able to do, suggesting alternative routes through the material and providing personal support. The teacher started the second lesson by giving students other material on time planning. During this lesson he promoted differentiation in a similar way. Finally, the teacher started the third lesson by giving students a choice between finishing the previous material or starting a new one, presenting a different situation that also required time planning. This group of students had no homework assigned during the observed lessons and the students were never allowed to take their notebooks home.

Group EE9M: When using material with this group of students, teacher E was giving them instructions different to those given by the material itself (first lesson) or asking the students to start from a different point than the initial point of the material (second lesson). These methods of using material generated several 'crisis' situations during lessons, as several students were following the material instructions instead of those given by the teacher. Teacher E gave these students a series of activities on reflections of shapes using coordinates. During the first and second lessons observed, teacher E tried to move the students from the use of concrete materials (mirrors) when doing reflections to the use of the relationship between the initial and final coordinates when a reflection was done. The adapted material developed by the school reinforced that point as well as the suggestions made by the teacher

to the students. Nevertheless, teacher E did not conclude or summarise the results of the topic for the whole group, but insisted that each student reach the conclusions by him/herself, suggesting some extra work at home for those students who did not conclude the topic.

Student's Group →	EE7X			EE10L			EE9M			EE11T		
Lesson → / Set III: Use of Material to:	1	2	3	1	2	3	1	2	3	1	2	3
Source of Activities	4	4	7	7ab	7ab	7ab	2	3	4	7	7ab	7ab
Introduce a new topic.	4	-	-	@	-	-	3	-	4	7ab	-	-
Introduce a lesson.	4	4	7	7ab	7ab	7ab	4/3	3	4	7ab	7ab	7ab
Reference in classroom.	1	1	1	7ab	7ab	7ab	1	3	1	7ab	7ab	7ab
Exemplification.	4	4	5	7ab	7ab	7ab	4	@	4	7ab	7ab	7ab
Conclude a topic.	-	-	@	@	@	@	-	6	-	7ab	7ab	7ab
Promote links between topics.	1	1	1	-	-	-	1	5	-	7ab	7ab	7ab
Promote differentiation.	@	@	@	@	@	7ab	@	@	@	7ab	7ab	7ab
Remedial material.	-	-	-	1	1	7ab	1	1	-	-	-	-
Source of homework.	@	4	@	@	@	@	4	@	4	7	7	7
Provide Feedback on Exercises.	1	1	1	1	1	1	1	1	1	1/7ab	1/7ab	1/7ab

table 8.5.3: Set III Headings - Case Matrix for Teacher E.

Group EE11T: This is the only group taught by teacher E that uses a textbook. The students are kept on the same chapter of the book and differentiation occurs because each student ends the chapter at a different point. Teacher E encourages the students to interact directly with the book, and the behaviour characterised as 'keep going' was observed during the lessons. It was also observed that teacher E asked one of the students who was ahead in the proposed tasks to help other students who were behind. Teacher E provided feedback and support during the lesson to the students and no crisis situations were observed. Notice also that this group had homework assigned every lesson, in a completely different way from the other groups: teacher E proposed that they should work for one hour on their maths in between lessons, but did not propose a specific task, letting the students decide what was better for them to do at home (either carry on with the class work or to revise for their GCSE exams). It was also observed that teacher E was providing extra support for those students wishing to improve their examination grades by offering revision lessons at lunch time for those willing to participate.

To complete the analysis of tables 8.5.2 and 8.5.3 it is necessary to talk about the 'matching questions' and the sub-categorisations: it can be observed that all activities developed in class and also the homework were matched with the main activity for all groups taught by teacher E. The tables show that different ways of using materials were observed

during lessons with groups **10L** and **9M**: the teacher offering the students a choice between two materials in the first group and using the material given to the students for different purposes with the second, generating some crisis situations with this behaviour. It is also notable that the behaviour classified as 'keep going' was observed with two groups: **10L** and **11T.**

Having analysed the main features observed in each group, a comparative study on how written materials were used is developed, using the Conceptual Matrix graphs for choice and use of materials respectively, as presented in figures 8.5.1 and 8.5.2.

Analysis of the Ways Teacher E Chooses the Materials.

The overall look of the graph in figure 8.5.1. shows two different ways to choose materials, that seem to be group related: either the teacher uses the textbook (group **11T**) or he uses a selection of printed and adapted materials (all other groups). The main source of material is also usually used as source of introductions and examples, but the teacher complements the material as reference in groups **7X** and **9M** with notes in notebooks.

'Conclusion of a topic' row shows that the material chosen by the teacher for groups **7X** and **10L** does not present conclusions to the topics, while with groups **9M** and **11T** conclusions are presented by the material. Group related choices of materials are also noticed in the 'promotion of links' and 'differentiation' rows: in the first, the data suggest that the teacher chooses materials (a) without the aim of promoting links with group **10L**, (b) with the aim of promoting the links himself with groups **7X** and **9M**, and (c) with the aim of allowing the students to read the links by themselves with group **11T**; in the 'differentiation' row, the data suggest that the choice of material is made to allow group **11T** to work in a differentiated way while the other groups of students work through the materials as a whole, without differentiation noticed within a group.

The 'remedial material' row shows that the teacher does not usually change the material when facing a 'crisis' (although once a new material was introduced and the students were given the choice of changing materials - group **10L**). This row also shows that the teacher had no crisis to face with groups **7X** and **11T,** indicating that his plans were successfully applied with these groups. 'Homework' row shows that the choice of materials for this activity also seems to be group related and close to the options made for the main source of material, except for group **10L**, to which homework was never assigned.

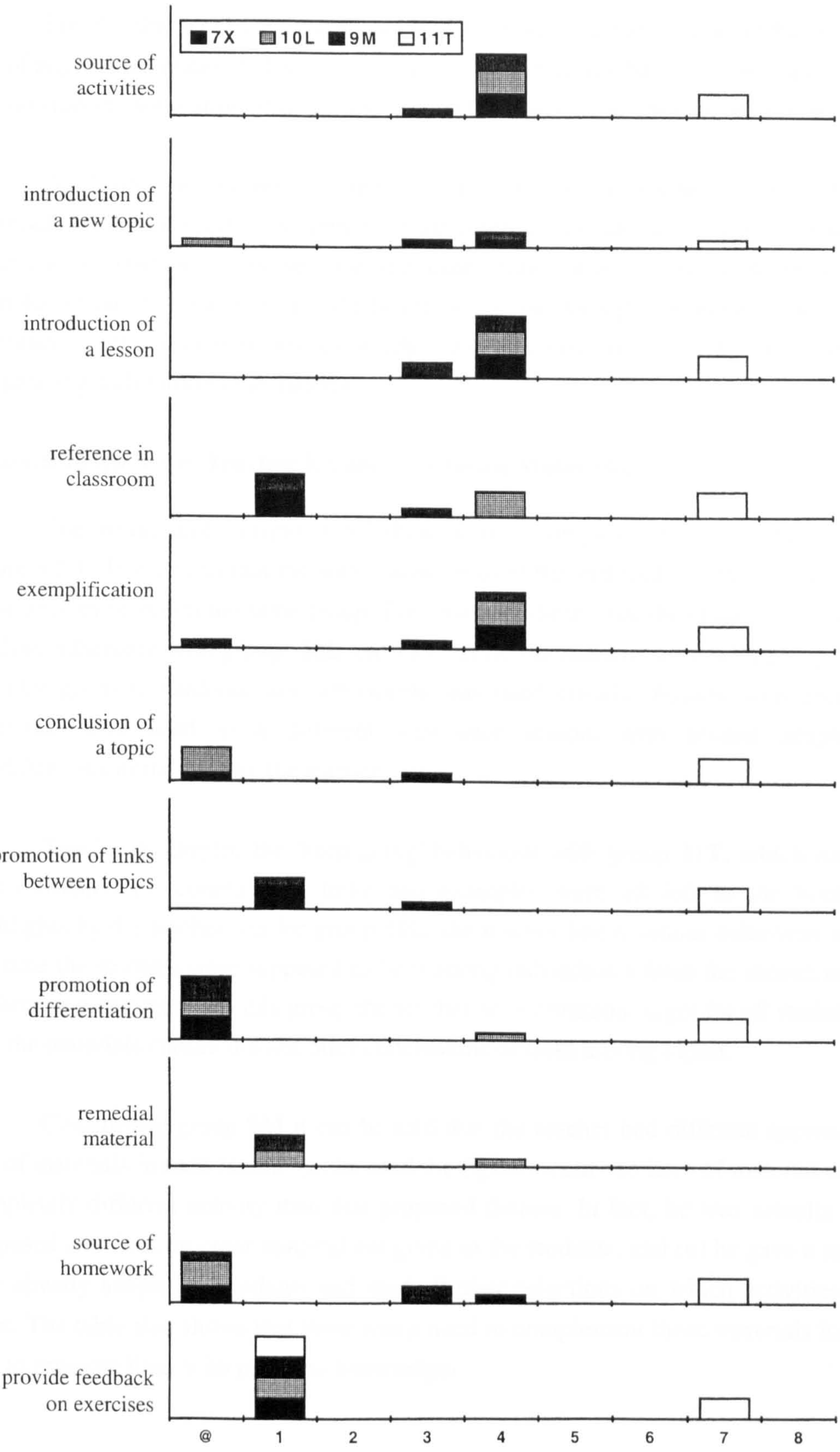


figure 8.5.1: graph of the conceptual matrix for choice of materials - teacher E.

Finally, the 'feedback' row shows that the teacher usually gave feedback without the aid of any written material. The only group that had feedback based on the material was **11T**, whose students were allowed to use the answer book whenever they thought it necessary.

Analysing the columns in figure 8.5.1 it seems that teacher E when choosing the textbook as an aid, used it as source for all activities in the class, except for his personal feedback on students' exercises. On the other hand, when using worksheets, teacher E complemented the material in whichever sense he thought necessary: providing extra references, links with previous knowledge, and remedial activities for those students not progressing well through the material.

Analysis of the Ways Teacher E Uses the Chosen Materials.

The overall look of figure 8.5.2 shows a more complex pattern than the one noticed in figure 8.5.1. This means that the way teacher E used the materials seems to vary in different situations, even within the same group. The first row shows that the chosen material was used in close adherence with groups **10L** and **11T**, while the material used with group **7X** was not initially given to students, and afterwards was used closely. Finally with group **9M** the materials were used in a different way each lesson, with several adaptations and modifications introduced by the teacher.

Teacher E adopted the 'keep going' behaviour with group **11T**, which means to say that introductions, conclusions, links and examples were all left to the book, with no highlights by the teacher. As for group **10L**, the teacher had a similar behaviour and most of the time the students were supposed to be working individually from the chosen material, the difference being that with this group the teacher set a common target for all students to reach and the materials chosen did not offer conclusions or links among topics.

Considering group **9M** it can be said that the teacher had different approaches to the use of materials in each lesson: (a) he used the figures from one item of material to develop a completely different activity than that proposed therein. In fact, he was actually basing the proposed activities on other material not given to the students; and (b) he gave a material that was already adapted to students and made further selections on which activities should be done. The table also shows that there was a need to complement these materials for reference and to promote links with previous knowledge.

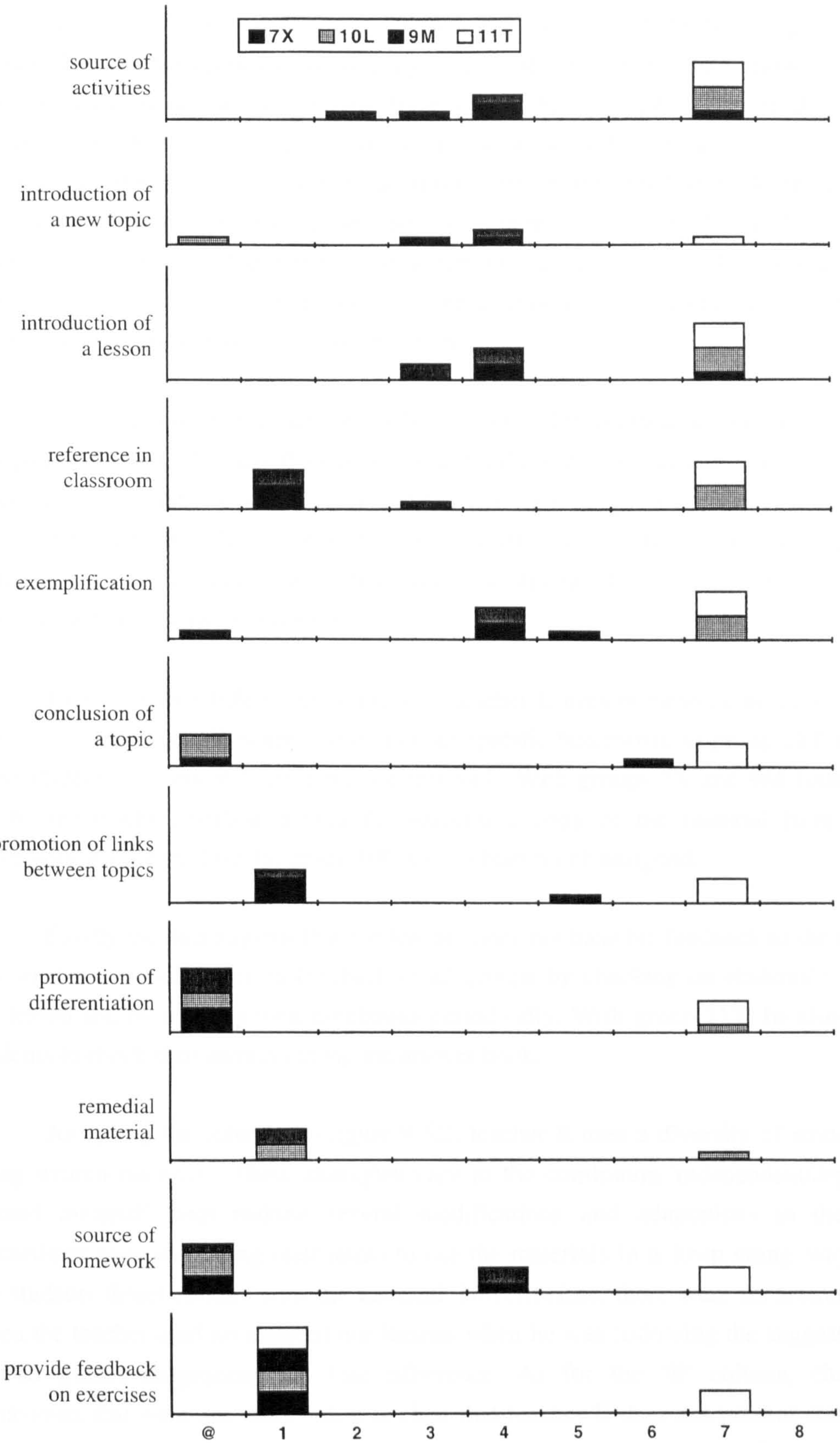


figure 8.5.2: graph of the conceptual matrix for use of materials - teacher E

The material used with group **7X** was not given to students during the first two lessons. The teacher proposed the activity as an open ended one, and allowed students to work for two lessons without giving them either the material or the final aims of the investigation. The teacher gave the material to the students during the final observed lesson, together with the aims, and used the material closely during this lesson. As the material did not contain any references, the teacher had to complement it by himself. He also provided the examples, adapted from the material. Although the teacher said that this particular piece of work was developed to be used in a future investigation based on computer environment, the teacher did not establish links with other topics.

The table also shows that the teacher does not differentiate among the students in all groups except for **11T**, where the teacher expects all the students to start at the same point but reach different points. It is also clear that teacher E's strategy when facing a crisis usually does not include any change of materials. Nevertheless, he did it once with group **10L**, justifying it by saying that it was difficult for these students to concentrate on the same piece of work for two lessons in sequence.

There are also differences in the way teacher E uses homework as a complementary activity to class work. Teacher E does not set specific homework to group **11T** but expects these students to work at home from the textbook. With groups **7X** and **9M** homework was set by the teacher, without giving the students a copy of the material from which the homework was taken. Finally, group **10L** has no homework assigned.

Finally the data suggest that the teacher does not base his feedback to the students on any written material, he gives feedback to all groups by checking on students' work during the lesson and by marking their notebooks periodically. With group **11T**, he also allows the students to check their answers using the answer book.

Analysing the columns in figure 8.5.2, teacher E uses a diversity of strategies when using written materials. These strategies vary in the continuum 'independent/dependent on printed material' from making several modifications and adaptations to the materials (sometimes even modifying their aims) to use the materials in a 'keep going' way, allowing the students direct contact with the material. Nevertheless, there were no lessons observed when the teacher used no material nor lessons when he was following the suggestions of the teacher guides suggestions in close adherence. As for the '@' column, characterising behaviours that were not observed, it is clear that teacher E does not present conclusions to investigations (group **7X**) and does not conclude the topics with group **10L**. Notice also that differentiation was not intended by the teacher with all groups, except for **11T**.

Summary of the Analysis:

Summarising the above analysis, it can be said that teacher E varies the materials he uses and also the way the chosen material is used. The way it was done seems to be group related, in the sense that (a) the material chosen for the 'top' group was a textbook, used closely, (b) the material used with the 'bottom' group was taken from resource material prepared for low attainers, the teacher giving them the choice of changing material whenever a material had to be used for more than one lesson period, (c) the material used to the 'mixed ability' group suggests that teacher E allows students to work through it as an open ended task for a period of time, defining the aims afterwards, and finally (d) the material was used with the medium group in a variety of ways, some of them generating confusion between the students who did not know which instructions to follow: the teacher's or the material's.

8.6. Analysis of Data Concerning Teacher F.

Teacher F is a particularly experienced teacher, working in school F, a middle school in Greater London. She has been a mathematics teacher since the late sixties, after completing an under-graduate course in Mathematics and a PGCE. Teacher F has changed from another school, in which she was head of the mathematics department, to School F after concluding her masters' degree in Mathematics Education. She says that she became interested in foundations in Mathematics while doing her masters' degree, and decided to work with younger students, specially with those considered low attainers. She is presently the head of the Mathematics department, and says that the way the School Scheme of Work is organised reflects her views on how it should be done.

School F has had an adopted textbook for years seven and eight: the NMP (National Mathematics Project) series. Nevertheless, the school scheme is not based on the textbook. It has a special way of presenting study programmes: instead of proposing study programmes for each school year, the school scheme proposes them by levels of attainment. So, the heading in each paragraph looks like this: 'Study Programme for Groups 7.X and 8.Y - working towards levels A/B'. In each paragraph, a sequencing of contents is proposed, in line with National Curriculum attainment targets, with no suggestions of materials or chapters from the adopted textbook. Teacher F says that the textbook covers almost all the content previewed in the school scheme of work, and can be used as resource for the development of the proposed activities.

The mathematics department also has a series of advice sheets complementing the teacher's guides for the adopted textbook in year seven available for their teachers. These sheets were produced by teacher F, and address the following issues: Number of Chapter, Topic, Number of Recommended Lessons, Aim of the Chapter, Knowledge Base Required, Skills Required, Additional Preliminary Work, Investigation Possibilities, Possible Difficulties, Follow up Work, Reinforcement for Lower Groups, Extension for Higher Groups, Resources Needed, Assessment Opportunities, Links with Other Subjects and Other Comments (always including homework suggestions).

Teacher F admits that some teachers in the department do not follow the suggestions made in the school scheme or in the complementary sheets, but choose to follow the textbook instead. She says:

> As you know, we use textbooks ... some teachers say they feel better just using the book ... because the students can follow the activities proposed there ... one can say that these teachers adapt the scheme to fit the book, instead of adapting the book to fit the scheme ...

Considering her personal position, teacher F says that she uses the book and/or the scheme in a way that is very much group related. She says that she uses (1) the NMP 'red track', the one for 'high attainers', with her year eight 'top group' (FF8T); (2) the 'blue track' , the one for 'medium' and 'low attainers' with her year eight 'medium group' (FF8M - the third out of four groups), with several complementations and 'lots of class teaching'; (3) the NMP for year seven with her 'mixed ability' group (FF7X) for about half of the lessons, with the other half being covered by other sources of materials or, more frequently, no material at all; and finally (4) no textbook or any other written materials with the 'low attainers' group (FF7L). Despite having such a variable approach as far as written materials are concerned, teacher F has some common attitudes towards all groups, reflecting her views on teaching mathematics: she says that (1) she believes that it is necessary to be sure students have the necessary pre-requisites before starting a new topic (she says she always starts by asking them questions, unless she is already sure that they have the requisites); (2) she also believes that some skills are necessary and regularly reinforces them (for example: whenever there is some spare time at the end of the lesson, after the students have 'packed away', she uses it to review multiplication tables or to play games using tables skills). Commenting on the particular use of the textbook with group FF8T, teacher F says that the students are comfortable using the book, even if she sometimes considers that 'I do not stretch them as much as I should'

Teacher F says that the school does not allow students to take the textbook home, 'because they come back destroyed ... we have tried and it did not work ... not even with the 'top' group'. She also says that all students are allowed to take their notebooks home. When asked about the position of the school about mixed-ability groups, she says:

> We do not believe in mixed ability ... it is not possible to have a good lesson if you have to prepare four or five different ones for the same group ... in year seven, the teachers set the students all together ... I mean, the 'top group' is 'mixed ability' for mathematics, ... because some students are there because of their English or History marks, not because of their Maths... but the students with serious learning difficulties are all in the 'bottom group' ... In year eight it is different: the students are set in groups by subjects.

Set I - Headings: Students' Grou → General Decisions	**FF7X**	**FF7L**	**FF8M**	**FF8T**
Main Source of Materials for Class Work	5	1	6	7
Progression of the Content	5	2	5	7
Materials given to students for reference	1	1	4	4

table 8.6.1 - Case Matrix for the first set of headings (general decisions) - Teacher F.

Table 8.6.1, the Case Matrix for the headings in general decisions (set I) taken by teacher F, summarised the introduction above, and is based on table 4f.1 in Appendix 4f.

During a week, all three mathematics lessons involving each of the groups mentioned above were observed. In the following section, a comparative study for the four groups of students during these lessons will be developed, considering the headings in sets II and III.

Comparative Study of the Headings in the Second and Third Sets for Teacher F's Four Groups of Students:

Tables 8.6.2 and 8.6.3 are the summarised versions of the Case Matrices for teacher F, each one relating to one set of headings. Based on these tables, an overview of the choice and use of written materials by teacher F with each group of students is presented. As there were lessons when two behaviours under the same headings were observed, two codes can appear in the table's cell. They were developed based on the tables of summarised data presented in Appendix 4f.

Student's Group →	FF7X			FF7L			FF8M			FF8T		
Lesson → / Set II: Choice of Material to:	1	2	3	1	2	3	1	2	3	1	2	3
Source of Activities	1	1	1	1	1	1	4	4	6	7	7	7
Introduce a new topic.	-	5	-	-	1	-	-	-	5	-	-	7
Introduce a lesson.	1	5	1	1	1	1	4	4	5	7	5	7
Reference in classroom.	@	1	1	@	@	@	4	4	7	7	7	7
Exemplification.	1	1	1	@	1	@	@	@	6	6	6	6
Conclude a topic.	-	-	@	1	-	@	-	@	-	-	3	5
Promote links between topics.	-	1	-	-	1	-	-	-	-	-	-	-
Promote differentiation.	@	@	1	@	@	@	1	1	@	7	7	7
Remedial material.	-	-	-	-	1	1	-	-	-	-	-	-
Source of homework.	4	4	1	@	@	@	4	4	4	@	@	1
Provide Feedback on Exercises.	1	1	1	1	1	1	4	4	1	6/1	1	1

table 8.6.2: Set II Headings - Case Matrix for Teacher F.

Group FF7X: Although the teacher was developing a topic that can be found in a chapter of the textbook with these students, she was not using the book. She decided to base their activities on work with concrete material instead: the students were measuring boxes, calculating volumes and constructing boxes with given capacity. The activities developed in class were different from those suggested in the book. Nevertheless, the teacher said she would expect students to be able to solve exercises from the book after the lessons. During the week, the students had also been working on a project sheet at home. This project sheet was used during the first lesson in 'volume', introducing the topic. It involves isometric

representation of 3D shapes and calculation of volumes. This sheet was not associated with the textbook series adopted, but matched closely with class activities. Teacher F planned the lessons ahead and no crisis was observed. There was also evidence of differentiation: although all students started at the same point, extension work was given for those who finished the task ahead of the others.

Student's Group →	FF7X			FF7L			FF8M			FF8T		
Lesson →	1	2	3	1	2	3	1	2	3	1	2	3
Set III: Use of Material to:												
Source of Activities	1	2	2	1	1	1	8	8	5	7	5	7
Introduce a new topic.	-	3	-	-	1	-	-	-	3	-	-	2/7
Introduce a lesson.	1	3	2	1	1	1	8	8	3	7	5	2/7
Reference in classroom.	@	1	1	@	@	@	2	2	2	7	7	7
Exemplification.	1	2	2	@	1	@	@	@	2	5	5	2
Conclude a topic.	-	-	@	1	-	@	-	@	-	-	8	2
Promote links between topics.	-	2	-	-	1	-	-	-	-	-	-	-
Promote differentiation.	@	@	1	@	@	@	8	8	@	@	@	@
Remedial material.	-	-	-	-	1	1	-	-	-	-	-	-
Source of homework.	7	7	1	@	@	@	7	7	7	@	@	1
Provide Feedback on Exercises.	1	1	1	1	1	1	8	8	1	5/1	1	1

table 8.6.3: Set III Headings - Case Matrix for Teacher F.

Group FF7L: Teacher F had carefully planned activities to enable these students to calculate areas of rectangles and right-angled triangles. The lessons were planned without using the support of any written material, but using coloured shapes and grids instead. The aim of the observed lessons was to move the students from counting squares to a multiplication strategy when calculating area of rectangles, and to generalise the results to areas of right-angled triangles, as 'half-rectangles'. Nevertheless, the teacher had to face an unexpected difficulty: during the second lesson, the students did not associate the ideas of 'halving' and 'division by two'. Teacher F quickly diagnosed the situation by asking questions. The students were able to say the half of several simple quantities, but were not able to say what they were doing to obtain the answers. Teacher F decided immediately to change the lesson: she gave the pupils calculators, introduced the link between halving and division by two, and asked them to do some activities using this idea. This remedial situation took the final part of the second lesson and part of the third, when the teacher went back to the work with rectangles, using the grids and moving the students again to the multiplication strategy. Although this was the last observed lesson, the researcher asked in the final interview with the teacher if the students

were able to successfully calculate areas of right-angled triangles in the following lesson, receiving a positive answer.

Group FF8M: Teacher F was developing an investigation on octagon loops, based on printed materials with these students during the first two observed lessons (respectively third and fourth for the investigation task). The main feature of these lessons can be summarised by saying that teacher F followed every suggestion of the investigation guide. All her inputs were matched with its suggestions, with the exception of writing a summary on the board, involving the steps to be followed during the investigation, which the students could use as reference. In the third lesson, the teacher returned to her own planned activities, starting a new chapter in the textbook. Teacher F added several personal examples, clarified the differences between average, median and mode (she thinks the book does not emphasise it), and selected a series of activities taken from the book. The homework given by the teacher during the observed lessons was to complete proposed steps in the investigation reports.

Group FF8T: Differently from all the other groups, these students used the textbook during all the observed lessons. Nevertheless, the use of the book was determined by the teacher: the students followed its activities as a group: all of them on the same chapter, working through pages and exercises chosen by the teacher. Differentiation occurred during the last lesson dedicated to the chapter, because some students were able to go further than others. The teacher used the extension material from the book (which is presented after what is called the 'core section'). Even using the book during most of the lessons, teacher F added several personal inputs: (1) she started the second lesson by giving a mental test (own produced), which summarised the results of the chapter; (2) during the third lesson she stopped the students' work through the book to promote some class teaching, commenting on their solutions and leading them to some generalisations on simple mathematical identities using symbolic language, such as p+p=2p. No crisis situations were observed: the students were able to work through the 'core' activities, with eventual help from the teacher, who had also planned extension work for those who finished the tasks ahead of the others. Two other particular features were noticed in this group: (1) these students were asked to read references in the book during the lessons, with time assigned for the activity; and (2) they were asked to revise their previous work as homework (using their notebooks, as the textbooks cannot be taken home)

To complete the analysis of tables 8.6.2 and 8.6.3 it is necessary to talk about gaps: there was no need to use the sub-categorisations (particularly, there were no 'keep going' situations) and there was no need to use (*), which means that all the activities developed in

class, as well as the homework given to the students, matched with the main activity developed in class in all observed lessons.

Having analysed the main features observed in each group, a comparative study on how written materials were used can be developed, using theConceptual Matrix graphs for choice and use of materials respectively, as presented in figures 8.6.1 and 8.6.2.

Analysis of the Ways Teacher F Chooses the Materials.

The overall look of the graphs in figure 8.6.1. shows that the group of students influences the choice of materials made by the teacher. While group **8T** seems to be concentrated in the higher numbers, suggesting the use of the textbook, group **8M** seems to be concentrated in the medium ones, suggesting complementation and adaptation of the textbook. Concentrated in code '1' are the groups **7X** and **7L**, suggesting lessons without using written materials. Differentiating these last two groups, the data suggest that **7L** appears in the '@' column more frequently than **7X,** suggesting that some activities are left out for group **7L**. The data in the first row reinforce the general impression above: the activities were (1) taken from the textbook in all lessons with group **8T**; (2) taken from the book or from printed materials for **8M**; (3) proposed by the teacher, without using written materials, for **7X** and **7L**. The difference between these two last groups can be observed in the 'introduction to the lesson' row: for group **7X,** the teacher was comparing the development of the topic in class with the way the topic was developed in the book; with no material considered for **7L**.

Without considering the rows in which the '@' column plays an important role, to be discussed when comparing the use of materials, two other rows appear relevant: 'Source of Homework' and 'Source of Feedback'. The first one shows that sources other than the textbook are used for homework, reflecting the school policy of not allowing students to take the book home. 'Source of Feedback' shows that, despite using textbooks and having the answer books available in the class at all times, the teacher does not use them very frequently. In fact, with group **8T,** while using the answer books during the first lesson to give feedback, the students showed that they do not trust the answer book. They did not believe that one of the answers given by the book was correct, and discussed it with the teacher, asking her to verify it. They were only convinced after the teacher had presented the solution to the question on the board.

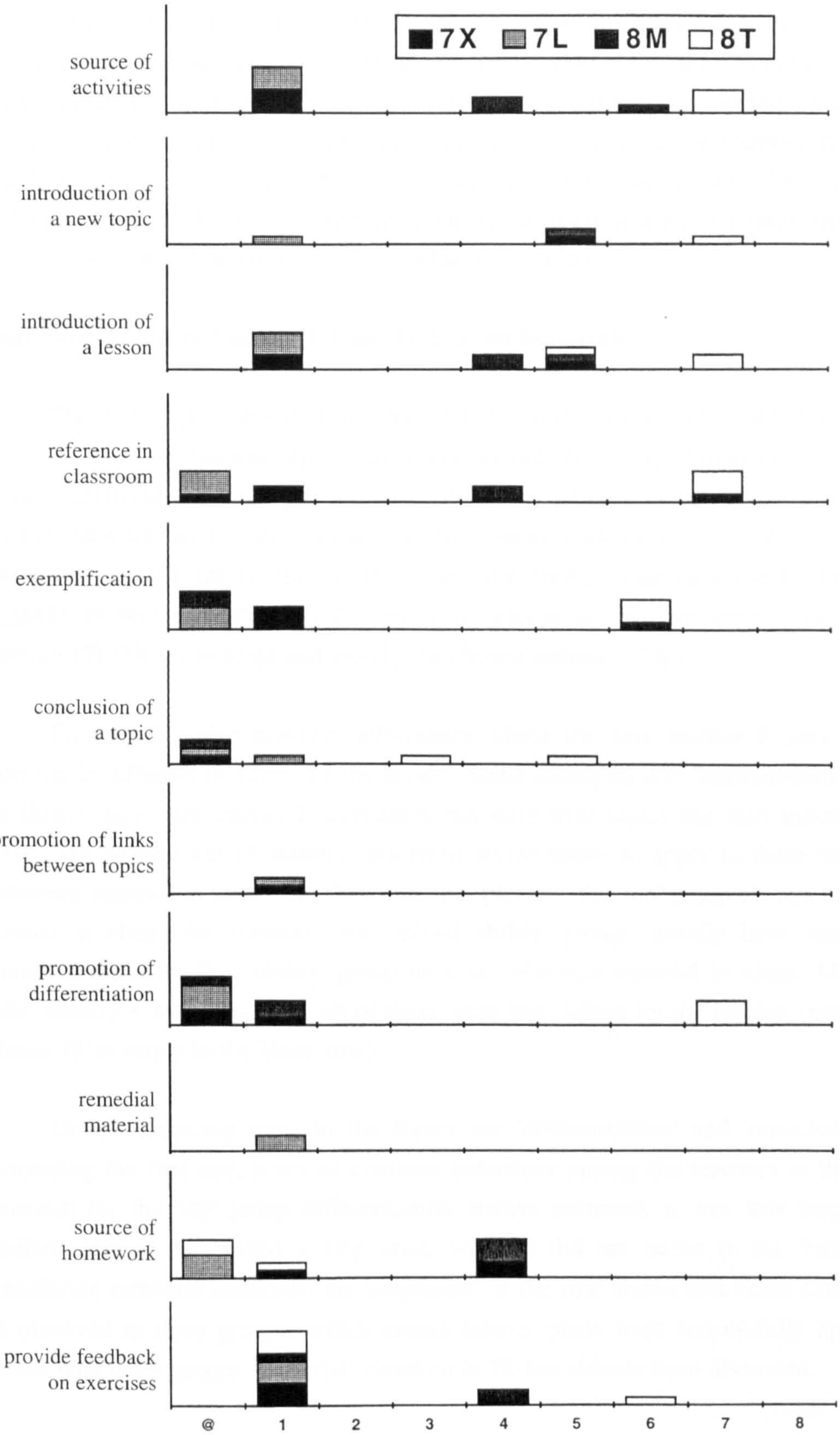


figure 8.6.1: graph of the conceptual matrix for choice of materials - teacher F.

Finally, columns '2' and '8' are completely empty in figure 8.6.1. In column '2', the absence can be analysed as a decision made by teacher F to not produce material. She prefers to use no material, in those cases when she does not want to use the resources available. The absence of cases in column '8' shows that teacher F does not use the textbook following its guidelines, (it can be said that she follows her own guidelines instead). The only material produced by teacher F was a mental test, which was used as a way of summarising results from the book, providing some conclusions for the students.

Analysis of the Ways Teacher F Uses the Chosen Materials.

The overall picture shown in figure 8.6.2 is very similar to the one shown in figure 8.6.1. Nevertheless, because use is being considered, the interpretation of the data in this figure is different from the previous one. From the general aspect of the graph, we can conclude that teacher F varies her uses of the chosen material in a group related way: for groups **8M** and **8T** the teacher seems to use the chosen material close to the way it is originally presented, while with the remaining groups the teacher seems either to use no materials (**7L/7X**) or to adapt and modify the chosen material (**7X**).

Figure 8.6.2 also provides information about the way teacher F uses the chosen materials in different moments of the lesson. Some examples are: 'introductions' (rows two and three) show that teacher F introduces not only new topics but also introduces every lesson. The general use of material described above seems to apply to these introductions. 'Reference material in class' row shows another picture: the 'top' group always has reference material in class; the 'medium' and 'mixed ability' groups usually have their notes as reference; while the 'low ability' group uses no reference material in class. Although few 'links' situations were observed, all of them were highlighted by the teacher (notice that the column '@' is empty in the 'links' row).

Other interesting rows in the figure are 'differentiation' and 'remedial materials'. Concerning the first one, a not so common behaviour among the teachers in the sample is observed: for the 'top' group differentiation always occurred, it was less frequent in the 'medium' and in the 'mixed ability' ones, while it did not occur in the 'bottom' group. Considering remedial materials, the 'emptiness' of the row shows that crisis situations were not observed in three groups, which means teacher plans were successfully applied in all lessons with these groups. The crisis situation in **7L** has already been discussed.

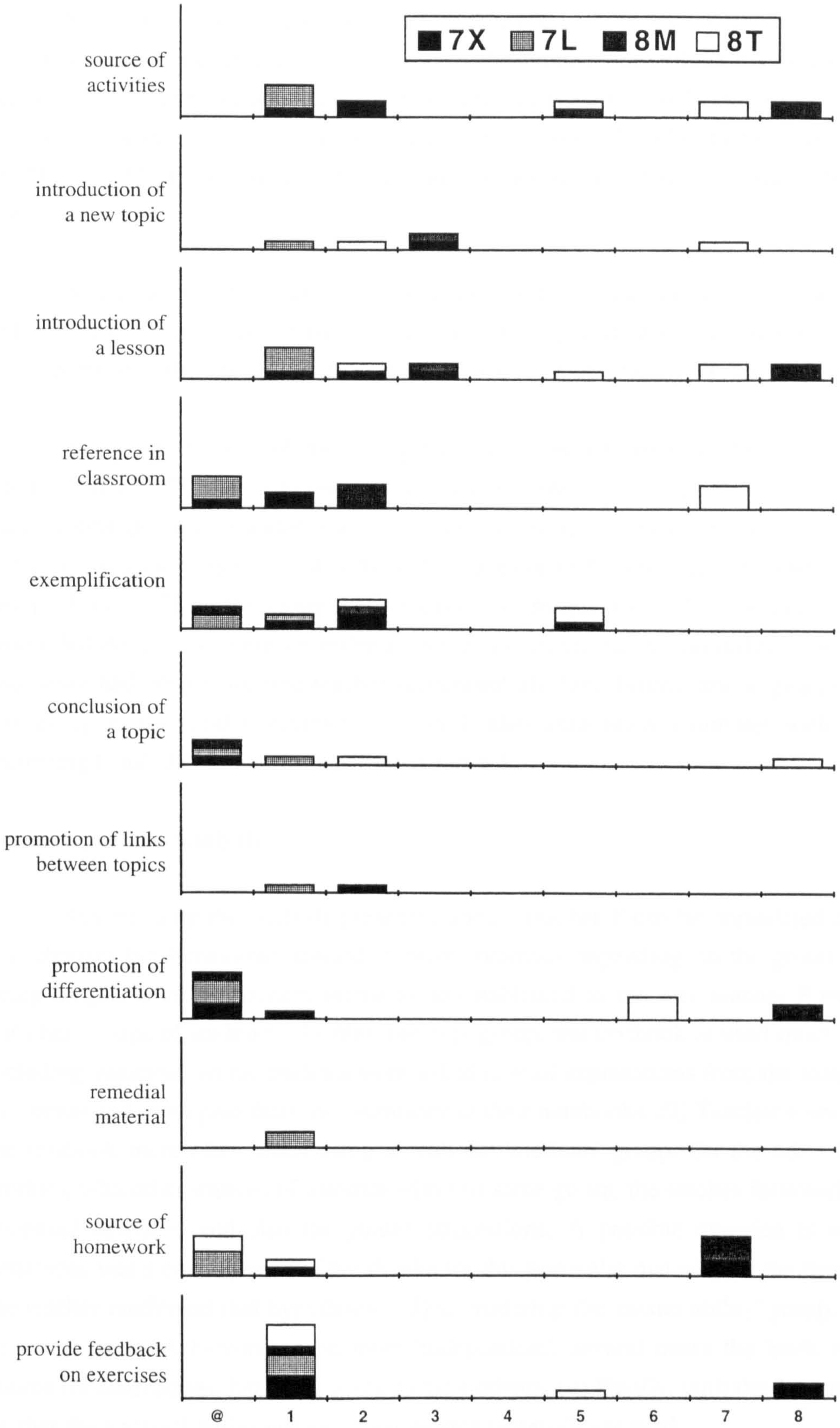


figure 8.6.2: graph of the conceptual matrix for use of materials - teacher F

Analysing figure 8.6.2 by columns, other aspects of the ways teacher F uses materials are highlighted. For example, she never uses a chosen material without giving a copy of it to her students (column '4', which is essentially characterised by this behaviour, is empty). Her more common strategies were: (1) not to use materials with **7L**, (2) adapt the chosen material for **7X** and **8M** groups, and (3) offer guidance to **8T** in their direct contact with the chosen material.

When use of homework sources is considered, the strategy seems to change: **7X** and **8M** have their homework set from a chosen material, while **8T** receives less guided tasks (such as revising the content for an assessment activity), and finally **7L** has no homework set.

Analysing the '@' column, group **8T** only appeared there by not having homework. Groups **8M** and **7X** appear in this column in three rows each, but notice that in these rows these groups also appear under other categories (for example: teacher F did not give examples in the investigation lessons with **8M**, but used them in the other lesson observed with this group). Finally, **7L** is the group that appears most frequently in this column: it is the only group that never had reference material in the classroom, never had differentiated activities and never had homework (the teacher recognised all three behaviours as general ones with this group in the final interview). Teacher F also used fewer examples with this group, preferring to ask them to perform activities instead.

Summary of the Analysis

Summarising the analysis presented above, teacher F can be considered as a teacher that changes her behaviours towards written materials depending on the group of students being taught. A certain pattern seems to be established in the way teacher F uses material with her groups of students: (1) With the 'top' group, the textbook is used most of the time, including situations where students were asked to read explanations from the material during the lesson and to prepare their own summary in their notebooks. (2) Teacher F seems to adapt the textbook more often when using it with the 'medium' group. On the other hand, while working with other sources of material with this same group, the teacher followed closely the proposed activities and also the guides suggestions. A possible question is whether this behaviour was a consequence of her dislike for this particular material (in the final interview, the teacher confirmed that hypothesis). (3) Considering the 'mixed ability' group, the method of using the book becomes even more 'independent': several times the book is used as a source for adaptations, but is not given to the students. (4) Finally, with the 'low ability' group neither the textbook series nor any other written materials are used.

In general, the data suggest that teacher F introduces the topics, and all other lessons in a carefully planned way. She usually summarises the topics developed, highlighting the principal results. The data also suggest that teacher F did not have a crisis with groups **8T, 8M** and **7X.** However, when faced with this kind of situation with group **7L,** she deals with it by diagnosing the problem and proposing remedial activities. Teacher F uses homework as a way of reinforcing lessons, proposing home activities that are related to class activities. Finally, she seems to be developing the studying skills of her 'top' group, by (1) giving them time to learn by reading the book; (2) asking them to write personal notes about what they read in their notebooks; and (3) asking them to revise previous work at home.

8.7. Analysis of Data Concerning Teacher G.

Teacher G is a teacher with well defined ideas about teaching and textbook usage. She is working in School G, a middle school in Greater London. Teacher G completed a Bachelor of Education and Mathematics eleven years ago and has worked in School G ever since. She is the head of the Mathematics department but also gives Information Technology lessons. She is responsible for several changes in both curricula, especially where integration between Mathematics and I.T. is concerned.

The school has adopted the SMP booklets as textbook for years seven and eight. The School Scheme of Work is organised according to student group and based is on the booklets. A 'typical' page of the school scheme has: the group it is set for, the half-term it is set for, the ATs of the N.C. being developed during that period with correspondent booklets, main learning objectives, number of lessons for each booklet, other resources to be used (including computer, audio, investigations, 'stretchers', etc.), extra curriculum activities in Mathematics, assessment (using the SMP proposed assessment with several additions), homework, and 'Tables and Mental Arithmetic' (this last based on a series of sheets developed by teacher G and on an extra series of books).

Teacher G says that each page of the school scheme is developed by the teacher responsible for the group, after discussion within the department. She also says that after the drafts are ready, she does the final review. When asked about the extra curricular activities, she says:

> This is mainly my idea. I usually find some time to look around for some new activities that I think will keep the pupils interested... the tree measurement activity using clinometers was a very successful one with years seven and eight this term. The students were really interested in learning how the clinometers work and to use them afterwards... I suppose they also enjoy these activities because most of them take place in the school grounds.

Another feature in the school is the number of Mathematics lessons in a week. Students in year seven have four lessons each week during the whole year. Students in year eight have four in the two first terms and three during the last one (which was the one when the observations took place). Year eight students are being transferred to an upper school, so they spend one half day each week in adaptation activities. Teacher G says

> Students have one more lesson per week than the usual, and this was not easy to be set... we convinced the head of the school that it was important to have this extra lesson so we could dedicate some time during the week to mental arithmetic and tables skills... usually, this extra lesson is dedicated to a mental test... but if the topic which is being developed during the week demands some extra work, I do not give them the test.

The school has four groups of students per school year and the groups of students in years seven and eight are all set by 'levels of ability'. Teacher G explains:

> It has been like this for a few years...before we had mixed-ability groups and they did not work well. First because the 'change' for the upper school was quite a shock...they had to cope with the new school and also had to cope with setting in ability groups...all at the same time, you see...and second because if you use SMP with a mixed ability group you have to keep it individualised almost all the time... and I do not think young students benefit from this excessive individualisation.

Teacher G is the only teacher in the sample who does not teach four groups of students. She only teaches in two groups: a year seven 'medium' group (GG7M: third out of four groups) and a year eight 'top' group (GG8T). The reason for including teacher G in the sample, even if she does not fit the ideal 'student's group definition' was her personal view on how the SMP booklets should be used. She says:

> I do not think that SMP was developed to be given to the students so they can work their way through the book...Actually, if you look into the guides it is clearly stated there that the books were developed to be used in teacher-led lessons... and this is the way I use the book. Before giving the booklets for the pupils, I do a bit of class teaching...and the whole group starts at the same point... different is the final point: some students can go further than others...but all of them work through the basic content.

During a week four lessons with GG7M and three lessons with GG8T were observed. In the next paragraph a comparative study between the groups will be developed. Table 8.7.1 summarises, using the meta-categorisation, teacher G's general decisions about written materials. It is based in table 4g.1, in Appendix 4g.

Set I - Headings: Students' Grou → General Decisions	**GG7M**	**GG8T**
Main Source of Materials for Class Work	6	6
Progression of the Content	6	6
Materials given to students for reference	1	1

table 8.7.1 - Case Matrix for the first set of headings (general decisions) - Teacher G.

Comparative Study of the Headings in the Second and Third Sets for Teacher G's Two Groups of Students:

Tables 8.7.2 and 8.7.3 are the summarised versions of the Case Matrices for teacher G, each one relating to one set of headings. Based on these tables, an overview of the choice and use of written materials by teacher G with each group of students is presented. As there were lessons when two behaviours under the same headings were observed, two codes can appear in the table's cell. Tables 8.7.2 and 8.7.3 were developed based on the tables of summarised data presented in Appendix 4g.

Group GG7M: Although this group is considered to be of medium attainment (the third out of four), some particular characteristics made this group different from the other 'medium' groups observed: (1) some students that ought to be in the second group are part of this group because they present behaviour problems, and (2) some students that ought to be in the last group, but are extremely well behaved, also form part of this group. So, this group can almost be considered to be 'mixed ability', without the brighter students in it. When examining the columns correspondent to this group in both tables, it is noticed that the first and final lessons were developed almost independently from written materials, while during the other lessons, the teacher developed two lessons simultaneously. She found out, during the first lesson, that some of the students did not have any lesson using LOGO in their I.T. classes. Teacher G had planned to conceptualise angles as turns using LOGO, thus she had to move away from this plan and concentrate the lesson on other strategies she expected to use. Teacher G decided that during the second and third lessons these students would use LOGO on the computer, instead of the booklet. Another interesting feature of this approach was the discussion teacher G promoted with the students at the end of each lesson, summarising the common results obtained from the two different approaches.

Homework was set once for this group of students. Teacher G assigned them to write at least five different facts about angles they had learned and/or reviewed during the lesson. The homework was set for the very next day (not a lesson day). Teacher G marked the homework before the following lesson, and used it to start the lesson. She wrote some of the facts the students had written on the board. These facts were discussed with the whole group and corrected when necessary, providing a summary of the previous lesson.

Student's Group →	GG7M				GG8T		
Lesson →	1	2	3	4	1	2	3
Set II: Choice of Material to:							
Source of Activities	1	8/4	7ab/4	1	1	8	7ab
Introduce a new topic.	1	8/4	-	1	1	-	-
Introduce a lesson.	1	1/8	7ab/4	1	1	8	7ab
Reference in classroom.	@	8/4	7ab/4	1	@	8	7ab
Exemplification.	1	8	@/4	1	1	8	@
Conclude a topic.	1	5	4	5	-	-	7ab
Promote links between topics.	-	5/4	5/4	-	1	-	-
Promote differentiation.	@	7ab/4	7ab/4	@	@	7ab	7ab/8
Remedial material.	1	4	4	-	-	-	-
Source of homework.	1	@	@	@	@	@	8
Provide Feedback on Exercises.	1	1	1	1	1	7a	7a/7ab

table 8.7.2: Set II Headings - Case Matrix for Teacher G

Student's Group →	GG7M				GG8T		
Lesson →	1	2	3	4	1	2	3
Set III: Use of Material to:							
Source of Activities	1	8/7	7ab/7	1	1	8	7ab
Introduce a new topic.	1	8/5	-	1	1	-	-
Introduce a lesson.	1	1/8	7ab/7	1	1	8	7ab
Reference in classroom.	@	8/7	8ab/7	1	@	8	7ab
Exemplification.	1	6	@/2	1	1	8	@
Conclude a topic.	1	2	3	2	-	-	7ab
Promote links between topics.	1	7	7	-	-	-	-
Promote differentiation.	1	@	@	@	@	@	8
Remedial material.	1	7	7	-	-	-	-
Source of homework.	1	@	@	@	@	@	8
Provide Feedback on Exercises.	1	1	1	1	1	7	7/7ab

table 8.7.3: Set III Headings - Case Matrix for Teacher G.

Group GG8T: When examining the columns correspondent to this group in tables 8.7.2. and 8.7.3, it is possible to notice a sequencing of lessons: (1) the first one was used to introduce the topic, using different activities from the ones proposed in the booklet to be adopted. Teacher G motivated her students to look into decimal numbers by playing a guessing game, using a sheet with number lines to allow students to record their tries. After this introduction, she made links with their previous knowledge on decimal fractions and on place value representations of numbers, concluding the lesson by comparing the values of measurements of objects using different units, one of the topics developed in the booklet planned for the following lessons; (2) the second observee lesson was the first using the booklet, so the teacher carefully introduced it, by reading the initial questions and asking the students to give some solutions to the exercises; and finally, (3) the third lesson was a continuation of the previous one, so the teacher did not introduce it, but made sure that by its conclusion all the students in the group had at least started the final section, by assigning as homework the completion of the booklet for those who did not get that far. All the students had been assigned the revision sheet associated with this booklet as homework.

To conclude the analysis of tables 8.7.2 and 8.7.3, two more features are analysed here: (1) answers to the 'match' questions; and (2) sub-categorisation. Both tables show that all the activities were matched with the main activity developed during the lesson. When the use of sub-categorisation is considered, teacher G carefully introduces the booklet, usually following the guides' suggestions. Once the students are already working through it, the teacher does not introduce the lesson, and expect the students to remember what they have done and to 'keep going' (sub-categorisation *ab*). Another use of the sub-categorisation can

be found in table 8.7.2, when the teacher used the answer book, reading its answers out loud to the group. The teacher neither discussed nor gave the students time to take note of the correct answers. Although this behaviour can be classified as a '7', it is different from other behaviours classified there. Teacher G was using the answer book in a way that can be considered incompatible with the guidelines offered by the book, and the sub categorisation (*a*) was applied.

To complete the picture on how teacher G chooses and uses written materials, two other data displays are used: the graphs of the conceptual matrix presented for teacher G in figure 8.7.1 for choice and 8.7.2 for use of materials.

Analysis of the Ways Teacher G Chooses the Materials.

Teacher G can be classified as a textbook user, who also uses own developed lessons and some extra printed material to complement the textbook. The influence of the group is also notable: (1) with group **8T** teacher G either does not use any material (one lesson registered in column '1') or uses the adopted textbook (two lessons mainly registered in columns '7' and '8'); while (2) with group **7M** teacher G either does not use any written materials (two lessons registered in column '1') or uses the textbook, complemented with other materials (two lessons mainly registered simultaneously in columns '4' and '7' or '8'). The first 'row', describing the choice of classroom activities, repeats the general impression just described.

Analysing 'introduction of a new topic' and 'conclusion of a topic' rows of figure 8.7.1, a difference between the groups was observed: with group **8T** teacher G introduces a topic only once, concluding it at the end of a series of lessons (in fact, in this case, the conclusion was left for the book); while with group **7M** teacher G 'breaks' the topic into small sub-topics, which are introduced and concluded each lesson. This difference showing in the 'choice' figure (and not only in the 'use' figure) suggests that the choice of materials was made to allow this 'breakage' of the topic. The 'remedial material' row suggests that the introduction of extra printed material for group **7M** was not initially planned by the teacher, but motivated by the needs of the students. Although this method of choosing materials cannot be considered as the usual way in which teacher G works with this group (see table 4g.2, Appendix 4g), another characteristic of hers when choosing materials was revealed by the way she solved the 'crisis' situation: instead of dealing with it at the moment, she preferred to plan a strategy for the next lesson.

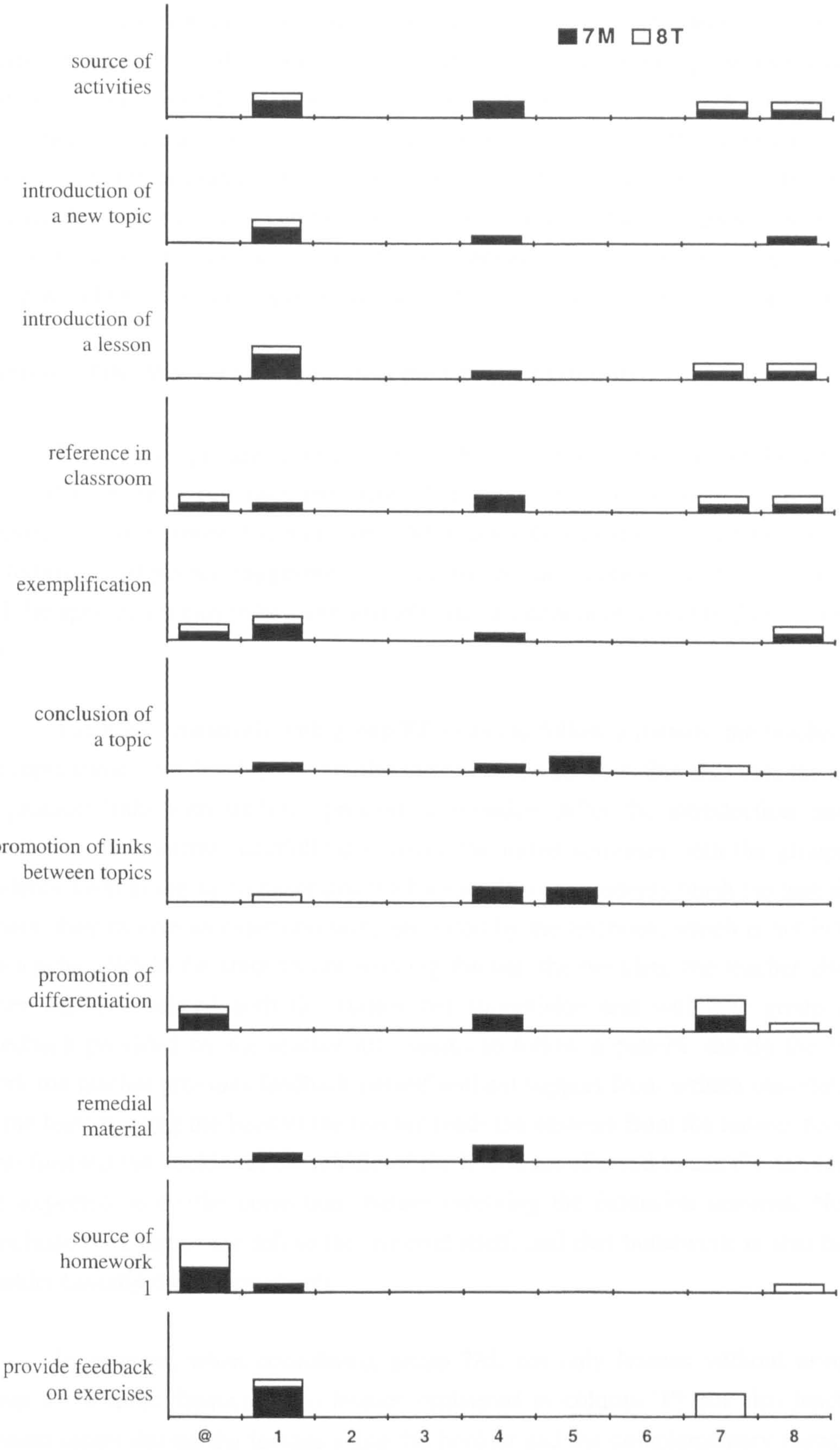


figure 8.7.1: graph of the conceptual matrix for choice of materials - teacher G.

The 'differentiation' row also reflects the way teacher G decided to deal with the 'crisis' situation: as she did not introduce the remedial situation during the lesson, omitting the part of the lesson based on LOGO instead (which means she had enough activities planned to allow her to do that), an extra series of materials transformed the following lessons into lessons with differentiated activities for the students. Another feature related to this particular way of choosing materials can be observed in the 'links between topics' row. It shows that teacher G chose the material to allow her to promote links between what some students were doing with LOGO on the computer and what the others were doing, using the booklet.

Analysis of the Ways Teacher G Uses the Chosen Materials.

The overall picture shown in figure 8.7.2 indicates that teacher G, either uses no material (column '1') or uses the material in a way that seems very much group-related. Columns '2' to '8' show that with group **8T** teacher G uses the material in close adherence, including use of guides' suggestions (all occurrences in columns '7' and '8'), while with group **7M** the approach seems to be more variable (occurrences in all columns from '2' to '8' , except '4').

The use of materials with group **8T** seems to follow a pattern: the teacher introduces the topic using own developed activities instead of the booklet. She also uses the introduction to promote links with students' previous knowledge. After the introduction, she gives the booklet to the students, carefully discussing the initial activities with the group. Then, the students 'keep going' up to the end of the booklet. If some students finish the task ahead of the others, they receive an extension task, proposed by the textbook, which is not introduced by the teacher. While the students are working through the booklets, the teacher also promotes other textbook related activities (under her supervision and with one group at a time). Feedback provided by the teacher also seems to follow a pattern: during the introductory work the teacher provides feedback herself without support from written material; at the end of the lessons using the booklet the teacher reads the answers from the answer book. Students who finished the booklet in the middle of the lesson are allowed to use the answer book, and are expected to do the corrections before receiving the extension material. Note that the conclusions of topics are left to the material itself, and that homework is also based on the booklet (usually the review sheet).

In contrast, when considering group **7M,** not only lessons without using materials seem to be more frequent (two lessons registered in column '1') but also teacher G adds several inputs during the lessons using the booklet and the complementary material (several teacher's behaviours registered in columns '2' to '6').

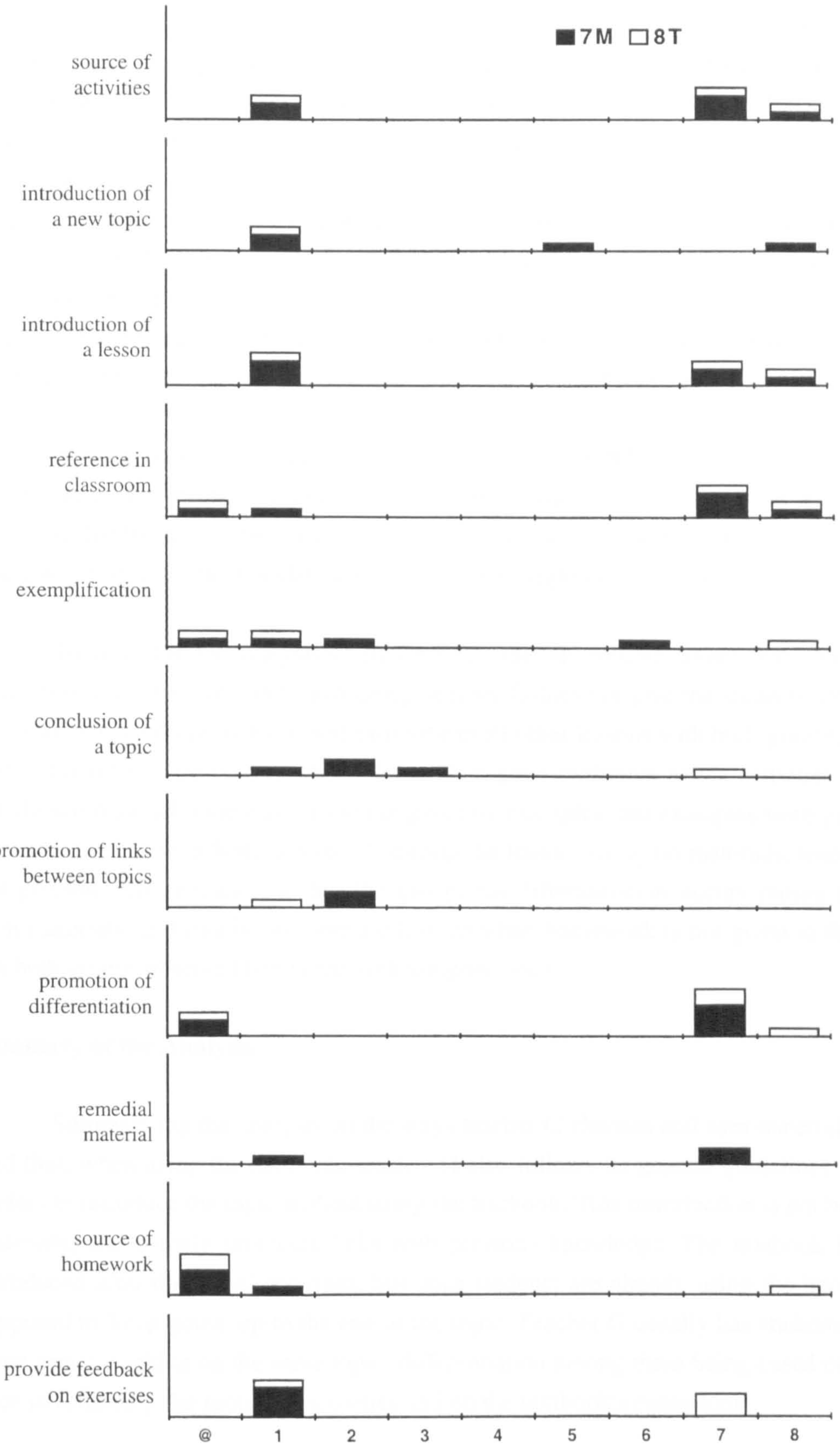


figure 8.7.2: graph of the conceptual matrix for use of materials - teacher G.

The way teacher G adds these 'inputs' has to be analysed. All the activities developed by the students during these lessons were taken from the chosen materials, in close adherence (no complementation or modification are introduced by the teacher). She also introduces the lessons close to the materials, which are also used as reference in class. This behaviour starts to change when examples are concerned: although during the observed lessons the examples given were close to the guide's suggestions for those using the booklet, for the other students, using the LOGO worksheet, the teacher adapted the material to provide examples. Teacher 'inputs' are specially noticeable in the 'conclusion' and 'promotion of links' rows, because the teacher finished each lesson promoting links between the two different activities developed by the students, and concluding the common results obtained (columns '2' and '3').

In contrast to the behaviour observed with group **8T**, homework was assigned to group **7M** by the teacher without using any written materials. Feedback for this group was also provided by the teacher (during all the lessons observed) without the aid of any material (the answer book for the booklet adopted was not brought to the class).

To conclude the analysis of figure 8.7.2, the '@' column shows that: (1) during the initial lesson on the topic with each group, teacher G does not give the students any reference material, but reference material was provided in all other lessons with both groups; (2) during one of the lessons (in both cases, the lesson that gave continuity to the activities taken from the chosen materials) the teacher did not give any examples, but examples were given during the other lessons with both groups; (3) during the lessons using no materials, teacher G does not promote differentiation within the group, but differentiation occurs during the lessons with materials; and finally (4) there are lessons when homework is not given to the students, but both groups observed had homework assigned once.

Summary of the Analysis

Summarising the analysis on the ways teacher G chooses and uses materials, it can be said that, when using the textbook, teacher G also follows its general guidelines. Teacher G prefers to introduce the topic without using the textbook. This introduction is not based on the materials, and usually promotes links with previous knowledge. The textbook is carefully introduced after the initial activities, but once students are already using the book, they are supposed to 'keep going' up to the end of the topic. Teacher G usually has students within the same group working on the same topic, differentiation among them being based on their own pace in following the proposed activities and on the textbook's extensions.

Although teacher G uses the textbook in a settled way which applies to all her groups, there was also opportunity to observe her behaviour when forced to move away from it when facing a crisis: she responded by planning new differentiated activities to complement the textbook and used other sources of materials to do so. In order to adapt to this uncommon situation, some different strategies were used: (1) she offered conclusions each lesson with group **7M** but expected the students to read the conclusions in the book by themselves with group **8T;** (2) she adapted the materials used several times with group **7M** while with group **8T** she followed the book closely; (3) homework and feedback were provided by the teacher with group **7M,** while both headings were dependent on the book with group **8T.**

8.8. Analysis of Data Concerning Teacher H.

Teacher H is the busiest teacher in his department: he not only teaches six groups of students, but is also second in the department and thead of key stage three mathematics in school H, a comprehensive secondary school which also runs a sixth form in the Midlands, UK. Teacher H completed a mathematics teaching degree sixteen years ago and has been working in school H for ten years.

School H has an adopted textbook: the Heinemann Series for years seven to eleven. There is one main textbook for years seven, eight and nine, with extension textbooks for those students needing 'stretchers', and for year ten and eleven there are three versions of the book: the foundation, intermediate and upper versions. The textbook series also offers worksheets, support worksheets, assessment, teacher notes and answer books for all years.

The school also has a scheme of work. The scheme is organised by topics for each year group and states that the textbook is to be used as the main resource, but also offers suggestions of materials for support and extension of work and, for specific topics, it suggests a change of text. It was noticed that the scheme became more dependent on the textbook for the upper years: in year seven, the school is piloting a computer system developed by a British University, and two half lessons out of three in a week are spent at the computer. These activities are completely individualised and managed by the computer system itself, with little interference from the teacher. The school scheme also suggests several alterations in the text to be used for years seven and eight. Support and complementation are suggested in almost all topics, based on another series of textbooks or different sources of printed materials. From year nine upwards students are set in eight groups according to 'attainment level'. The top group uses the upper course of the textbook, the next two groups use the 'intermediate', followed by three groups using the 'foundation' course. The school scheme is strongly based on the textbook in all cases, including the order in which topics are presented. Finally, the two 'bottom' groups use the S.M.P. green series (in fact, the scheme of work for these groups in unique for years nine, ten and eleven, and the students are expected to use one of the green books after another, individually).

When asked his opinion about the way the books are used, and the school scheme is set, teacher H says:

> In fact I participated in the elaboration of the school scheme... so I basically agree with it... The younger students need more variety of materials... they work better this way... and the computer system we are piloting has presented excellent results in complementing the basic arithmetic and number skills of the students... The students in year nine upwards work better when set in groups... we have tried to use the textbook with the bottom group... but it did not work well, so we have decided

> to split the bottom group in two... each one with less than ten students ... and we give these students the opportunity of working at their own pace... you are going to see that they really work at completely different paces...

The school scheme also states that the department believes that each student should have a copy of the textbook for the whole year. The students are supposed to care for the book, and this policy is independent of year group or student ability level.

Teacher H says:

> We lose a few volumes each year... they are destroyed or lost... but even though we believe that the benefits of giving each student a copy of the textbook is worthwhile... It sometimes lead to some problems... for example, at the moment all copies of the year ten upper series are with the students... and two pairs of students are actually sharing a book... it means that I usually do not have the material to prepare my lessons... because the only copy of the teacher guide is lost and there are no copies of the textbook left at school... How I manage this situation?... well... usually while the students are working through the exercises, I borrow one copy of the book and have a look what is coming next... it means that the lessons sometimes are not well prepared at all, but I still prefer the students to keep the book... so they can use it at home for their personal revisions... these students will be facing exams next year, you know...

Finally, when asked about the progression of the contents, teacher H says that it is based on the school scheme in all cases, 'which means to say that it is basically decided by the school for years seven and eight and by the book for years nine, ten and eleven'. During a week, all three of teacher H's lessons with a year seven 'mixed-ability' group (HH7X), one of the two year ten 'bottom' groups (HH10L), a year nine 'medium' group (the second out of the six groups that use the textbook series) and the year ten 'top' group were observed. Table 8.8.1, the Case Matrix for the headings in general decisions (set I) taken by teacher H, summarised the introduction above, and is based on table 4h.1 in Appendix 4h. In the following section, a comparative study for the four groups of students during these lessons will be developed, considering the headings in sets II and III.

Set I - Headings: General Decisions **Students' Group→**	**HH7X**	**HH10L**	**HH9M**	**HH10T**
Main Source of Materials for Class Work	5	7	7	7
Progression of the Content	5	7	7	7
Materials given to students for reference	8	8	8	8

table 8.8.1 - Case Matrix for the first set of headings (general decisions) - Teacher H.

Comparative Study of the Headings in the Second and Third Sets for Teacher H's Four Groups of Students:

Tables 8.8.2 and 8.8.3 are the summarised versions of the Case Matrices for teacher H, each one relting to one set of headings. Based on these tables, an overview of the choice and use of written materials by teacher H with each group of students is presented. As there

were lessons when two behaviours under the same headings were observed, two codes can appear in the same table's cell. They were developed based on the tables of summarised data presented in Appendix 4h.

Student's Group →	HH7X			HH10L			HH9M			HH10T		
Lesson →	1	2	3	1	2	3	1	2	3	1	2	3
Set II: Choice of Material to:												
Source of Activities	4ab	1	1/6	7ab	7ab	7ab	2	2	2	7	7	7
Introduce a new topic.	-	5	5	7ab	7ab	7ab	2	-	2	5	5	5
Introduce a lesson.	4ab	5	5	7ab	7ab	7ab	2	2	2	5	5	5
Reference in classroom.	4	7	7	7ab	7ab	7ab	1/2	1/2	1/2	7	7	7
Exemplification.	4ab	1/5	1/5	7ab	7ab	7ab	2	2	2	7	7	7
Conclude a topic.	4ab	-	5	7ab	7ab	7ab	-	-	2	6	6	6
Promote links between topics.	4ab	5	5	7ab	7ab	7ab	2	2	2	6	6	6
Promote differentiation.	4ab	1/4ab	1/4ab	7ab	7ab	7ab	@	1	@	@	@	@
Remedial material.	-	-	-	1	1	1	-	-	-	-	-	-
Source of homework.	@	1	1	@/7	@/7	@/7	@	2	@	7	7	1/7
Provide Feedback on Exercises.	4b	1/4b	1/4b	1/7b	1/7b	1/7b	1	1	1	7b	7b	1/7b

table 8.8.2: Set II Headings - Case Matrix for Teacher H.

Group HH7X: As stated in the introduction, the school is piloting a computer system with the objective of revising the contents for key stage two. The system is auto-managed, which means to say that students interact with the computer straight away, with little interference by the teacher. Each student works for twenty five minutes at the computer, on activities proposed individually to them, based on their answers to previous exercises. The teacher has divided the group into two sub-groups, by level of ability (in fact, he said that at the beginning of the year the sub-groups were mixed ability and he went on changing students from one sub-group to another until they could be considered as the 'bottom half' and the 'top half' of the group). Each of these sub-groups works for half of the lesson at the computer, while the teacher works with the other group. The students have been working in this way for the whole year and by the time of the observed lessons it all happens quite naturally: half of the students go straight to the computers and start work, without asking any help from the teacher. The other half sit in the 'middle' of the classroom, and work on activities proposed by the teacher. The system stops by itself after twenty five minutes work, and the students tell the teacher they are finished. The teacher asks the students to change places, and the ones in the 'middle' go to the computers and vice-versa. It was observed that the teacher kept the whole group on the same topic, but differences in instruction were noticed between the two halves. For example, while using SMP, the students in the top-half were working through the extension booklet, while the others were working through the 'core' one. During the lesson

introducing the concept of 'mean', the teacher proposed more exercises for the top-half than for the other students. The teacher stated that in his opinion the level of difficulty of these extra exercises was higher than the common ones.

Student's Group →	HH7X			HH10L			HH9M			HH10T		
Lesson → / Set III: Use of Material to:	1	2	3	1	2	3	1	2	3	1	2	3
Source of Activities	7ab	1	2	7ab	7ab	7ab	6	6	6	7	7	7
Introduce a new topic.	-	2	2	7ab	7ab	7ab	5	-	5	5	5	5
Introduce a lesson.	7ab	2	2	7ab	7ab	7ab	5	5	5	5	5	5
Reference in classroom.	7	2	2	7ab	7ab	7ab	2	2	2	5	5	5
Exemplification.	7ab	1/2	1/2	7ab	7ab	7ab	6	6	6	6	6	6
Conclude a topic.	7ab	-	2	7ab	7ab	7ab	-	-	5	5	5	5
Promote links between topics.	7ab	2	2	7ab	7ab	7ab	5	5	5	5	5	5
Promote differentiation.	7ab	1/7ab	1/7ab	7ab	7ab	7ab	@	1	@	@	@	@
Remedial material.	-	-	-	1	1	1	-	-	-	-	-	-
Source of homework.	@	1	1	@/7	@/7	@/7	@	6	@	7	7	1/7
Provide Feedback on Exercises.	7ab	1/7ab	1/7ab	1/7ab	1/7ab	1/7ab	1	1	1	7ab	7ab	1/7ab

table 8.8.3: Set III Headings - Case Matrix for Teacher H.

Group HH10L: This group uses the SMP green series on a regular basis. During the observed lessons, three different volumes were being used, because each student follows the series at his/her own pace. During all observed lessons, a routine was followed: the teacher started the lesson by asking the students to carry on with their work. He also asks if some student has finished a section of the book or if there are students in need of help. Teacher H start so by providing SMP assessment activities for those students who had finished a section of the book and by helping those students who asked for extra support. The teacher also verified the work of the other students during the lesson, and set homework for those who were finishing one section (usually the teacher asked these students to do the revision exercises so that they could have their assessment in the next lesson).

Group HH9M: Teacher H commented that he used the textbook during the whole year with these students but not during the observed lessons, and the notebooks of the students show exactly that. Nevertheless, during the observed lessons, the teacher was developing an experimental activity designed to integrate mathematics and science. Teacher H explained that the science teachers would be asking the students to plot linear graphs from experimental work during the following week, and the teachers in that department had asked the teachers in the mathematics department to do some work on plotting linear graphs and calculating gradient before the experiment took place. One of the teachers in the mathematics department

had designed a series of six worksheets starting from plotting graphs given a table and a scale and finishing by calculating gradients of the graphs of straight lines. During the observed lessons, the teacher used the worksheets, selecting exercises and providing extra explanations. Homework was set once, also using the worksheets, and the textbook was not used during these lessons.

Group HH10T: The main characteristics observed during these lessons were the options given by the teacher to the students. The group was kept together and the teacher started the lessons by introducing the topic that was going to be developed during the lesson. The introduction was based on the book, and the examples given were selected from the ones presented in the book. The teacher gave the students the option on how many practical exercises they wanted to do on the topic, by asking them to do at least two items from the first two exercises proposed by the book. After that, he suggested that they should work through the 'wordy problems', and complete the topic at home. Teacher H often uses the question asked by one student to complement the explanations for the whole group. The students are supposed to verify their answers using the answers provided at the end of the textbook. Twice it was observed that the students thought their answers were correct and the answers provided by the book were wrong. In these cases, they ask the teacher to solve the problem.

To conclude the analysis of tables 8.8.2 and 8.8.3, two more features are analysed here: (1) the answers to the 'match' questions; and (2) the sub-categorisation. Both tables show that all the activities were matched with the main activity developed during the lesson. When the use of sub-categorisation is considered, it is clear that teacher H uses the 'keep going' strategy during all lessons with group **10L**, and eventually with group **7X**, when they are expected to finish some activity that started in the previous lesson. The sub-categorisation was also applied in all groups (except for **9M**) when considering the way feedback was given to the students. It is possible to conclude that teacher H allows students to use answer books by themselves, and it probably did not occur with group **9M** because there was no answer book to be used.

To complete the picture on how teacher H chooses and uses written materials, two other data displays are used: the conceptual matrix graphs, presented in figure 8.8.1 for choice and 8.8.2 for use of materials.

Analysis of the Ways Teacher H Chooses the Materials.

The overall look of figure 8.8.1 shows that teacher H uses a variety of teaching materials. Nevertheless, it has to be considered that teacher H admitted (and the students' notebooks confirmed) that group **9M** was not having a typical week, as they also usually use the textbook as the main source of class activities. The choice of material shows that teacher H follows the suggestions of the school scheme when using materials with the students, but also shows the school scheme to be changeable. Teacher H says that if the experience of introducing extra graphic work with year nine proves to be successful, it will be included in the school scheme for the next year.

The overall look of the figure also shows that the way materials are chosen seems to be group related. In fact, the school scheme suggests a different textbook for group **10L** and it seems that this choice is reflected in the way the choice of materials is made for the different activities developed in the classroom: While for group **10L** teacher H uses the textbook as main source for introductions, examples, conclusions and links, with groups **7X** and **10T,** which were also observed using the recommended textbook, teacher H chooses to complement the textbook for all these activities, including own created examples in the case of group **7X**.

'Reference Material' row shows that whenever teacher H is using the recommended material for the development of the lesson, no other reference is given to the students. On the other hand, when using the material developed in the school with group **9M** teacher H complemented the reference given by the material with his own references.

The group also seems to influence the decisions made on choice of materials to promote differentiation. This row in figure 8.8.2 shows that teacher H chooses the material in order to promote differentiation with group **10L**. One of the materials used with group **7X** helped the teacher to promote differentiation, but he also differentiated the activities for this group even when the material chosen did not promote the differentiation itself. On the other hand, the material chosen to be used with groups **9M** and **10T** by teacher H seems to suit the whole group instead of promoting differentiation on the activities developed by the students during observed the lessons.

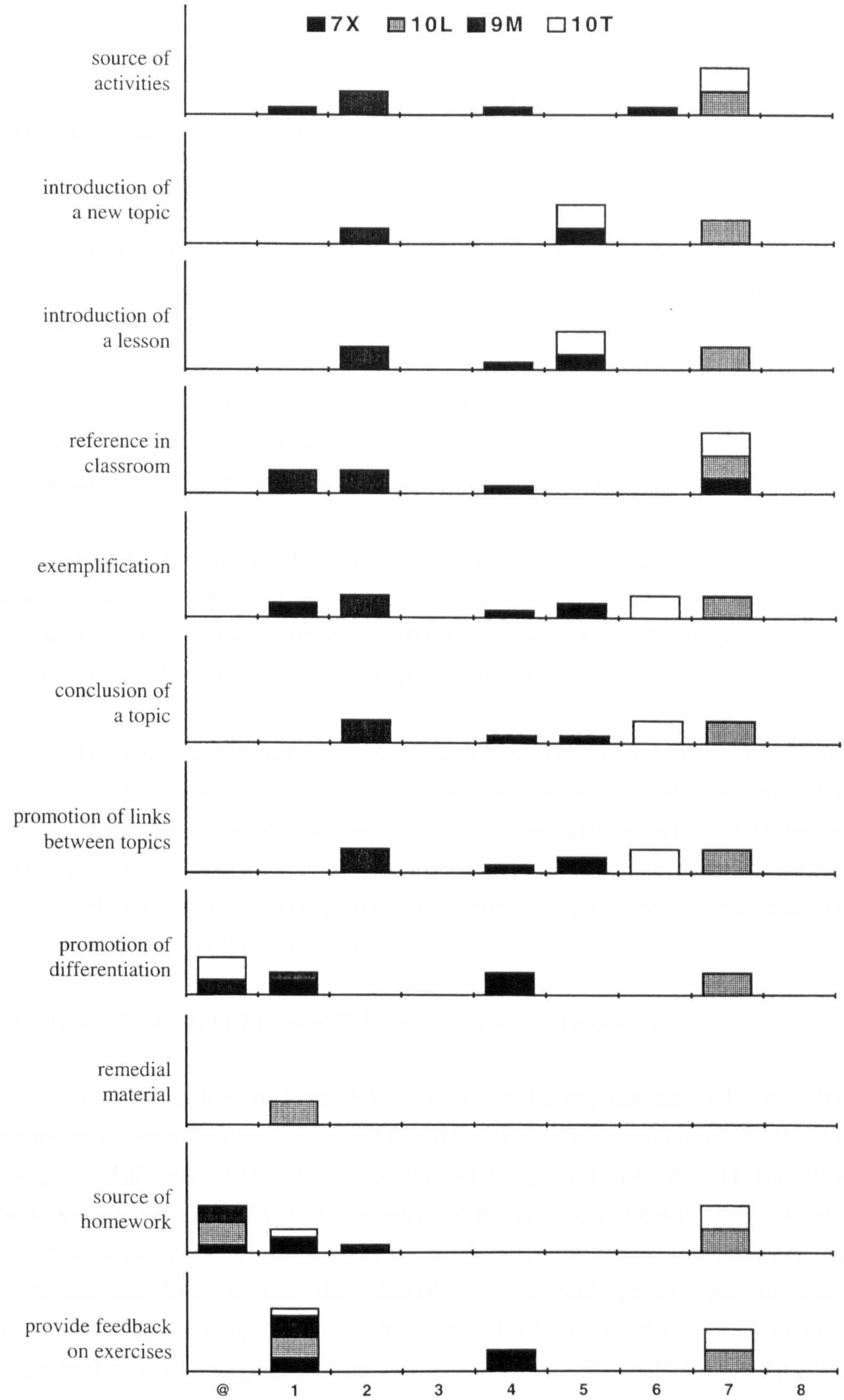


figure 8.8.1: graph of the conceptual matrix for choice of materials - teacher H.

'Remedial Material' row shows that the teacher did not introduce materials to respond to 'crisis' situations. In fact, this row shows that teacher H is usually successful in applying his planned materials with the students, and 'crisis' situations were only observed with group **10L**, when teacher H decided to provide extra support himself, without changing materials.

On the 'source of homework' row it can be observed that all students working with teacher H had homework assigned at least once a week. Students in group **10T** had homework assigned every lesson observed, while students in group **10L** had homework assigned individually, implying that different students were assigned homework during different lessons, usually when the student finished a chapter. With this group it was observed that the two students who were using the G3 book (all other students were using either G4 or G5) were the only ones that did not have homework assigned during the week, which means to say that they were the only ones that remained on the same chapter for the whole week.

Finally, the 'feedback material' row shows that the answer books are used by the teacher whenever they are available, but teacher H also provided feedback without the aid of the answer book. In fact, with group **10T** the students asked for feedback from the teacher whenever they disagreed with answers provided by the book.

To conclude this analysis it is interesting to notice that group **9M** is the only example in the sample where a material developed by the school not based on printed materials was observed. Teacher H seems to have chosen the material as basis for almost all activities developed during the lesson, including homework assigned to the students. Nevertheless, teacher H has chosen to complement the reference given by the material with his own references written on the blackboard.

Analysis of the Ways Teacher H Uses the Chosen Materials.

The overall look of figure 8.8.2 shows that the way the material is used by the teacher seems to be group related. The use of the materials can be considered as 'close to the material' for group **10L**, complemented by the teacher for groups **9M** and **10T** and either 'close' or 'independent' for group **7X**. It is interesting to notice here that the use of material classified as 'close' with group **7X** is related to the use of the SMP series as complementary material to the textbook, and SMP is also the adopted textbook with group **10L**. In fact, during the interviews, teacher H said that he believes that SMP presents the contents in a clear and well organised way, which could explain why teacher H asks his students to 'keep going' whenever this particular series of books is being used.

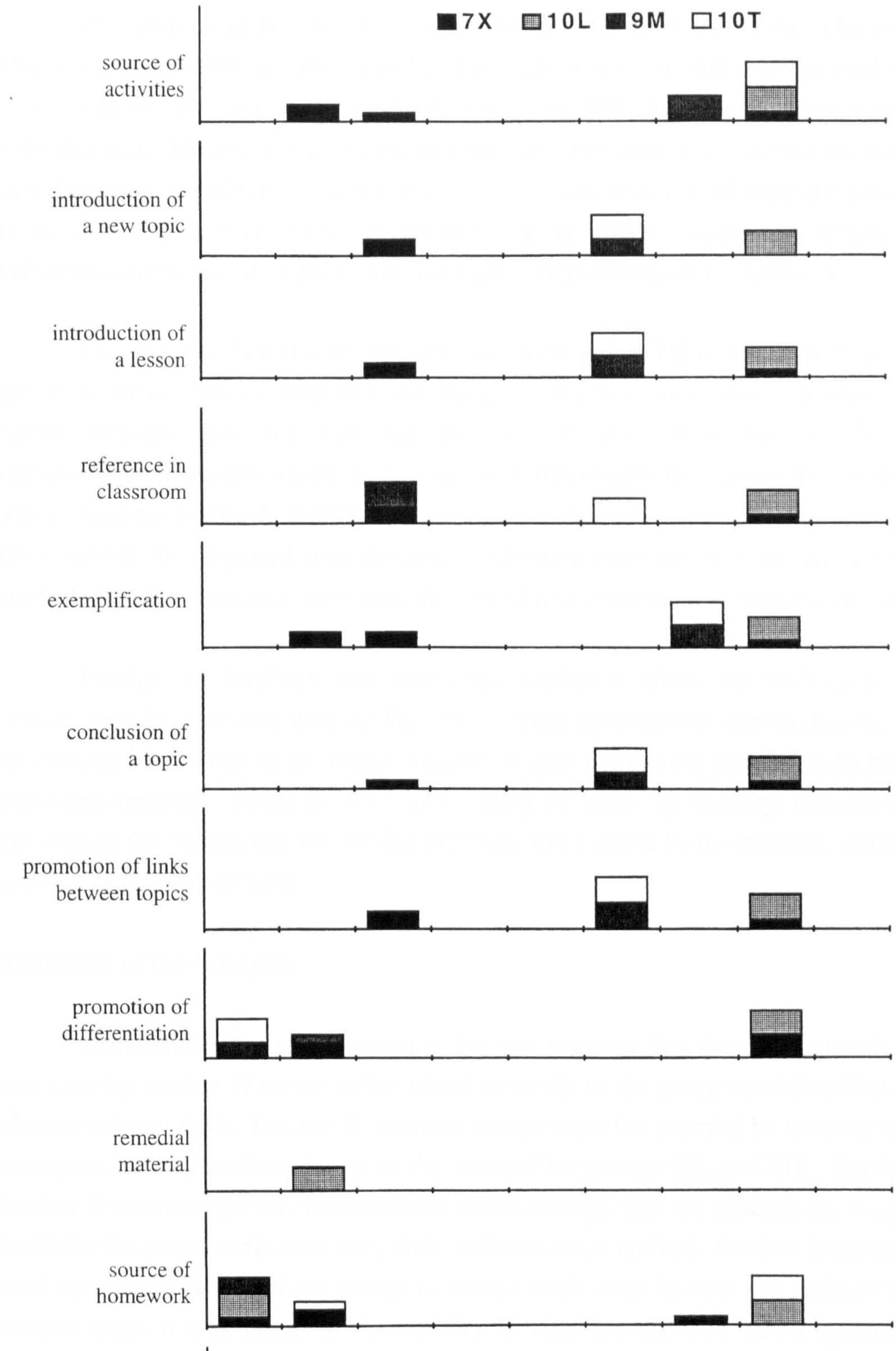


figure 8.8.2: graph of the conceptual matrix for use of materials - teacher H.

The analysis of how teacher H uses the material produced by the school with group **9M** is also interesting. In fact, it can be observed that the material is being used in a similar way to teacher H's use of the textbook with group **10T**. Teacher H selects activities from both materials. He also complements and provides introductions, conclusions, examples and links between related topics. Notice that teacher H needed to complement the school material to be used as reference, while for the textbook he usually makes very simple selections, indicating that he seems to prefer the way topics are presented in the textbook.

The way teacher H uses the textbook with group **7X** is different from the pattern above. As he uses the material with half the group at a time, he is able to develop the contents further with the more 'able' half than he is with the other. It is also clear that teacher H introduces more modifications and complementations with this group than with the others while using the textbook. During the two lessons observed where contents were taken from the textbook, he proposed own developed activities most of the time, and only when the students finished these activities were they asked to use the book as a source of exercises.

Finally, the 'feedback' row shows that teacher H allows the students to consult the answer book in their own time (in fact, the adopted textbook for year ten has the answers to all exercises at the end of the book). Teacher H does not use the answer books himself when providing feedback, which he does in a variety of ways: by marking exercises, by giving answers on the blackboard, by solving exercises when asked by the students, and by checking answers in the answer book.

Summary of the Analysis

Summarising the above analysis, the data suggests that the way materials are chosen and used by teacher H seems to be related not only to the group considered but also to the chosen material itself. Teacher H seems to complement the material he is using by providing examples and explanations based on the material for groups **9M** and **10T** . For these groups teacher H seems to prefer class teaching as the strategy and the students are supposed to do basically the same work, with very little differentiation noticed. Another interesting strategy used by teacher H with these groups is related to the way he uses the students' difficulties: several times it was observed that teacher H used the question asked by one student to provide extra explanations for the whole group.

On the other hand, with group **10L** teacher H works in a completely individualised way, and no class teaching was observed. With this group his strategy seems to be to allow the students to work through the material at their own pace, with eventual 'input' from the

teacher whenever the students face a 'crisis'. This was the only group in the sample that had individualised homework.

Finally, with group **7X** teacher H seems to use written materials in a variety of ways. During the observed lessons, teacher H asked the students to 'keep going' with their individualised work when using SMP as a complementary material. He also provided complementary explanations and activities when basing his teaching on the adopted textbook, as well as asking the students to use the textbook as source of activities. Teacher H adds more 'inputs' with the half-group considered 'more able', and allows the students to work at their own pace at the computer system the school is piloting.

8.9. General Summary Analysis: Categorising the Ways Written Materials are Used by the Teachers.

In this chapter, many aspects relating to teachers' decisions concerning choice and use of written materials were described. In this section, those parts which could be considered relevant to categorise the teachers in the sample as users of such materials are summarised. One could not expect to take into account all the aspects highlighted in this section to generate the final categorisation, due to the small size of the sample. Nevertheless, these aspects constitute an important part of the results in the present work.

General Decisions Taken by the Teachers

First of all, it was observed that all teachers in the sample used printed materials as resources for their class work. Even those who considered themselves as non-textbook users, used printed matter most of the time, and no teacher used only 'own produced' materials. The analysis of the data also suggests that the way these teachers chose materials (i.e. dependence or not on a textbook) and the ways they made modifications and adaptations on the chosen materials (i.e. ability or not to act independently of material) are two different issues, and not necessarily related.

The line between those who consider themselves non-textbook users and those who do consider themselves textbook users seems to be a thin one. Note that some choices of materials made by teachers E and F are quite similar, despite the fact that they consider themselves to be in different groups. Also, a line cannot be draw between those who do not use any written materials (or who use only own produced materials) and those who rely on printed matter for their class work. In fact, almost all the teachers at some point either used some of their own (or school) produced material to complement the printed materials used or were observed using no written material at all for class work.

As far as teacher's planning is concerned, except for teacher G, who had a written plan for those lessons without written materials, teachers in the sample did not write their lesson plans. Although some teachers mentioned the school scheme of work as a major source for planning, most of them relied on short notes, their previous experiences and their choices of materials as plans for their lessons. It must be said here that some of these lessons did seem to be carefully planned, and the choice of material played a central role in these plans: the sequence of lessons observed with the T-group of teacher B is a good example on how carefully planned lessons can be even if no written plan was presented. On the other hand, there were also a few occasions when lessons were observed and the teacher recognised that he/she was uneasy working with the chosen material (teacher A's T-group or teacher H's M-group can be considered as examples).

Overview of the Adopted Strategies for Use of Written Materials in Classroom

Strategies in choice and use of materials will be discussed further in this work during the analysis of data relating to the second research question, in the light of research on teacher's decision making. Nevertheless, some important aspects of these strategies were quite influential in the present analysis, and are summarised in this section.

Although 'keep going' lessons were first described in this work when reporting the behaviour of some textbook users in the sample, they were also observed in lessons in which the teachers did not use textbooks. Nonetheless, these strategies were far more frequent among teachers who adopted textbook series. Among the teachers observed, only teacher F (a textbook-user) was never observed asking students to 'keep going'.

The 'keep going' classification was an important tool to describe teacher's behaviour when using written materials, and was used in a variety of situations to fulfil different purposes. Teacher C adopted this kind of behaviour with all her groups of students most of the time. Teacher B was observed adopting it with all groups but the 'top'. Teacher E was observed adopting it regularly only with the 'top' group, while Teacher H adopted this behaviour regularly only with the 'low-attainers' group. Teachers A, D, and G oscillated between teacher-led lessons and 'keep going' lessons.

Differences were also observed in the ways 'keep going' was used as strategy. While in some instances the teacher seemed to use an approach in which students were 'expected to manage their learning on their own' (Brophy and Good, 1983, pg. 339), on other occasions teachers established 'targets' that should be fulfilled in a certain period of time. In this case, teachers kept these groups of students 'together', at least as far as assessment was concerned. Whenever a teacher who did not use textbooks was observed using 'keep going' strategies, this meant that some of the students never had the chance to finish the proposed tasks, because the whole group of students was supposed to change to a different material at the same time.

Concerning how the teachers spend their own time during lessons, several different behaviours were also observed. Although these behaviours are related to choice and use of written materials, these matters do not provide a complete explanation for them, as different behaviours could be associated with similar choices and, on the other hand, similar behaviours were observed associated to different choices of materials:

(1) although the teachers in the sample who did not adopt textbooks introduced each new activity carefully, this behaviour was also observed within several groups taught by teachers using textbooks. On the other hand, teachers who used a textbook and introduced lessons tended to be more concerned with pre-requisites and links with other topics than those not using textbooks. Notice also that those teachers who used textbooks and did not introduce lessons could rely on the book to provide students with the necessary links and pre-requisites for development of the topic;

(2) marking students' notebooks with the aid of the answer book during lessons was an activity developed only by textbook users in the sample. Nevertheless, several teachers who used textbooks did not mark during lessons, spending most of the 'seatwork segment' (Koehler and Grouws, 1992) of the lesson helping students who faced difficulties or verifying students' work, in a way similar to the non-textbook users;
(3) although several teachers were observed when they introduced topics, few were observed emphasising the main concepts when developing the topic, summarising the principal results obtained or asking the students to think about conclusions they could draw from the activities developed. As Good and Biddle (1988) argued, most students would 'benefit from high-quality teacher statements about *mathematics* content [authors' italics]' (pg. 132). If it is considered that textbooks usually provide conclusions for the topic, it seems that those students who did not use textbooks had less chance to draw conclusions from their activities. In the light of the theories of Vygotsky, Ausubel and Piaget, this observation leads to an hypothesis that these students (without the aid of a textbook) would have less chance to apply their newly acquired knowledge in new situations.

An implication that could be extracted from the publications reviewed in chapter four advising teachers on written materials is that textbooks are not 'flexible' enough. In other words, it is suggested that teachers who did not use textbooks were more likely to change materials when necessary. However, the results of this research suggest that there are no real links between change of materials and use of the textbook as main source. On the one hand, teachers E and F seemed to adapt the chosen materials to their objectives. For instance, teacher F decided whether or not to use the textbook at each topic: she was observed both using it closely and not using it at all (in one of the observed groups, she developed a complete topic, which did exist in the adopted textbook, without using it during the lessons). Teacher E was observed giving his L group a choice between two different materials which he considered equivalent in fulfilling his objectives. Also, he was observed using a printed material for a different objective than the one for which it was originally designed. On another occasion, he was also observed using a textbook activity closely. On the other hand, teachers A and C seemed to be bound by their original choice of materials. Teacher A was never observed moving away from the suggestions made by the school scheme of work. Even when he had to introduce remedial material to complement an unhappy choice, this was designed to fulfil the original objectives of the chosen worksheets, on which it was strongly based. Teacher C used the textbook most of the time, and all material used to complement the textbook was strongly based on it. The other teachers in the sample can be considered as textbook users who 'on occasions' moved away from it, either because their previous experience showed that the chosen textbook was not the best material to be used in the situation (teachers B, D, and H) or because they had to face a crisis situation by changing or complementing the chosen material (teachers B and G).

Reference Materials

A hypothesis that could account for a real difference between textbook users and non textbook users concerns whether or not reference materials could be taken home. But, although the students whose teacher adopted a textbook could have the advantage of keeping the textbook, this was not necessarily true, due to different school policies. Teacher H's school policy clearly stated that each student should be the 'owner' of his/her book. On the other extreme, no group of students in teacher G's school was allowed to take the textbook home regularly. Whether or not students keep reference materials seemed to be one decision on which school context factors affected teacher's decisions, as suggested in the literature (Millett and Johnson, 1996; Hoyles, 1988).

Where 'low attainers' groups of students were concerned, the teacher's attitude about taking material home seemed to be even more radical: the students in some of these groups were not allowed to take even their notebooks home (teachers E and F L-groups are examples). Several teachers in the sample stated that they did not believe that these students would use the materials at home, and most of them were concerned about loss of, or damage to the material. This did mean that these low attainers had no homework assigned and were not expected to revise at home. It is interesting to notice that the loss of material was not a real problem in teacher H school, in which students were considered responsible for their material. This leads to a hypothesis that teacher's and school's expectations seems to 'affect student's attitudes' as stated by Good and Brophy (1997, pg. 105).

No teacher in the sample relied completely on a single textbook series. As most textbooks adopted in schools were printed before implementation of the N.C., they did not include enough MA1 activities. Most teachers in the sample based their 'investigations' on printed materials which were introduced to them during 'in-service' courses. Most of these materials were not related to the textbook adopted by the school. Note that it was not possible to find two teachers in the sample who had a similar behaviour when developing these activities in class. While some teachers believed that investigations should be developed preferentially in mixed-ability groups, others believed that better results were achieved in groups set by attainment levels. Some teachers based the development of a complete year's curriculum on investigations, others kept these activities to the minimum allowed by the N.C.. Some teachers chose the investigations they were going to use carefully, others used investigations not selected by themselves. Nevertheless, it became clear that each teacher in the sample had chosen a set of investigations to be developed with their students beforehand, which was not likely to be easily changed (even if the reasons for a particular choice appeared to be completely different in each case).

Analysis of Strategies Adopted with Different Groups of Students

The fieldwork for this research was based mainly on schools in which the younger students were kept in mixed ability groups and the elder ones were grouped according to their

levels of performance. One of the objectives of this research was to verify whether or not there were differences in the way written materials were used within the different groups of students taught by the same teacher. In fact, this research shows that teachers in the sample could be classified in three groups according to these criteria:

(1) The majority of the teachers in the sample seemed to change the way they used written materials according to the group considered. Teachers B, D, E, F, and H used written materials in different ways, depending on the group considered. Their strategies were summarised at the end of their respective sections in this chapter.

(2) Teachers C and G resorted to the aid of written materials in a way that did not seem to be group related. They made up their minds on how adopted textbooks would be used and consistently applied it with all their groups. Even when changes to the materials had to be made, these teachers were concerned whether the objectives of their first choice would be fulfilled.

(3) Teacher A varied the use of written materials in his classes in a way that did not seem group related. He adopted several strategies in different groups in a way which depended more on the topic considered and the material chosen than on the group.

When teaching mixed ability groups, teachers presented different behaviours and strategies. These could be as different as

(1) using the fact that the group was mixed ability to 'challenge' the 'low-attainers' by keeping them working together with the 'high attainers' and giving them the same written material or

(2) set completely individualised tasks most of the time, or

(3) split the group in sub-groups, considering the level of performance.

It has also to be considered here that some of the 'mixed ability' groups in the sample did not include the 'low attainers', because they had already been moved into a smaller group.

As far as the 'low attainers' groups were concerned, the data also show different strategies and solutions being used by the teachers, varying from individualised work using written materials (or alternating individualised work using written materials with other activities) to teacher-led lessons without using any written materials. These groups usually had a different treatment from others in the same school: they were smaller and it was observed on two occasions that the teacher had the aid of an auxiliary teacher. It was also observed that these students were not usually allowed to take printed materials home (as discussed before, sometimes they were not even allowed to take their own notebooks home). Apart from teacher F, who clearly had special strategies for this group, and was concerned with issues such as their self-confidence when doing mathematics and improvement of their level of achievement, no other teacher in the sample seemed to have designed any special strategy for using materials with these groups, apart from those intended to keep them on task (teacher E had some ideas concerning homework, but did not put them into practice).

Except for teacher F, who was observed developing a complete topic before moving to another, it seems to be a common belief amongst the teachers in the sample that low attainer students cannot concentrate on the same task for several consecutive lessons, and these students were frequently observed being moved by the teachers from a task to another without regard as to whether or not they had finished the initial task. Most of the time, this behaviour implied that the students had to go back to the same task again later (if there was an adopted textbook) or the task would remain unfinished (if there was no textbook). Analysis of notebooks confirmed that most teachers in the sample seemed to alternate Number work and Shape and Space work with these students most of the time (even for the elder students). Little was done to develop the other ATs in the N.C., forcing the hypothesis that it would be nearly impossible for one of these students to improve his/her personal level of achievement and move to a 'higher' group, as the 'gap' between what is done in his/her group and what is done in the 'medium' ones seemed to widen throughout the course.

The 'medium' groups in this research can be considered the least homogeneous of all, in the sense that they varied from the second in a school which has nine groups per year to the third of four possible ones. The data seem to point to the conclusion that these groups were less likely to receive any special written material from their teachers: teaching them seemed to require fewer decisions and strategies than teaching the 'top', 'low' or 'mixed' groups. Usually the solution for these groups was to keep them in activities parallel to the ones developed by the 'top' groups, excluding the most 'sophisticated' or challenging problems (for the 'high medium' groups) or emphasising every day problems related to mathematics, with the most challenging topics taken out altogether (for the 'low medium' groups). Although these differences can be easily observed in the different 'tracks' of the textbooks adopted, they could also be noted for those teachers in the sample who did not adopt a textbook.

Different solutions were also observed when analysing the ways materials were used with 'top' groups. Some teachers in the sample seemed to believe that these students did best when working in a completely individualised way, allowing each one to achieve his/her best individual result, while others seemed to believe that these students should benefit from each other's progress and difficulties and that better results could be achieved if the group was kept together (or, at least, working through a given task in small groups). Usually, these students had a special set of materials dedicated to them: they were the only groups in teacher E's school using textbooks and, in schools adopting textbooks, they were the only group using the 'higher track' of the textbook series. Most schools had extra worksheets specially prepared for them. The concept of 'stretching' seemed to be applied to these students more frequently than to the others. When referring to these students, teachers in the sample did not use expressions such as 'each one working at the best of his/her ability', 'they must be made to feel comfortable with their task' or 'asking them to go further could affect their confidence',

frequently used in justifying why other groups in the school were not equally challenged. Instead, it was common to hear teachers say that 'some extra material' (or some extra planning) was needed to 'attend the demand of the most able' or that 'these students benefit from taking the material home'.

Concluding the present analysis, it is important to state that all the teachers in the sample were volunteers, so they can be considered confident teachers, who believed in the way they choose and use written materials for classroom work. Most of them had participated in their schools' decisions about written materials and none of them were against the choice. made by the school. Even so, the differences discussed above in their decisions on how to use written materials are impressive. As far as choice of material is concerned, this research also shows some interesting differences: (a) the 'non-textbook user' E moved to use a textbook series for the 'top' group, justifying it with his belief that these students could benefit from a '...more structured and organised way of work in this particular textbook...', (b) the 'textbook user' F regularly avoided use the textbook with some of her groups either because '...the topic can be better developed than it is in this book...' or because '... these particular students do not benefit much from using a book...' (c) Apart from teacher G's school, all other schools where SMP 11-16 is the main textbook series adopted had other textbooks for particular cases: teacher B's school used another textbook for years seven and eight and teacher C and D's schools used remedial textbooks for low attainers, justified by their belief that the '...SMP series is not suitable for these students...'. (d) In a completely opposite position, teacher H's school did not use SMP series except with low attainer groups, saying that from '...all textbooks, SMP green track series is the best suited for low attainer students...'.

Categorising the Ways Written Materials are Used by the Teachers.

When considering the data generated in the present work, it becomes clear that at least two questions concerning (1) choice and (2) use of written materials need to be asked when considering teacher's behaviours towards written materials:

(1) Concerning choice: What is the main source of material chosen by a teacher? Does the same choice of materials apply to all observed groups? The answer to this question generates three fields for categorising teachers' attitudes:

C1. Teachers who use textbooks' series for all their groups of students.

C2. Teachers who use textbooks for some groups and do not use them for others.

C3. Teachers who do not use textbooks with any of their groups of students.

(2) Concerning usage: Does the teacher use the chosen material consistently, independently of the group considered? If not, is there evidence that differences in usage depend on the group considered? Or are there other factors involved? When considering the answer to this question, another three fields for categorisation are obtained:

U1. Teachers who use chosen materials consistently, independently of the group of students considered.

U2. Teachers who use chosen materials in different ways, with evidence that changes depend on the group of students considered.

U3. Teachers who vary the use of chosen materials in ways which are not dependent on the group of students considered but on other criteria.

The final categorisation was obtained by examining the groups formed when answering both questions above. Table 8.9.1 provides a visual aid to the final categorisation. Nine categories are defined. In each category, a specific pair (choice of materials, use of chosen materials) is contemplated, so that teachers who are classified in each one can be recognised by their behaviour in both these aspects:

use of the chosen material → choice of materials ↓	Consistent in use, independent on the group.	Changes in use are dependent on the group	Changes in use are not group related
Textbooks series for all groups.	**(C1, U1)**	**(C1, U2)**	**(C1, U3)**
Dependent on the group	**(C2, U1)**	**(C2, U2)**	**(C3, U3)**
No textbook for all groups	**(C3, U1)**	**(C3, U2)**	**(C2, U3)**

Table 8.9.1: Categorisation of teachers by choice and use of written materials.

Category **C1,U1**: teachers who adopt textbook series with all groups of students, and use these materials consistently, in the sense that there are no real changes either due to the group of students considered or to any other factor.

Category **C1,U2**: teachers who adopt textbook series with all groups of students, but use these materials in different ways, depending on the group considered.

Category **C1,U3**: teachers who adopt textbook series with all groups of students, and use these materials in different ways, where changes are due to factors other than the group of students considered.

Category **C2,U1**: teachers who alternate their choice of materials between textbook series and other sources, but use these materials consistently, in the sense that there are no real changes either due to the group of students considered or to any other factors.

Category **C2,U2**: teachers who alternate their choice of materials between textbook series and other sources, and use these materials in different ways, depending on the group considered.

Category **C2,U3**: teachers who alternate their choice of materials between textbook series and other sources, and use these materials in different ways, where changes are due to factors other than the group of students considered.

Category **C3,U1**: teachers who do not adopt textbook series, and use the chosen materials consistently, in the sense that there are no real changes either due to the group of students considered or to any other factors.

Category **C3,U2**: teachers who do not adopt textbook series, and use the chosen materials in different ways, depending on the group considered.

Category **C3,U3**: teachers who do not adopt textbook series, and use the chosen materials in different ways, where changes are due to factors other than the group of students considered.

The set of categories described fits all possible answers to the general questions posed above. In this sense, it can be used to classify teachers in general, not only the ones in the present sample. Table 8.9.2 presents the categorisation for the teachers in the sample. The categorisation is based on these teachers' observed behaviours with the different groups of students taught, supported by their interview results and analysis of supplementary sources of data, and requires no further explanation. Notice that the 'gaps' show clearly that teacher A was a singular teacher among those observed. Although there are several empty cells in the table, it is not impossible to imagine teachers who would fit those categories. It can be envisaged that further research, involving a large number of teachers, would close gaps in this table.

use of the chosen material → / choice of materials ↓	U1	U2	U3
C1	C G	B D H	
C2		E F	
C3			A

Table 8.9.2: Categorising teachers in the sample as users of written materials.

It is also important to observe that several aspects of the analysis were not summarised in this section. For example: teachers did present different behaviours in relation to the 'headings' (such as 'introduction of lessons', 'conclusions', 'generalisation of a topic', etc.). As the final categorisation is based on general decisions made by the teacher rather than classroom strategies, these issues were not examined here. A discussion on the links between the strategies on choice and use of materials adopted by the teachers in the sample and the results of the respective post-tests is presented in the following chapter.

CHAPTER 9

LINKING STRATEGIES OF CHOICE AND USE OF WRITTEN MATERIALS AND THE RESULTS OF THE TESTS

> ... although students' outcomes are indisputably the ultimate dependent variable in research on teaching, researchers need to question what outcomes are to be examined, and how data are to be gathered and interpreted .
>
> Romberg and Carpenter, 1983, pg. 861

This chapter is dedicated to investigate the second question posed in this work, namely: *Do particular identified ways teachers use written materials in their classroom work relate to better achievement of teacher's short term aims by the pupils?*

One objective of this chapter was to complete the picture of how teachers in the sample did use written materials for classroom work. In order to do so, aspects of usage of written materials other than those used to generate the categorisation in the previous chapter were analysed, namely those defined in chapter 7, based on previous research on teacher's decision making. The definition of strategies took into account not only observed lessons, but also data collected during interviews with the teachers in the sample, when they were asked to discuss further their reasons for a particular choice or use of written materials. Another objective was to establish links between observed strategies in using written materials and the achievement of teacher's short term aims. In order to do so, the different strategies observed were re-examined in the light of teachers' decisions on the use of written materials. Teacher's expectations about students' performance were also considered.

It has been decided that the results of the tests would be used in this work only as indicators of performance, in order to verify whether different aspects of the use of materials observed could be associated with different levels of performance among the groups. The complete tables of tests results are presented in appendix five. As previously argued, each test was strongly linked to the mathematics content developed by the teacher for that particular group during the observed lessons. Whenever written materials were used for class work, the test was based on them. Thus, the tests should be considered as immediate post-tests for the topics developed during the observed lessons.

Each test was examined beforehand by the teacher, who had to approve it before it was applied to the students. In view of these considerations, it was initially hypothesised that students in the sample should be able to perform at a high level in these tests, independently of the 'kind' of group which they belonged to. Not only had the pupils been previously classified into the groups by their own teachers but the teaching had also been developed

accordingly, by experienced teachers who knew beforehand that their lesson would be observed and their students tested on their short term aims.

In this sense, there was no reason to expect that one 'type' of group would perform better than others in these particular tests. Nevertheless, some variations in performance within the same group could be expected if a mixed ability group was taught as a whole. As will be seen, although the majority of the results of the tests cannot be considered poor, some of them are not as good as initially hypothesised and as should be expected for immediate post-tests, in which students were asked to repeat questions done during the observed lessons.

The small number of teachers in the sample does not allow the present research to discuss the effectiveness of one observed strategy in use of written materials, as other variables can be influencing the results (see comments in chapter three about research results reviewed by Brophy and Good, 1983). Even if the strategies were adopted by more than one teacher, the constraints imposed by testing methodology do not allow comparison or cause-effect conclusions to be drawn when examining these links.

In section 9.1, case matrices linking strategies and the results of tests for each type of group are presented. The main aspects of usage of written materials for classwork by each teacher in the sample are also highlighted. Each section, from 9.2, to 9.5 is dedicated to one specific strategy (or decision orientation on use of written materials), as described in chapter seven. The occurrences are discussed, as well as the similarities and differences among them. The results of tests associated with the strategy are also displayed and commented on. Finally, in section 9.6 the analysis of the data related to the second research question appears.

9.1. An Overview of Strategies Adopted by the Teachers in the Sample.

The main objectives of this section are (a) to provide examples of actual ways teachers in the sample used written materials and (b) to generate hypotheses on how written materials could be successfully used for class teaching, by linking the observed strategies with students' performance in the test. These are important aspects of the analysis that are not contemplated in the following sections, where the strategies on use of written materials are grouped according to similar decisions made by the teachers, as described in Chapter Seven.

In order to achieve the proposed aims, the data was summarised and grouped in two different ways: (a) according to the ability level of the groups of students and (b) according to the groups taught by the same teacher. The first grouping process allowed an overview of the differences and similarities in strategies adopted by different teachers within groups of students that were considered at a similar level of ability, and the second allowed to observe the differences and similarities in the strategies adopted by the same teacher within different ability level groups of students. Both ways of looking at the data provided useful information, complementing each other.

Summary Study according to the Ability Levels of the Groups of Students

The Mixed-Ability Groups

Seven groups were classified by their schools as **mixed-ability** groups (teacher G did not teach any). Nevertheless, the 'definition' of a mixed-ability group changed from one school to another: (1) Broad sense mixed-ability groups, where students were allocated by chance (AA8X, DD7X, EE7X, and HH7X); (2) mixed-ability groups from which the 'low attainers' group of students had already been taken out (CC8X and FF7X); and (3) a group which was considered mixed ability by its teacher because it was formed by joining together groups already set in perceived different levels of attainment (BB10X).

The description of decisions associated with written materials, teacher's performance expectations and strategies in choice and use of written materials observed in mixed-ability groups were summarised in table 9.1.1, together with the results of the tests.

One of these groups was observed while working through a complete single mathematical topic (FF7X), but the others were observed either while working through more than one topic (AA8X, DD7X, EE7X and HH7X), or in different tasks simultaneously on an individual or sub-group basis (BB10X, CC8X and second part of the observed lessons in DD7X). For those groups observed working through more than one topic, there was always some change either to choice or use of written materials. To reflect this situation, tests given to these students had two parts. For the groups working through more than one task simultaneously, different tests had to be given to account for the differences observed. In the case of individualised tests, they all had the same number of items and their result was treated as if it were a single test. For BB10X, two different tests were given, one for each

sub-group, and these results are presented separately, to account for the differences in the adopted strategies.

mixed- ability	**A8**		**B10**		**C8**	**D7**		**E7**		**F7**	**H7**	
feature →	Part 1	Part 2	Red	Yellow	Samp.	Part 1	Part 2	Part 1	Part 2	-	Part 1	Part 2
Decision Orientation	**W SaIn**	**W SaCo**	**T DiCo**	**T DiCo**	**T DiCo**	**W SaIn**	**T DiCo**	**W SaIn**	**W SaIn**	**W SaIn**	**T SaIn**	**T SaIn**
Expectations	**I**	**R**	**I**	**H**	**I**	**I**	**I**	**I**	**H**	**R**	**I**	**I**
Choice of Materials												
Introduction	3	1	7ab	7ab	7ab	3	7ab	1	4	5	4	5
Classwork	3ab	1	7ab	7ab	7ab	3ab	7ab	1	4	1	4ab	6
Conclusion	@	1	7ab	7ab	7ab	@	7ab	@	@	@	4ab	5
Differentiation	3ab	@	7ab	7ab	7ab	@	7ab	@	@	@	4ab	1
Mathematical Links	@	1	7ab	7ab	7ab	@	7ab	1	1	1	4ab	5
Use of Materials												
Introduction	7	1	7ab	7ab	7ab	8	7ab	1	4	3	5	2
Classwork	7ab	1	7ab	7ab	7ab	8ab	7ab	1	4 / 7	1 / 2	7ab	1 / 2
Conclusion	@	1	7ab	7ab	7ab	8ab	7ab	@	@	@	7ab	2
Differentiation	7ab	@	7ab	7ab	7ab	7ab	7ab	@	@	@	7ab	1/7ab
Mathematical Links	@	1	7ab	7ab	7ab	@	7ab	1	1	2	7ab	2
Levels of Questions	**4 / 6**	**4 / 5**	**6 / 8**	**7 / 8**	**4 / 5**	**5**	**4 / 5**	**4 / 5**	**4**	**5**	**4**	**4**
Results of the tests - Percentage of students in considered ranges of performance												
50 % or more of test items correct	**64**	**77**	**88**	**92**	**75**	**83**	**78**	**94**	**90**	**84**	**97**	**66**
70% or more of test items correct	**52**	**32**	**35**	**50**	**42**	**65**	**48**	**35**	**90**	**37**	**55**	**28**
Test Performance	**L3**	**L2**	**L2**	**L2**	**L2**	**L2**	**L2**	**L2**	**L1**	**L2**	**L2**	**L3**

Table 9.1.1 - Case Matrix for Mixed Ability Groups

The best result among the mixed ability groups in the sample was teacher E's year seven group. The test applied to group EE7X had two parts, each associated with a different strategy of choice and use of written materials. This group had excellent levels of performance in part 2 questions, which are those related to the investigation task developed during the observed lessons. Teacher E declared during the interviews that he was particularly fond of developing investigations in mixed ability groups. In his opinion, these groups are 'a good environment to develop investigations' because 'the students can work in small groups, each one of them mixed-ability itself '. This organisation 'allows the better ideas to be shared and discussed by all students'. On the other hand, as the students' activities were classified as level four in AT1 (see appendix five), it could be argued that these students were not working through a particularly difficult piece of investigation on numbers.

The Low Attainer Groups

Seven groups in the sample were classified by their schools as **low-attainer** groups (teacher G did not have one). The description of decisions associated with written materials, teacher's performance expectations and strategies in choice and use of written materials observed in low-attainer groups were summarised in table 9.1.2, together with the results of the tests.

Teacher A's low attainer group had different characteristics from the others. Since his school had only three groups per school year, several students in teacher A's 'low' group were at the same level as students in other teachers' 'medium' groups.

low attainers	**A9**	**B9**		**C8**		**D10**	**E10**	**F7**	**H10**
feature →	-	Part 1	Part 2	Part 1	Part 2	-	-	-	-
Decision Orientation	**W SaIn**	**W SaCo**	**T DiCo**	**T DiIn**	**T DiIn**	**T DiIn**	**W SaIn**	**W SaCo**	**T DiCo**
Expectations	**R**	**R**	**L**	**L**	**L**	**L**	**R**	**H**	**L**
Choice of Materials									
Introduction	4	1	7ab	4	4	6ab	1 / 4ab	1	7ab
Classwork	4	1	7ab	4	4 / 1	6ab	4ab	1	7ab
Conclusion	@	@	7ab	@	@	6ab	@	1	7ab
Differentiation	1	1	7ab	4ab	4ab	6	4b	@	7ab
Mathematical Links	@	1	7ab	@	@	6ab	@	1	7ab
Use of Materials									
Introduction	4	1	7ab	7ab	7ab	7ab	1 / 7ab	1	7ab
Classwork	4 / 6	1	7ab	7ab	7ab	7ab	7ab	1	7ab
Conclusion	@	@	7ab	@	@	7ab	@	1	7ab
Differentiation	1 / 2	1	7ab	7ab	7ab	7ab	7ab	@	7ab
Mathematical Links	@	@	7ab	@	@	7ab	@	1	7ab
Levels of Questions	**3/4/5/6**	**3**	**3 / 4**	**2**	**3 / 4**	**3 / 4**	**4**	**2/3/4/5**	**4**
Results of the tests - Percentage of students in considered ranges of performance									
50 % or more of test items correct	**55**	**100**	**55**	**93**	**74**	**83**	**74**	**100**	**78**
70% or more of test items correct	**22**	**55**	**18**	**47**	**47**	**25**	**42**	**100**	**56**
Test Performance	**L3**	**L2**	**L3**	**L2**	**L2**	**L2**	**L2**	**L1**	**L2**

Table 9.1.2. - Case Matrix for Low Attainer Groups

Three of the low attainer groups were observed while working through a single mathematical topic (AA9L, EE10L and FF7L). The others were observed either while working alternately between two topics (BB9L and CC8L) or while working individually from the textbook (BB9L part 2, DD10L and HH10L). For those groups observed working through more than one topic, there were differences in strategies either in choosing or using materials, and the corresponding test were in two parts. In the case of individualised teaching,

the tests were also individual, even though all tests given to a particular group had the same number of questions and their results were treated as if a single test had been given to the whole group. The were no need to test these groups by sampling, due to the small number of students in the majority.

The outstanding result of teacher F's low attainer group suggests that the methodology employed there should be better analysed as a possible recommendable way of working with this type of group. Five issues are brought to attention when analysing these lessons:

(1) they were carefully planned, with a clear aim to be achieved by all students (the aim was not too ambitious, she worked 'one step at a time');.

(2) the conclusions and main results of the topic were reinforced by the teacher, and applications of these results were given to the students as exercises;

(3) teacher F was extremely careful during the observed lessons in making sure that all pre-requisites were mastered by the students, so they could achieve the final aims;

(4) teacher F was worried with issues such as the students' self-confidence, attention and interest;

(5) teacher F made sure that all students received personal attention during each lesson (possible not only because it was a small group but also because teacher F had an auxiliary teacher to help her).

It must be said here that although teacher F was not using any printed material, these students ended up with highly organised notes from these lessons, in which the main results were highlighted and applications were developed.

The Medium Attainer Groups

The description of decisions associated with written materials, teacher's performance expectations and strategies in choice and use of written materials observed in medium-attainer groups were summarised in table 9.1.3, together with the results of the tests.

As commented before, 'medium' groups can hardly be considered as a 'type'. The definition of these groups allowed a wide range of students to be classified as such. The eight 'medium' groups in the sample were: AA10M (second out of three groups in the same school year), BB11M (third out of five), CC10M (second out of five), DD9M (second out of five), EE9M (second out of four), FF8M (second out of four), GG7M (third out of four) and HH9M (second out of nine). One can expect a group which is the second out of nine groups in the same school year to be quite different to the third group out of four. Nevertheless, note that most 'medium' groups in the sample were the second ones in their respective school years.

medium attainers	A10	B11		C10	D9	E9	F8	G7		H9
feature →	-	red	blue	-	sample	-	-	Textb.	Logo	-
Decision Orientation	W SaIn	T DiCo	T DiCo	T DiCo	T DiCo	W SaCo	W SaIn	part 1: TSaCo	WSaCo WSaCo	W SaIn
Expectations	R	R	L	R	R	R	R	R	R	R
Choice of Materials										
Introduction	1	7ab	7ab	7ab	7ab	4/3	5	1	1	2
Classwork	4/1	7ab	7ab	7ab	7ab	4/3	4/6	8/7ab	1/4	2
Conclusion	@	7ab	7ab	7ab	7ab	3	@	1/5	1/4	@
Differentiation	@	7ab	7ab	7ab	7ab	@	@	7ab	4	@
Mathematical Links	@	7ab	7ab	7ab	7ab	3	@	5	4	2
Use of Materials										
Introduction	1/4	7ab	7ab	7ab	7ab	3/4	3	1	1	5
Classwork	4/1	7ab	7ab	7ab	7ab	3/4	5/8	8/7ab	1/7	6
Conclusion	@	7ab	7ab	7ab	7ab	6	@	1/3	1/3	5
Differentiation	@	7ab	7ab	7ab	7ab	@	@	@	@	@
Mathematical Links	@	7ab	7ab	7ab	7ab	1/5	@	1/7	1/7	5
Levels of Questions	3/4/5/6	6	6	6/7	5/6	4	4/5	4/5	4/5	6/7
Results of the tests - Percentage of students in considered ranges of performance										
50 % or more of test items correct	33	100	44	100	84	79	60	66-46	66-83	74
70% or more of test items correct	0	75	0	89	42	39	20	0 -31	0 -83	37
Test Performance	L4	L1	L4	L1	L2	L2	L3	L3/L4	L3/L1	L2

Table 9.1.3 - Case Matrix for Medium- Attainer Groups

Three of these groups' teachers were observed using textbooks (teachers B, C and D), while another three were observed using other printed materials instead (teachers A, E, and F). Teacher H was using school developed material, and Teacher G was expecting to use only the textbook, but had to change her plans. In her group, some students were using the textbooks while others were using printed materials and computers to develop the same topic. Some tests had to be sub-divided (teachers B and G), because the teacher divided the group in two sub-groups. One group was tested by sampling: DD9M, because the observed lessons were completely individualised.

The best result in this type of group is associated with a group tested by sampling. It was tested that way because the teacher was adopting a individuallised strategy, based on textbook use. These students also had regularly assigned homework and were allowed to take their textbooks and notebooks home.

The High-Attainer Groups

Eight groups in the sample were classified as 'top' ones. For this particular type of group, teacher E, a non-textbook user, moved from his usual behaviour and adopted a

textbook. The description of decisions associated with written materials, teacher's performance expectations and strategies in choice and use of written materials observed in high-attainer groups were summarised in table 9.1.4, together with the results of the tests.

high attainers (top)	A10	B9	C1	D9	E11	F8	G8	H10
feature →	-	-	sample	-	-	-	-	-
Decision Orientation	W SaIn	T SaCo	T DiCo	T DiCo	T SaIn	T SaIn	T SaCo	T SaCo
Expectations	R	H	I	I	I	R	H	H
Choice of Materials								
Introduction	4	5	5ab	7ab	7	7	1	5
Classwork	4/3	6/3	5ab	7ab	7/7ab	7	8/7ab	7
Conclusion	3ab	6	5ab	7ab	7ab	3	7ab	6
Differentiation	4b/3	@	5ab	7ab	7ab	@	8/7ab	@
Mathematical Links	4	5	5ab	7ab	7ab	@	1	6
Use of Materials								
Introduction	6	5	7ab	7ab	7	5/7	1	5
Classwork	5	7/8	7ab	7ab	7ab	5/7	8/7ab	7
Conclusion	7ab	5	7ab	7ab	7ab	2/8	7ab	5
Differentiation	6	@	7ab	7ab	7ab	@	@	@
Mathematical Links	5	2	7ab	7ab	7ab	7ab	7ab	5
Levels of Questions	5/6	7/8	7 to 10	6 & 8	6 to 10	6	5/6	10
Results of the tests - Percentage of students in considered ranges of performance								
50 % or more of test items correct	17	89	100	93	96	57	100	96
70% or more of test items correct	03	86	92	83	52	30	100	81
Test Performance	L4	L1	L1	L1	L2	L3	L1	L1

Table 9.1.4 - Case Matrix for High Attainer ('Top') Groups

The group taught by teacher C had already completed all volumes in the adopted textbook series, and was working through some school-prepared revision material. As this material was strongly based on the textbook and on the GCSE exams related with this particular series of textbooks, it cannot be considered that the teacher had moved away from the textbook. So, for the purposes of this research, seven of these groups were considered textbook users.

Note that, in contrast to all others, the 'top' groups had the results of their tests highly concentrated in the first classification. This indicates that these groups generally performed at the highest levels, and, consequently, better than the other groups. The only group of this type tested by sampling was teacher C's.

Summary Study of Groups Taught by the same Teacher

Teacher A:

Regarding the groups of students taught by **teacher A,** it can be said that none of his groups performed really well in the tests given. Group 8X was the most successful in achieving teacher A's aims, although there were students in the group who clearly did not achieve the teacher's basic aims, independently of choice and use of material.

In this sense, it can be said that his use of materials with mixed ability groups does not cater for lower attainers within the group. During the observed lessons, teacher A adopted a similar strategy in using materials for investigations with groups 9L and 10M, achieving better results with 'low attainers' than with 'medium' attainers, an even more surprising result considering that the investigation developed in 10M was supposed to be used in their GCSE marks.

Considering the 'top' group 10T, test results suggest that students did not benefit much from the several different ways teacher A used printed worksheets, his own produced material or the computer as an auxiliary aid during the observed lessons, as they were only able to perform well in the initial questions of the task, related to pre-requisites.

Teacher B:

In order to summarise the results of **teacher B**'s student groups, it is important to remember that two were actually divided into two sub-groups each, consistently using different tracks of the adopted textbook in the same class.

In both these groups (10X and 11M) students using the 'higher' track of the textbook performed better in their tests than those using the 'lower' track did (see figures 5b.1 and 5b.2 in appendix 5b). In fact, the 'blue sub-group' in 11M had the worst result among teacher B's students, and one of the worst results in all the sample, while the 'red sub-group' in this same group performed at the highest level. These results suggest the hypothesis that the students using the 'lower' track did not benefit from sharing the lesson with other students who were using a 'higher' track.

When considering group 9L, it was noticed that two different teaching strategies were used. Both strategies were tested, and students in the group performed much better in the part of the test related to activities developed in an 'interview strategy'. led by the teacher on an individual basis than in the part of the test related to individualised work from the textbook. This result seems to indicate that individual interaction with the teacher can be a very positive strategy to be used with 'low' groups.

Group 9T performed extremely well in the test (in fact, one of the best results of all the sample, and the best result among teacher B's groups), which seems to indicate that a good strategy for using materials with 'top' groups was adopted. This strategy can be summarised as: teacher-led classes supported by the textbook, complemented by other

materials, whenever the teacher's previous experiences using the book indicated it could be considered appropriate. This class strategy was combined with frequent homework (usually taken from the textbook).

Teacher C

The results of **teacher C**'s students are some of the best in the sample, and they certainly reflect highly organised lessons using the textbook, in which students worked hard. These lessons were followed by regularly assigned homework. All her groups performed within the higher levels.

Teacher C used the textbook or its complementation in an individualised way with all her student groups, checking their work regularly. Nevertheless, it was possible to apply a common test to all students in groups 8L, as all of them were working at the same topics in ATs 2 and 4 (even if changes from one AT to the other were individualised, and more revision work was given to students who had difficulties in division).

Group 10M was regularly tested as a whole by the teacher, which means to say that they were all 'kept together'. The test given to these students had five different versions, each including a common part, followed by different sets of questions, to account for differences observed in classroom.

On the other hand, groups 8X and 11T had only a sample of their students tested, as the work observed during their lessons was completely individualised. It is important to observe that teacher C declared that group 11T used to be kept together, up to the point at which they began their revision work for the GCSE examinations. At the time of the observations they were being encouraged by the teacher to do as many revision sheets as possible in order to be better prepared for the exams.

Teacher D:

All groups taught by **teacher D** performed within the higher levels, and she also used the individualised teaching strategy most of the time. Teacher D's 'top' group had the best result among her students. They also worked towards a given goal in a certain period of time, as they were tested regularly by the teacher with a common test.

On the other hand, although group DD9M was also supposed to be kept together by a common test, during the observed lessons their work was so differentiated that it was necessary to test the group by sampling. There was evidence that these students did not interact with the other students and the teacher in the same way as other groups which were 'kept together' did.

Teacher D had an auxiliary teacher helping her with the 'low' group, which means to say that these students received more attention from a teacher than usual, and this is probably reflected in their results on the test.

As for group DD7X, teacher D interrupted their individual work using the textbook in order to give them a common task, based on her own developed material. This material was designed to replace the textbook, on a topic in which the teacher herself considered the material displayed in the textbook unsatisfactory. The students in this group were tested for both activities, and the results obtained by the group in the 'common work' questions were better than the ones obtained in the 'individualised work' questions. This result leads to a hypothesis that, even if individualised work using written materials led to good results, teacher led lessons supported by written materials may be even more effective, as the same teacher working with the same students in teacher led lessons achieved a better result.

Teacher E:

Supporting the premise that the results in the tests did not allow any conclusion as to whether or not it is better to use a textbook, **teacher E**'s groups have results that can also be considered among the best in the sample. Teacher E was a non-textbook user, who moved away from this position when the 'top' group was concerned.

Considering teacher E's groups' results, it is possible to conclude that none of his choices of materials, or observed strategies, could be considered ineffective. His best results were achieved with groups 7X and 11T. Group 7X had the best result in the sample among 'mixed ability' groups. Group 11T had the second best result among E's groups, although its performance is not one of the best when compared with other 'top' groups in the sample.

Analysing the other groups' results, it was noticed that three out of 19 students in group 10L were not even capable of starting the test questions. This indicates that these students did not benefit much from the series of observed lessons. Considering group 9M, it can be said that although most students were able to perform again the tasks they did during the observed lessons, all of them had to use their initial strategies, supported by concrete materials (mirrors).

This result indicates that the students had not moved towards generalisation, which seemed to be one of the teacher's main aims during the observed lessons. Only seven out of 28 students were able to do the question that required generalisation in the test (the others did not even try it). On the other hand, it was the only opportunity to observe a teacher (apart from the 'top' groups) attempting to move the students towards generalised results or encouraging them to draw conclusions (out of MA1 activities).

Teacher F:

Teacher F also changed materials and strategies while working with different groups. In fact, although teacher F considered herself a textbook user, she was only observed using the textbook with one of her groups. This group (8T), together with the group in which she was following an 'investigation' material (which she did not choose herself) closely (8M), were responsible for the poorest performances among her student groups.

On the other hand, teacher F confidently moved away from printed materials with her other two groups, and these actually performed better in their respective tests than her groups which used textbooks. Her 7L group had the best result among all 'low attainers' groups in the sample. In fact, it was one of the best results in all the sample, reflecting a carefully planned and executed series of lessons.

To account for the fact that the topic developed with group 7X without using materials could be developed using the textbook, some of the questions in this test were taken from the textbook. It was observed that the students performed in these questions as well as they did on the ones taken directly from their classroom activities. This seemed to confirm teacher F's hypothesis that they learned more this way than using the book, as some of the most difficult questions done in class could not be matched by any activity proposed in the textbook.

Teacher G:

Although **teacher G** was classified as one of those teachers who did not easily change their particular way of using the textbook, she had to face a crisis situation with group 7M. She had to move away from her plans with part of the group, and used most of the lesson time to teach Logo to this sub-group.

The test given to this group reflected the way teacher's decision to integrate the two 'sub-groups' during lessons. It had a common part, based on the common discussion promoted by the teacher by the end of each lesson, followed by two different series of questions, each one related to material used in lessons by the sub-groups: Logo or Textbook. A comparison was made (see figure 5g.1 in appendix 5g), and results indicated that both sub-groups performed at the same level in the common part of the test. On the other hand, students working through the Logo worksheet performed much better in their specific part (learning the movements of the turtle in Logo) than those working through the textbook (learning angles).

On the other hand, teacher G's 'top' group had one of the best results in the sample, which seems to indicate that her way of using the textbook was highly successful with top groups but not so with medium groups. In fact, students in the medium group who had a different treatment than the 'usual' performed better in the test.

Teacher H:

The decisions about using materials made by **teacher H** were group related. The most successful one seemed to be the way he used the textbook with his 'top' group. Teacher H was working through problems applying the sine and cosine rules (considered level 10 in the N.C.) in teacher-led lessons, supported by the textbook. The group displayed an excellent performance level.

On the other hand, his way of using printed materials with the low attainers did not differ much from other completely individualised lessons observed with this kind of group, and neither did the results.

Teacher H used teacher-led strategies again when applying the school developed material with group 9M, but in this case the result was not good. One possible explanation could be that neither the teacher nor the students felt really comfortable using the material, which had recently been developed by another teacher within the department, and never tested before.

Although group 7X was split in two halves, according to perceived ability level, the test results did not reflect this division, and were spread continuously. During each of the observed lessons, these students were engaged in two completely different activities, one 'led' by the computer and another led by the teacher. This way of using lesson time did not account for any great difference in results and group HH7X had a level of performance comparable with other 'mixed' groups (and also with other of teacher H's groups).

To conclude this section, it is important to observe that the analysis of the data summarised in this section allowed the discussion of particular strategies adopted by the teachers in the sample, providing the present work with useful examples of actual ways these teachers used written materials with different ability level groups of students. It also allowed these strategies to be linked with students' performance in the test, generating hypotheses on how written materials could be successfully used to support class teaching.

In the following paragraphs, the links between different decision orientations towards use of written materials and the results of the tests are presented. Each section (from 9.2 to 9.5) is dedicated to the analysis of one specific set of strategies (or decision orientation on use of written materials), according to the methodology described in Chapter Seven. The results of tests associated with each strategy are also displayed and commented on. The analysis of the data related to the second research question is completed in section 9.6.

9.2. Strategy 'SaCo' : Same material and Complete Tasks

As already discussed, the study of the strategies is not intended to be comparative. Due to the size of the sample and to the constraints on testing methodology, the analysis developed can only be seen as an attempt to describe the links between different ways teachers in the sample used written materials and the results obtained by their students in immediate post-tests. It is important to notice that a 'strategy', as defined in chapter seven is not one strategy adopted by one teacher, but a set of strategies, with a common decision orientation. These strategies were adopted by different teachers, with different groups of students. As stated previously, other variables influenced test results, and one cannot expect a complete cause-effect explanation of results based on these decisions.

Table 9.2.1 presents all occurrences of the '*SaCo*' strategy. A particular observed way of using written material was classified into this set of strategies whenever the teacher gave the same material to all students, and expected them to complete the proposed tasks before moving to another topic.

Strategy 'SaCo': Same Material and Complete Tasks.

Choice Orientation	**W**	**W**	**W**	**W**	**W**	**T**	**W**	**T**	**T**	**T**
Students Group	**AA8X part 2**	**BB9L part1**	**FF7L**	**E9M**	**GG7M Whole part 1**	**GG7M sub: text part2**	**GG7M sub: logo part2**	**BB9T**	**GG8T**	**H10T**
Expectations	**R**	**R**	**H**	**R**	**R**	**R**	**R**	**H**	**H**	**H**
Choice of Materials										
Introduction	1	1	1	4/3	1	1	1	5	1	5
Classwork	1	1	1	4/3	3	8/7ab	1/4	6/3	8/7ab	7
Conclusion	1	@	1	3	1	5	1/4	6	7ab	6
Differentiation	@	1	@	@	@	7ab	4	@	8/7ab	@
Mathematical Links	1	@	1	3	1	5	4	5	1	6
Use of Materials										
Introduction	1	1	1	3/4	1	1	1	5	1	5
Classwork	1	1	1	3/4	3	8/7ab	1/7	7/8	8/7ab	7
Conclusion	1	@	1	6	1	3	1/3	5	7ab	5
Differentiation	@	1	@	@	@	@	@	@	@	@
Mathematical Links	1	@	1	1/5	1	7	1/7	2	7ab	5
Levels of Questions	**4/5**	**3**	**2 to 5**	**4**	**4/5**	**4/5**	**4/5**	**7/8**	**5/6**	**10**
Results of the tests - Percentage of students in considered ranges of performance										
50 % or more of test items correct	**77**	**100**	**100**	**79**	**66**	**46**	**83**	**89**	**100**	**96**
70% or more of test items correct	**32**	**55**	**100**	**39**	**0**	**31**	**83**	**86**	**100**	**81**
Test Performance	**L2**	**L2**	**L1**	**L2**	**L3**	**L4**	**L1**	**L1**	**L1**	**L1**

Table 9.2.1 - Conceptual Matrix for Strategy 'SaCo'.

The first strategy to be analysed considered that the same written material was used during the same period of time for all the students in the group, who were supposed to complete the tasks, at least up to a certain point. Nevertheless, this does not mean to say that all students finished the task at the same point (some students finished the 'basic' proposed tasks earlier, and were observed working on extensions), but it does mean that no student left the 'basic' tasks intended by the teacher unfinished. Whenever a teacher opted for a 'SaCo' strategy, level of expectation was high or regular. Neither individualised nor low levels of expectation were associated with this set of strategies.

All teachers who decided to adopt this strategy within a group of students had to act as 'active teachers' (Brophy and Good, 1983, pp. 343-344), providing explanations and asking questions. Whole group discussions were also observed during these lessons, except for teacher B, who was working the material on an individual basis. It was also common for these teachers to use the question posed by one student during seatwork time to discuss the answer to the proposed question with all students. Small group and whole group interactions were also observed during these lessons, and it was rare to observe one student who did not discuss exercises with others, even if working on an individual basis.

There seems to be no clear relation between adopting this strategy and the choice of material made by the teacher. Both textbook and non-textbook users adopted strategies classified as 'SaCo'. From the ways written materials were used during the observed lessons, it is important to observe that teachers introduced almost all lessons. Conclusions and mathematical links were also offered by the teacher in most cases. The differentiation row shows that these students developed the same basic material. Nevertheless, teacher G's students who finished the proposed task ahead of the others were offered some extension material extracted from the textbook.

Figure 9.2.1 shows the frequency graph relating to test results. As previously stated, due to methodology constraints it is not possible to be conclusive. It can only be hyphotesised that this group of strategies can generally be associated with good level of performance. It is also important to note that some of the best results of tests were associated with this set of strategies.

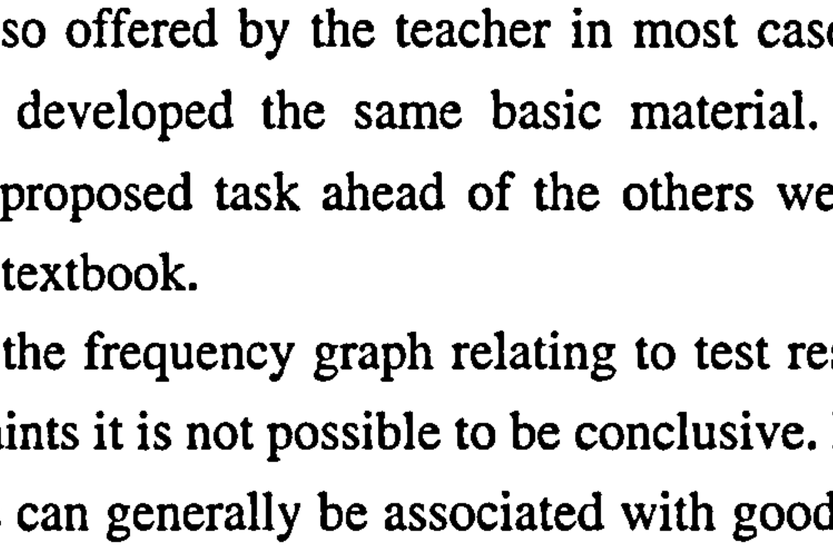


Figure 9.2.1: Frequency Graph of results associated with 'SaCo'.

9.3. Strategy "SaIn': Same Material and Incomplete Tasks.

Table 9.3.1 presents all occurrences of the '*SaIn*' strategy. A particular way of using written material was classified into this set of strategies whenever the teacher gave the same material to all students, but did not expect them to complete the proposed tasks before moving to another topic.

Strategy 'SaIn': Same Material and Incomplete Tasks.

Choice Orientation	**W**	**W**	**W**	**W**	**W**	**T**	**T**	**W**	**W**	**W**	**W**	**W**	**W**	**T**	**T**
Students Group	**A8X p. 1**	**D7X p. 1**	**E7X p. 1**	**E7X p. 2**	**F7X**	**H7X p. 1**	**H7X p. 2**	**A9L**	**E10L**	**A10M**	**F8M**	**H9M**	**A10T**	**E11T**	**F8T**
Expectations	**I**	**I**	**I**	**H**	**R**	**I**	**I**	**R**	**R**	**R**	**R**	**R**	**R**	**I**	**R**
Choice of Materials															
Introduction	3	3	1	4	5	4	5	4	1/4ab	1	5	2	4	7	7
Classwork	3ab	3ab	1	4	1	4ab	6	4	4ab	4/1	4/6	2	4/3	7/7ab	7
Conclusion	@	@	@	@	@	4ab	5	@	@	@	@	@	3ab	7ab	3
Differentiation	3ab	@	@	@	@	4ab	1	1	4b	@	@	@	4b/3	7ab	@
Mathematical Links	@	@	1	1	1	4ab	5	@	@	@	@	2	4	7ab	@
Use of Materials															
Introduction	7	8	1	4	3	5	2	4	1/7ab	1/4	3	5	6	7	5/7
Classwork	7ab	8ab	1	4 / 7	1 / 2	7ab	1 / 2	4/6	7ab	4/1	5/8	6	5	7ab	5/7
Conclusion	@	8ab	@	@	@	7ab	2	@	@	@	@	5	7ab	7ab	2/8
Differentiation	7ab	7ab	@	@	@	7ab	1/7a b	1/2	7ab	@	@	@	6	7ab	@
Mathematical Links	@	@	1	1	2	7ab	2	@	@	@	@	5	5	7ab	7ab
Levels of Questions	**4/6**	**5**	**4 / 5**	**4**	**5**	**4**	**4**	**3 to 6**	**4**	**3 to 6**	**4/5**	**6/7**	**5/6**	**6to10**	**6**
Results of the tests - Percentage of students in considered ranges of performance															
50 % or more of test items correct	**64**	**83**	**94**	**90**	**84**	**97**	**66**	**55**	**74**	**33**	**60**	**74**	**17**	**96**	**57**
70% or more of test items correct	**52**	**65**	**35**	**90**	**37**	**55**	**28**	**22**	**42**	**0**	**20**	**37**	**03**	**52**	**30**
Test Performance	**L3**	**L2**	**L2**	**L1**	**L2**	**L2**	**L3**	**L3**	**L2**	**L4**	**L3**	**L2**	**L4**	**L2**	**L3**

Table 9.3.1 - Conceptual Matrix for Strategy 'SaIn'.

From table 9.3.1 it is notable that this strategy in using written materials has some clear differences from the previous one. Observe that, although lessons are still introduced by the teacher in almost all cases, classwork became more individualised, and differentiation more common. When considering conclusions and mathematical links, it is clear that these features became less frequently observed in these series of lessons. In most cases, these features were not presented at all, and in others the students were supposed to gather these kinds of information from the textbook.

The 'keep going' strategy became much more frequent, and teachers' expectations shifted from high-regular ('SaCo' strategy) to regular-individualised. The teacher usually

acted as a 'facilitator' in these observed lessons, and whole group discussions were uncommon. Different students were observed working at different points of the task. Although it was possible to observe some students working in pairs, group work also became rarer. Note also that this set of strategies was observed among both textbook and non-textbook users.

Figure 9.3.1 shows the frequency graph for results of associated tests. It can be hypothesised that this group of strategies cannot be generally associated with a level of performances as good as the ones associated with the 'SaCo' strategy. Instead, it seems to be associated with a regular level of performance, in which most students are able to complete the post-tests in a way that can still be considered satisfactory. The 'L1' results were not frequent in the matching post-tests.

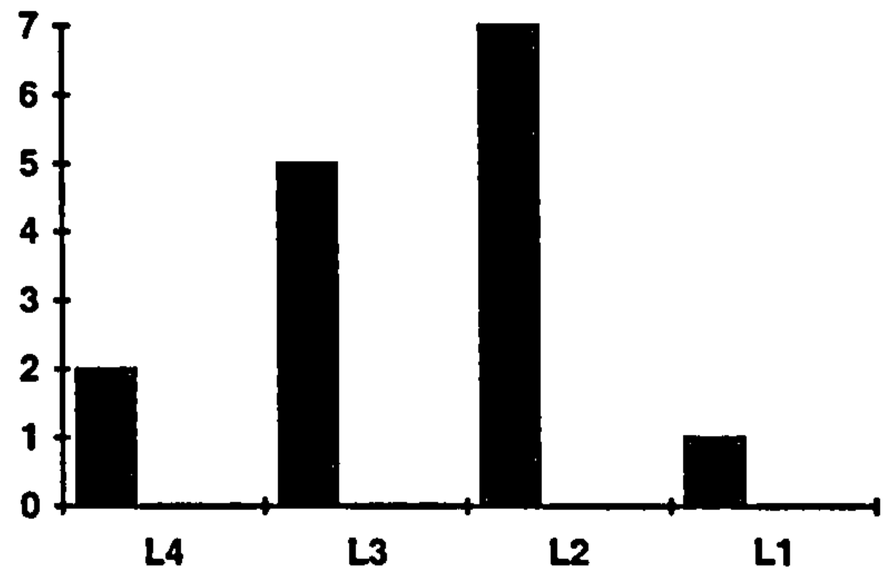


Figure 9.3.1: Frequency Graph of results associated with 'SaIn'.

9.4. Strategy 'DiCo': Different Materials and Complete Tasks.

Table 9.4.1 presents all occurrences of the '*DiCo*' strategy. One particular way of using written material was classified into this set of strategies whereby the teacher gave different materials to different students, and expected them to complete the proposed tasks before moving on to another topic. Notice that strategies 'Di..' are always associated with the use of textbook, perhaps because it would be very difficult to manage students in the same group working on different sets of worksheets at the same time.

Strategy 'DiCo': Different Material and Complete Tasks.

Choice Orientation	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**	**T**
Students Group	**B10X red**	**B10X yellow**	**C8X**	**D7X part 2**	**B9L part 2**	**H10L**	**B11M red**	**B11M blue**	**C10M**	**D9M**	**C11T**	**D9T**
Expectations	**I**	**H**	**I**	**I**	**L**	**L**	**R**	**L**	**R**	**R**	**I**	**I**
Choice of Materials												
Introduction	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	5ab	7ab
Classwork	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	5ab	7ab
Conclusion	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	5ab	7ab
Differentiation	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	5ab	7ab
Mathematical Links	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	5ab	7ab
Use of Materials												
Introduction	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab
Classwork	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab
Conclusion	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab
Differentiation	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab
Mathematical Links	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab	7ab
Levels of Questions	**6 / 8**	**7 / 8**	**4 / 5**	**4 / 5**	**3/4**	**4**	**6**	**6**	**6/7**	**5/6**	**7 to 10**	**6 & 8**
Results of the tests - Percentage of students in considered ranges of performance												
50 % or more of test items correct	**88**	**92**	**75**	**78**	**55**	**78**	**100**	**44**	**100**	**84**	**100**	**93**
70% or more of test items correct	**35**	**50**	**42**	**48**	**18**	**56**	**75**	**0**	**89**	**42**	**92**	**83**
Test Performance	**L2**	**L2**	**L2**	**L2**	**L3**	**L2**	**L1**	**L4**	**L1**	**L2**	**L1**	**L1**

Table 9.4.1 - Conceptual Matrix for Strategy 'DiCo'.

All the occurrences in this table can be described as 'keep going' strategies. During the observed lessons, the students worked on an individual basis from the textbook, each one at his/her own pace. Although it was possible to observe students working in pairs, group work was never observed, and whole group teaching was rare. Nevertheless, during these lessons, students were always 'on task', and the teachers did provide constant feedback on their exercises (usually supported by the answer book). In most cases, during these observed lessons, teachers acted primarily as 'managers'.

As far as teachers' expectations are concerned, it could be considered that the teacher had high expectations for only one sub-group. In most of the cases, the teacher demonstrated regular or individualised expectations and even low expectations in a few cases. Notice also that table 9.4.1 seems to indicate a connection between the low level of expectation in this set of strategies and results obtained by students in post-tests.

Figure 9.4.1 shows the frequency graph for the associated test results. Although the set of data does not allow a conclusive answer, it can be hypothesised that this group of strategies is associated with a level of performance at least as good as, or most probably, even better than those associated with the 'SaIn' strategies. In fact, it appears that, generally speaking, results are closer to the ones displayed by the 'SaCo' strategy.

It can be hypothesised that this set of strategies seems to be generally associated with regular to high performance levels, in which most students are able to complete post-tests in a satisfactory manner, with some very good results.

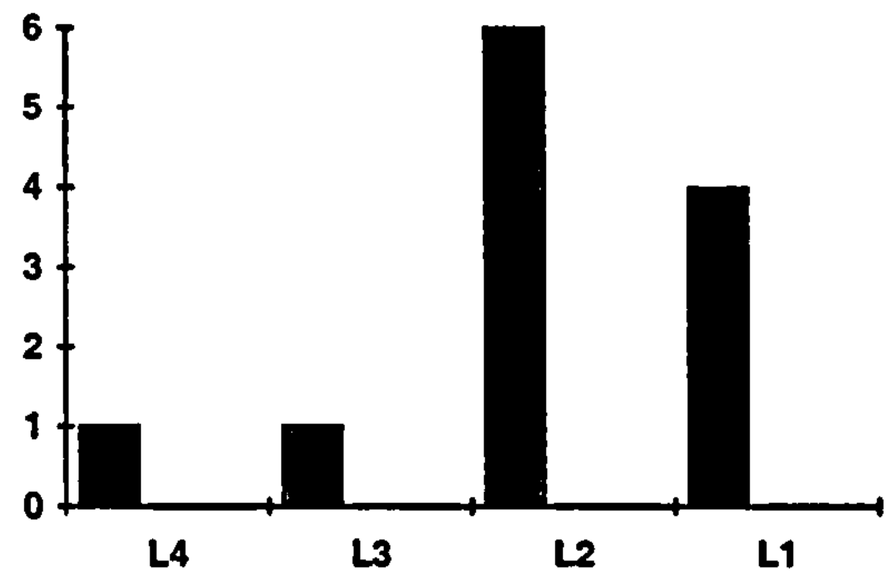


Figure 9.4.1: Frequency Graph of results associated with 'DiCo'.

9.5. Strategy 'DiIn': Different Materials and Incomplete Tasks.

Table 9.5.1 presents all occurrences of the '***DiIn***' strategy. One particular way of using written material was classified into this set of strategies whenever the teacher gave different materials for different students, not expecting them to complete the proposed tasks before moving them to another topic. As in the previous case, all occurrences of this strategy were associated with use of textbooks. It has to be said that, in contrast to what was observed in the occurrences of 'SaIn' strategies, in these observed lessons tasks did not remain unfinished, because the teacher asked the students to return to the same activity after a while.

Strategy 'DiIn': Different Material and Incomplete Tasks.

Choice Orientation	**T**	**T**	**T**
Students Group	**C8L part 1**	**C8L part 2**	**D10L**
Expectations	**L**	**L**	**L**
Choice of Materials			
Introduction	4	4	6ab
Classwork	4	4 / 1	6ab
Conclusion	@	@	6ab
Differentiation	4ab	4ab	6
Mathematical Links	@	@	6ab
Use of Materials			
Introduction	7ab	7ab	7ab
Classwork	7ab	7ab	7ab
Conclusion	@	@	7ab
Differentiation	7ab	7ab	7ab
Mathematical Links	@	@	7ab
Levels of Questions	**2**	**3 / 4**	**3 / 4**
Results of the tests - Percentage of students in considered ranges of performance			
50 % or more of test items correct	**93**	**74**	**83**
70% or more of test items correct	**47**	**47**	**25**
Test Performance	**L2**	**L2**	**L2**

Table 9.5.1 - Conceptual Matrix for Strategy 'DiIn'.

On the other hand, this strategy was only used with low attainer groups, and teachers demonstrated a low level of expectation in each case. Students were kept on task during observed lessons, the teachers provided feedback for their questions and, in both cases, they had homework assigned weekly. Nevertheless, these teachers kept changing the material the students were working on, based on the belief that they were not able to concentrate for a

long period of time on the same task. It can also be said that the main role of the teacher was managerial during these observed lessons.

The frequency graph showed in figure 9.5.1 seems to indicate that these students were able to perform in a satisfactory level at immediate post-tests, but this assumption is based on the smallest number of cases in all sample. Although more research in needed to confirm that hypothesis, part of the explanation for these results could be that these students were asked to perform very simple tasks (notice the levels of the questions in table 9.5.1) in the immediate post-tests.

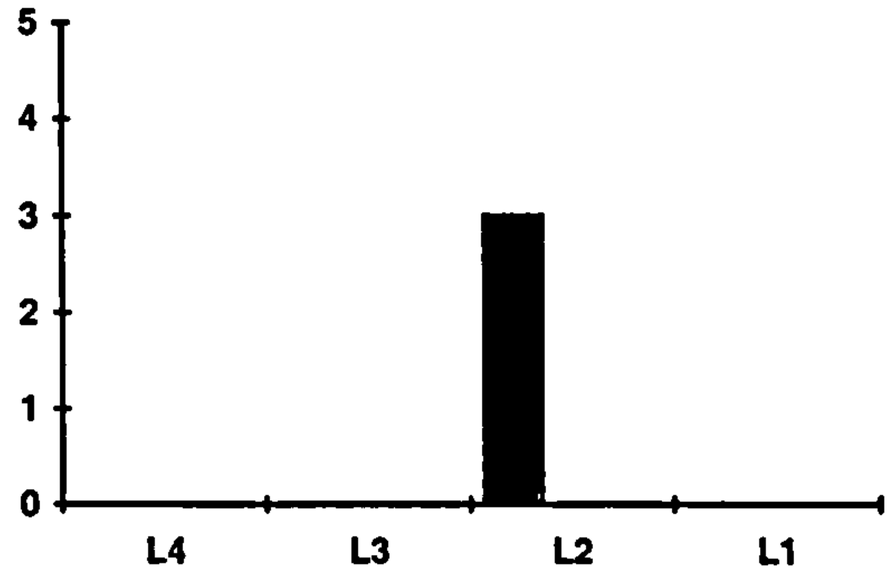


Figure 9.5.1: Frequency Graph of results associated with 'DiCo'.

Summary

Although the size of the sample does not allow a conclusive answer, the data gathered generated some hypotheses that should be verified in a larger sample of students. The main ones are:

(1) Strategies associated with a particular decision orientation could not be considered successful or unsuccessful in all occurrences, confirming the hypothesis that other variables (teacher, content, etc.) are also influencing the students' achievement.

(2) The set of strategies in which the teacher expected the whole group of students to complete a proposed set of tasks before moving to another piece of mathematical work appeared to be the most successful one. The expectations of teachers in the sample in these cases were never low, nor individuallised. This decision orientation also seemed to be linked to a role for the teacher in which active teaching is considered important.

(3) The two sets of strategies in which students work individually from textbooks, completing or not the tasks proposed before moving to another piece of work, were linked in the present sample to results that appear to be almost as successful as the previous one, except for the number of occurrences of 'excellent' results. This leads to a hypothesis that this kind of strategy keeps students on task, and allows teachers to act primarily as managers, checking students work during lessons. As a consequence, students are able to reproduce these activities in immediate post-tests at satisfactory levels.

(4) The set of strategies in which all students start to work on the same material at the same point but finish the proposed tasks at different points was associated with a spread levels of results. From the data gathered in classroom observation, a possible explanation could be that several students did not have the opportunity of finishing even what should be considered the 'basic' tasks proposed. This set of strategies was associated in the present sample with a role of the teacher that showed 'teacher as facilitator'. This analysis leads to a hypothesis that strategies in which several students do not reach the final tasks, usually missing the mathematical conclusions related to the developed activities, are responsible for the poorest results in students' immediate post-tests.

9.6. Analysis of Different Choices of Written Materials

In the previous chapter, different options made by teachers when choosing written materials for classroom work were discussed in detail. In this chapter, an attempt was made to identify links between strategies adopted by the teachers and the test results. To complete the present analysis, a summary analysis of the data related to each different choice of material is presented.

1 - There were occasions when a teacher decided not to adopt any written material. In these cases, the associated strategies were linked with different levels of performance. Two instances were observed among the low attainer groups, and both can be considered highly successful strategies, although it must be said that the success of the interview strategy adopted by teacher B was diminished by the lack of another successful strategy for the time remaining, when the students were not being interviewed. The overall impression about not using materials, in this sample, is that: first, it seems not to be a common choice among these teachers; and second, it could be considered an option to be made by an experienced teacher with successful results.

2 - No teacher in the present sample could be considered a regular producer of his own materials, at least in relation to the main lesson material. This option was resorted to only in special situations, with best results when based on previous experience of ineffective material displayed by the textbook. In these cases, the teacher knew beforehand what was ineffective, and had planned the material to improve the instruction in advance. On the other hand, cases where material produced was of a remedial nature, or a piece completely independent of the textbook, did not appear to be equally successful.

3 - Using printed materials other than textbooks was the main option available to teachers in this sample who did not adopt textbook series. On the other hand, it was not a common option among textbook users. A wide range of strategies was associated with this option, with a correspondingly wide range of results. One of the characteristic features of this kind of material is that it was given to the whole group and had to be finished at the same time.

4 - Finally, because of the way the sample was defined, textbooks series were the most common option adopted by these teachers. Several strategies were associated with their use, and related results seemed spread. The 'keep going' strategy, described earlier in this research report, can be mainly associated with textbooks (only once was

it observed when the teacher was using other sources of materials). Whenever this strategy was adopted, the teacher's role was reduced to management of the class, and very little interaction between the teacher and the students was observed. The results obtained by the groups when this strategy was applied were not necessarily poor, and the overall impression is that, depending on other teacher characteristics, this strategy could be even more successful than other choices illustrated herein. The high performance attained by some groups of students with this treatment was not easily foreseen, one possible explanation being that the possible advantages of adopting a textbooks series, already discussed in this report, could have compensated for the low levels of interaction between teacher and students.

It is also important to note that, as the present analysis looked for examples of successful strategies of using different sources of written materials, some other issues were raised, which can be considered as hypotheses for future research:

(1) The results of tests associated with the top-groups were usually better than all other group results. This begs the question of why medium and low-groups were not, in general, able to perform their allocated tasks as well as the as the top-groups did, even considering that the contents and the teaching had already been adapted to their levels.

(2) Whenever a group was divided by the teacher in levels of ability, those considered better (and, consequently, more 'challenged') performed better, sometimes with impressive differences between the two sub-groups.

Conclusion

To conclude this chapter it is important to say that the hypothesis raised by the analysis of links between observed strategies and the results of tests point in a direction that promotes the role of the teacher as an active instructor, and the use of whole group strategies. These directions were also pointed out in previous research (Brophy and Good, 1983). On the other hand, the results discussed in the chapter also seemed to confirm, as far as decisions on written materials are concerned, Brophy and Good (1983) stated that no generic skills could be directly correlated with better achievement, as different levels of results were obtained from similar orientations on the use of written materials.

The links discussed in this chapter also seemed to suggest that the results in the immediate post-test of those students who learned directly from the textbook were not necessarily amongst the poorest ones. This hypothesis seems to point in the same direction as more recent work on the need for support materials for teachers (see, for example, Clarke et al., 1996).

It must be repeated that no attempt was made to establish any criteria of 'quality' on the work developed by the teachers in the sample. In other words, a 'good' result in the immediate post-test does not necessarily mean that 'good quality mathematics' was achieved by the students. Nevertheless, the hypotheses raised in this chapter appear to confirm the importance of teacher's decisions in the process of class teaching, as discussed in more recent pieces of literature (Good and Biddle, 1998; Good and Brophy, 1997; Askew et al., 1997; Love and Pimm, 1996).

CHAPTER 10

THE RESULTS IN THE LIGHT OF THE REVIEW OF LITERATURE

> Many educators have contended that textbooks define the curriculum. However, recent research challenges this simplistic view and suggests that teachers act as decision makers, modifying the curriculum.
>
> Good and Brophy ,1997, pg. 105

10.1. Results and Review of Previous Research.

Davey's study (1988) discussed teacher's perceptions on how they use textbooks, and their opinions on why their use of such material was kept from being more effective. The present research complements her work as far as mathematics secondary teachers are concerned. Using classroom observation (as opposed to teachers' opinions only), it was possible to study in depth the matter of use of written materials, and particularly the use of textbooks. The results of the present study show that, as far as mathematics secondary teachers in the sample are concerned, some characteristics of use seem to be even less frequent than was pointed out in her work. For instance: (1) 'ask the students to read the text orally in the class' was never observed and (2) 'give the students time to read the text' or 'teach the students how to use the textbook' were characteristics that could be observed only once (one lesson on teacher F's T-group) in more than 90 lessons observed (of which the majority were based on some printed material). Another characteristic of use that was extremely rare to observe was 'I change texts when I see students cannot read them well'. Change of texts was only observed three times (teacher A's T-group, teacher B's M-group and teacher E's L-group), pointing to the conclusion that it is not a common practice among secondary mathematics teachers.

On the other hand, partly due to the experience of teachers in the sample and partly due to the way mathematics textbooks are designed in England, the use of different materials with different students was far more frequent than pointed out by Davey's research, as different attainment level groups would be using different written materials for all teachers researched in the present work. Finally, the present research seems to confirm Davey's conclusions as far as readability and content are concerned: the secondary mathematics teachers in the sample did not seem to be worried about readability of the adopted materials. Instead, they appeared to be more concerned with whether or not the content was properly covered by the chosen materials.

Ball and Feiman-Nemser's study (1988) looked at inexperienced primary teachers, while the present work looked at experienced secondary mathematics teachers, so any comparison between these two pieces of research has to be carefully made. Nevertheless, the

present research has confirmed that own produced materials are not necessarily better than commercially produced materials. Although some series of lessons in which no printed materials were used could be associated with excellent results in the corresponding test (for example: teacher F's L-group), at least in two observed series of lessons (teacher A's T-group and teacher H's M-group) materials produced by the teacher (or by the department) were used, and both the observation of the lesson and the results of the test have shown that these strategies were not especially successful. On the other hand, strategies associated with materials produced by experienced teachers to complement the textbook (whenever a teacher's previous experiences has shown that the textbook was not the better option) seem to be associated with excellent results (teacher B's complementation for her T-group, teachers C and D's homework and revision materials, and teacher D's complementation material for her X-group).

The results of the present research suggest that, even if they are designed by experienced teachers, own produced materials are more likely to be successful if they complement other materials. To design written materials for instruction is not an easy task: it requires more than experience and content knowledge. Teachers should be aware of these difficulties, otherwise they would not feel encouraged to make future attempts, if their first ones were not completely successful. It is possible to hypothesise the inference that inexperienced teachers are likely to be more successful if they have been oriented during their training courses in how to use textbooks with criteria, 'without making that the ultimate goal' (Ball and Feiman-Nemser, 1988, p. 422).

Although the researcher was able to obtain a copy only after the data for the present research had been analysed, the '*Evaluation of the Implementation of the National Curriculum Mathematics at Key Stages 1, 2 and 3*' report (Askew et al., 1993), especially those studies emphasised in the related paper by Millett and Johnson (1996), in which the issue of commercial schemes (textbooks) plays a major role, is linked with this work. A summarised comparison between the raised questions and findings from these pieces of research and the raised questions and findings of the present work is presented here. The research results highlighted by Millett and Johnson (1996), as well as the issues raised in their article are going to be discussed further, as they match closely the purposes of the present research.

Millett and Johnson's (1996) article highlighted a classification of teachers taken from Askew et al. (1993) as *scheme-driven* planners, *scheme-assisted* planners and *low-scheme* planners (see also section 3.3 in the present work). Although their classification was mainly related to the way teachers said they planned their lessons, some conclusions on the way commercial schemes would be used in each case were inferred by them from the interview data (Askew et al., 1993) .

The classification of secondary mathematics teachers produced here is different. It is not related to the documents teachers used to plan their work, or to the documents used to produce the school scheme of work. Instead, it is related to the ways materials were both chosen and used for classroom work.

In a first attempt, the teachers in the present sample would seem to be easily classified into Millett and Johnson's categories. Teachers B, C, and D could be considered as *scheme-driven* planners, teachers F, G and H *scheme-assisted* planners, while teachers A and E could be considered as *low-scheme* planners. However, classroom situations that would not fit the general description of this categorisation were readily observed. Just to give some examples: teacher B would not fit the description of a *scheme-driven* teachers' practice while working with her T-group, nor would teacher F fit the *scheme-assisted* teachers' practice with her L-group and teacher E also would not fit the description of l*ow-scheme* teachers' practice while working with his T-group.

These results suggest that a more complex categorisation is needed, mostly because the majority of the teachers in the present sample did change either their choices of materials or the ways the chosen materials were used from one classroom situation to another. These changes were usually linked with the group of students, but it was also possible to observe a teacher who changed the way the materials were used in a way that seemed material related, instead of group related (teacher A). In this sense, it can be said that the categorisation presented in the present work refines that presented in Askew et al. (1993) and highlighted by Millett and Johnson (1996), not only because the same teacher was considered while working with different groups of students but also because it was based on classroom observation.

Although the case of teachers that 'made extensive and sometimes uncritical use of a commercial scheme, and were using this scheme as a mediator of the N.C.' or '*scheme-driven* planners [interviewed] did use a narrower range of resources than both *scheme-assisted* and *low-scheme* planners' (Millett and Johnson, 1996, p. 64) where also observed for teachers in the present research, it is important to note that those *same* teachers, while working with other groups made very critical use of the chosen scheme, adding complementary materials whenever they felt it was necessary, and that their students subsequently achieved excellent results on the associated tests.

On the other hand, some teachers using own (or school's) produced materials were not so successful. These results suggest that the issue is more complex than it seems at first glance. Perhaps other mathematics educators might consider the position adopted in this research, of analysing the *strategies* used by the teachers in different situations, instead of analysing the teachers themselves. This position makes it possible to offer classroom based advice on use of materials (textbooks, other printed materials or even own produced ones) to teachers. The data also points to the conclusion that the real advantage of using alternative

materials instead of commercial schemes is not so clear cut as it seems to be suggested in Millett and Johnson, (1996, p. 73)

The association of the 'keep going' strategy with the use of textbooks was observed in the present research. Nevertheless, once more, the issue seems to be more complex than a simple equivalence between the 'keep going' strategy and the use of textbooks. A 'light' version of the 'keep going' strategy was observed even among those teachers who did not use textbooks, while some textbook users were never observed adopting this strategy. From this research point of view, the real problem was the reliance on individualised work, more than the material chosen, as discussed in Chapter Nine. The data have shown that there are teachers adopting group-teaching strategies with mixed-ability groups, so, the findings of the present research point in a different direction than 'all the KS3 interview teachers working with mixed-ability classes were using the strategy of individualised work'. (Millett and Johnson, 1996, p. 68).

Finally, if it cannot be denied that pressure to use commercial schemes is hard on teachers (Millett and Johnson, 1996, p. 72), it seems that pressure **not** to use them could be as hard. This seems to stem specially from mathematics educators, who have been systematically insisting that teachers would be better off without using textbooks, as will be discussed in the section 10.3.

The influence of teacher's decision making research in the present work is quite clear: the strategies described in chapter seven are based on previous research on this matter. The data used to generate the described strategies were gathered from teacher's interviews and observed lessons. Of special importance to the present work were: the pieces of research developed by (1) Askew et al. (1997), as it established connections between teacher's practical class behaviour and their belief orientations, and (2) Fennema et al. (1989 a, b), from which the model of analysis for the second research was derived.

Other pieces of work were important in defining the direction of the present work. These comments were made in section 3.5, and reviewed when defining the research project, so there is no need to comment on them again.

10.2. Results in the Light of the Theoretical Framework.

When linking the results of this work to the theoretical framework for this research presented in Chapter Two, it is clear that the evaluation and classification of the different ways teachers in the sample used written materials for classroom work, done by the researcher, used the teaching and learning theories described there as a basis. Also, in defining what should be observed during the lessons (chapter five), there was a clear interference on the views on education defended in chapter 2.

During the analysis of the data, what Kilpatrick (1987, p. 11) described as a possible misinterpretation of 'constructivist' methodology: constructivism from a 'theory of knowledge acquisition' to a 'theory of teaching or instruction' was used as background. In fact, this research made an attempt to verify whether or not the role of the teacher, as described by Kilpatrick (1987, p. 12), has been substantially reduced from what should be expected.

Several other aspects commented on during the analysis are particularly linked with the theoretical framework for the present research. The next paragraphs describe the main ones.

Although the teachers in the sample were not asked if they agreed with a particular theory of education, Constructivism was the only theory mentioned by them (two interviews). Some of the teachers expressed beliefs such as 'the teacher should not disclose results to the students'. As a consequence, they admitted to repeating similar tasks with the same students over and over again, in the hope that at some point, the language and/or the main results associated with the proposed tasks would be 'discovered' and assimilated by the students.

In groups where the 'not tell' strategy was used regularly, it was not uncommon to observe students giving each other the answers to the exercises, because the teacher would not do so. This situation was particularly observed in groups not using textbooks, perhaps because the students who had a textbook could use it to look for the main results and/or the answers to their exercises. Also, it was not uncommon to observe students showing a readiness to learn mathematical algebraic language, which was not presented by the teacher (for example, see teacher A M-group), in a similar process to that described by Ausubel as a 'repudiation of the very concept of culture' (Ausubel, 1963, p. 85).

On the other hand, strategies that could be associated with the ideas of emphasising pre-requisites, as described by Ausubel and Gagné, were often paired with successful teaching in our sample. These strategies were associated with some of the best results in the sample, and were specially common among the top groups (those which did use textbooks). This does not mean that these were the only students to gain from these strategies. In fact, the data suggest that groups of students of all levels would benefit from this kind of teaching strategies. Also, although textbooks can be considered an important source of learning organised in such a way that pre-requisites are considered first, they are not the only ones.

The data have shown teachers working without the aid of a textbook series (or even without the aid of any written materials) in such a way that pre-requisites were always reviewed beforehand (see for example the data for teacher F's L-group).

Note that among the theories reviewed for this research, Social Constructivism and Vygotsky's theory are the only ones which take into account the importance of social interaction in the learning process. This research adopts Kilpatrick's (1987) point of view: Constructivism is not a theory of teaching or instruction, but a theory of knowledge acquisition. Vygotsky emphasised the importance of a child's social interaction not only with other students but also with the teacher. The results of the previous chapter suggest that few strategies adopted by the teachers allowed these interactions. Nevertheless, whenever these strategies were adopted, the associated result appeared to be at the highest levels.

10.3. Results and Advice Offered to Teachers on Written Materials.

The main objective in reviewing the literature in Chapter Four was to justify the need for research on the use of written materials, based on classroom observations, supporting the view that there are not enough pieces of research written to uphold the advice offered by mathematics educators on the matter. In the end of Chapter Four, a summary was presented (section 4.5), in which the general conclusions that could be gleaned from the pieces of work written to advise teachers were discussed. It was argued that the advice on textbooks has frequently changed during the past twenty years. In this section, a brief discussion is presented, considering only the main points raised in section 4.5.

The teachers in the sample of the present research all have more than ten years teaching experience. As can be concluded from the variety of strategies in using written materials adopted, their solutions can be quite different. In fact, there was evidence (taken from teachers E and F's interviews) that even in the same school different solutions were likely to be adopted by different teachers, with some mathematics teachers following different strategies in choice of materials than those supported by the Head of the Department. So, it is apparent that UK teachers can be quite independent even in the face of excessive radicalism in advising the teachers not to use textbooks, as expressed in publications such as Better Mathematics (1987). This kind of advice is in contrast to teachers' needs for support materials to handle the pressures of their daily work (as is admitted in more recent works).

On the other hand, the findings of the present research may point to the fact that those teachers who did take this sort of advice seriously were not necessarily successful in achieving their teaching objectives. As the majority of the advice found in the literature was not research based, it is possible that the needs of teachers were not taken into account. It can also be considered that if advice given to teachers is to be useful, it should take into account their needs for quality classroom materials.

CHAPTER 11

CONCLUSIONS

You can work it out by Fractions or by simple Rule of Three,
But the way of Tweedle-dum is not the way of Tweedle-dee.
You can twist it, you can turn it, you can plait it till you drop,
But the way of Pilly-Winky's not the way of Winkie-Pop!
Rudyard Kipling -'The Jungle Book'

11.1. Summary of the Development of the Research

The present research had two aims: first, to generate a categorisation of the teachers in the sample as users of written materials for classroom work; and, second, to verify whether some observed use of written materials could be associated with better achievement of teacher's short term aims. So, a study with two aspects had to be developed.

The core of the methodology of data collection related to the first aspect was classroom observation. The categorisation should reflect observed strategies on the use of written materials, instead of teachers' interpretations of their own behaviours. The data included information taken from other sources such as interviews, school scheme of work, teacher's personal planning and students' notebooks. Eight experienced teachers, six 'textbook users' and two 'non-textbook users', had their lessons with different groups of students observed during a week. These groups of students included mixed-ability groups and groups set by levels of attainment: high, medium and low attainers.

To generate categories, the design took into account two major aspects: (1) how dependent the teacher was on printed materials (series of textbooks in particular) on his/her choice of materials for classroom work and (2) once the material was chosen, how dependent on it was the adopted strategy of use. The differences in the ways each teacher in the sample chose and used written materials for different groups of students were analysed to verify if they could be used as basic variables for categorisation.

Concerning the second aspect, tests were designed to investigate links between choice and use of written materials and students' achievement of teacher's short term aims. They were applied to each group just after the observed lessons. In the case of individualised strategies, a series of tests was applied to a sample of students.

During the analysis of observed lessons, several strategies of choice and use of written materials were identified. The additional analysis of these strategies took into consideration not only data from observed lessons but also data gathered from teacher's interviews, where they had a chance to state affirmatives demonstrating their decision orientation towards the use of written materials for different groups of students. These strategies were linked to the results of the tests, searching to verify whether or not there will be a connection between these variables.

11.2. Contributions to Mathematics Education

Main Features of the Analysis

The analysis of the data related to the first part of this study took into consideration three aspects of the ways secondary mathematics teachers have been using written materials for the class work: (1) the general decisions that are taken for the whole year (general planning), (2) the particular choice of materials for the observed lessons and (3) the particular use of the chosen materials during the observed lessons.

The ways materials were used by the teachers in each observed group were placed in two continua: (1) the one related to **choice** of materials started from 'independent from written materials' and had 'dependent from textbooks and teacher's guides' on the other extreme; and (2) the one related to **use** of the chosen material also started from 'independent from written materials' and ended in 'dependent on the chosen material and its guides'. As a consequence of the dissociation of choice and use, the concept of teacher's dependence on printed materials had to be reviewed. Which teachers can be considered more 'independent': those who had planned to base their teaching on a single series of textbooks, but were eager to move away from it (or to complement the material) whenever their previous experiences had shown it was necessary, or those who used different printed materials for different topics of the curriculum, but were not likely to modify these choices?

The data collected for the second part of the present study raised the problem of deciding how it would be possible to compare students' achievement. There were no 'similar' groups in the sample. Those of comparable age had different levels of attainment, and vice-versa. Even for the few groups that could be considered 'similar' (for example: mixed-ability groups in year seven), the contents developed by their respective teachers during the observed lessons were completely different. As a consequence, different tests were applied, each one of them linked to the observed work developed by the teacher. In the event, the achievement of teacher's short term aims was a rational criterion to be used, because all students in these groups were expected to fulfil their teacher's objectives for the observed lessons. In order to made sure that the tests matched the teacher's objectives, each teacher had the final word about them, before they were applied to his/her students.

The analysis of data related to the second part of the study took into consideration that:

(1) No evaluation of the mathematical contents developed with each group of students could be taken into account when analysing the results of the tests. As the tests had to be designed based on the use of written materials for classroom work during observed lessons, the researcher had no control on levels of difficulty of the questions. Nevertheless, a measurement of the level of difficulty of the tests had to be included, otherwise meaningless

final results could be overlooked - results such as 'very easy tasks are related with good performance', which would not lead to any improvement in teacher's strategies on the use of written materials. The solution to the problem was to classify each question using the English National Curriculum's Attainment Targets and Levels, which the teachers themselves have been using as parameters for their class work.

(2) The results of the tests were considered as indicators of performance. One of the main objectives of this part of the study was to refer to some successful examples of classroom strategies when applying different sources of written materials.

Teachers' decision orientation in using written materials were analysed, based on observed lessons and data gathered from a series of informal interviews developed after these lessons. The main aspects considered when defining teachers' decision orientations were: (1) whether or not the teacher considered it important that all students in the same group were using the same written material and (2) whether or not the teacher considered it important that all students finished the proposed tasks before moving to another mathematical activity.

Another aspect considered was the level of expectation of the teacher for each group he/she taught. The analysis of data showed that teachers had different decision orientation towards written materials for different groups of students. The set of possible orientations was then matched against the results of the tests to verify whether or not links could be established.

The Research Results

The first central result of this study was the categorisation of teachers as users of written materials for classroom work. This categorisation was based on the conclusion that the same teacher could change his/her choice of materials and/or his/her strategy of use from one classroom situation to another. There was evidence that changing from one group of students to another was the most frequent reason why a teacher changed strategies.

Nevertheless, several other issues were raised by this study, that cannot be neglected:

(1) The same printed material could be used differently by different teachers, or even by the same teacher in different situations. This contrasts with the common belief that textbooks are always determinant (or even imposing) on the associated practice. Notice that this finding was mostly based on use of textbooks - the most frequent tool to be observed in different situations, due to the definition of the sample proposed in the present work.

(2) On the other hand, the influence of the chosen material (textbooks, other printed materials or own produced materials) could not be denied: similar materials were sometimes found to be used in a similar way with different attainment level groups. But there was indication that individualisation was most frequently associated with the use of textbooks. According to the observations, not every time that the teacher produced his/her own material resulted in

improved teaching. But, in the present sample, improved results on tests were always observed whenever an experienced textbook user decided to complement it with materials of his/her own based on previous unsuccessful experiences using the book.

(3) The inflexibility in changing materials was not so strongly related to the use of textbooks as commonly believed. Some teachers who adopted textbooks were observed changing materials, or providing complementary materials for the students while it appeared that some teachers who did not adopt textbooks became dependent on the choice of material proposed in the school scheme of work.

(4) From the previous conclusions, it can be argued that at least some of the problems usually associated with the **choice** of a textbook series as basis for instruction might be actually be related to the way teachers **use** these (or any other) materials.

When data from the first study were used to compare groups of similar attainment levels, aiming to analyse the strategies adopted, several other issues were raised:

(1) Individualised strategies were not the only ones adopted with mixed-ability groups. Several strategies of use of materials were adopted with these groups, including those working from a textbook series. For instance, some teachers in the sample believed that group work, gathering students of different levels of attainment, could be beneficial to all of them.

(2) Some teachers in the sample had strong beliefs about low-attainers. The most common ones were: (1) these students could not concentrate for long periods of time on the same activity, and (2) it was worthless to assign homework or to allow these students to take reference materials home. These beliefs were reflected in the teachers' practices. Nevertheless, it was observed with other teachers that low attainers were capable of work on the same topic for the whole week, without loss of interest. Moreover, some of these students were doing homework whenever it was assigned and were keeping the reference material taken home in good condition.

(3) It has to be considered that 'medium' did not define one level of attainment, but several different ones. These students constituted the majority in the upper years of their respective schools, and the tasks proposed to them seemed to depend on 'sub-levels'. Some textbooks would have more than one track to be offered for 'medium' students, depending on how 'high' or 'low' they can be considered within these sub-levels.

(4) The top-groups were likely to receive special attention as well as special materials in almost all observed cases. There was only one case in which the material used for the 'top' group was neither specially designed for them nor complemented by extension materials. This case was also the only top group in which a crisis was observed during the lessons: several students did not work on the designated tasks, and others had difficulties in completing them.

For the second part of the study, teachers' decision orientation towards written materials were analysed. The different orientations, together with teachers' levels of expectations for each group of students were then matched against the results of tests in immediate post-test, to verify whether or not links could be established.

The size of the sample does not allow conclusive answers but, at least as far as achievement in immediate post-tests is concerned, the analysis of data gathered generated some conclusions, that can be considered as hypotheses to be verified in a larger sample of students. As seen in chapter nine, the main ones are:

(1) No strategy associated with a particular decision orientation could be considered successful or unsuccessful in all occurrences. Other variables seem to be also influencing the students' achievement.

(2) The set of strategies in which the teacher expected the whole group of students to complete a proposed set of tasks before moving to another piece of mathematical work appeared to be the most successful one. In these cases, expectations of teachers in the sample were never low, nor individualised. This decision orientation also seemed to be linked to a role for the teacher in which active teaching is important.

(3) Strategies in which students work individually from textbooks, completing or not the tasks proposed before moving to another piece of work, were linked in the present sample to results that appear to be almost as successful as the previous one, except for the number of occurrences of 'excellent' results. This leads to a hypothesis that this kind of strategy keeps students on task, and allows teachers to act primarily as managers, checking students work during lessons. As a consequence, students are able to reproduce these activities in immediate post-tests at satisfactory levels.

(4) The set of strategies in which all students start to work on the same material together but finish the proposed tasks at different points was associated with a spread levels of results. From the data gathered in classroom observation, a possible explanation could be that several students did not have the opportunity of finishing even what should be considered the 'basic' tasks proposed. This set of strategies was associated in the present sample with a role of the teacher that privileged 'teacher as facilitator'. This analysis leads to a hypothesis that strategies in which several students do not reach the final tasks, usually missing the mathematical conclusions related to the developed activities, are responsible for the poorest results in students immediate post-tests.

As the second part of the study looked for examples of successful strategies of using different sources of written materials, some other issues were raised:

(1) The results of tests associated with the top-groups were usually better than all other group results. This begs the question of why medium and low-groups were not, in general, able to perform their allocated tasks as well as the top-groups did, even considering that the contents and the teaching had already been adapted to their levels.

(2) The results of the tests indicated that level of performance was not necessarily related to choice of material. It seemed that the strategy adopted when using a given material was also important.

(3) All teachers in the sample had variations in the levels of performance for different groups, indicating that different strategies adopted by the same teacher (or the same strategy adopted by the same teacher with different groups) were not equally effective.

Another feature of this study was that some issues permeated the analysis of the effectiveness of adopted strategies in all levels:

(1) Whenever a group was divided by the teacher in levels of ability, those considered better (and, consequently, more 'challenged') performed better in their allocated tasks, sometimes with impressive differences between the two sub-groups.

(2) An effective strategy occurred whenever a teacher who adopted a textbook series used other materials (either to replace or to complement the textbook) in group strategies. These decisions were usually based on previous unsatisfactory experience using the textbook, as stated by the teachers themselves during interview.

11.3. Discussion.

The present research was designed to investigate some questions on the use of written materials, textbooks in particular, for mathematics classroom work. It is expected that the findings of this research can be used: (1) as information for teacher training; and (2) to suggest that more research is needed on the use of written materials (at least for mathematics in Secondary School).

Revision on Views on Use of Written Materials

It is expected that some issues raised in this research can complement current views on the use of written materials for classroom work, discussed in chapters three and four. Different practices were associated with different choices of materials, suggesting that the strategy adopted by the teacher is at least as important as the chosen materials themselves.

One consequence that can be inferred from the present research is that the day-to-day work of the teacher is not at all easy. Even if one ignored all the administrative work and the responsibilities in dealing with students and parents out of classroom situations, as well as the implementation of the National Curriculum and other planning tasks, the teacher still has to deal with his/her lessons for different groups of students, requiring choices of curriculum development and corresponding strategies.

This research has indicated that the way that the mathematics curriculum is developed in the classroom is strongly dependent on the role of the teacher. Moreover, there was evidence that even experienced teachers have problems in dealing with so many different situations and, as a consequence, their teaching with different groups is not equally effective. There was also evidence of teachers (e.g.: teacher E) who would not implement some of their ideas, because they could not spare the time necessary to plan the activities, even if they were to be developed using existing printed materials as support. As for the materials used, there was evidence not only of teachers who were not satisfied with part of the developed contents in the adopted textbooks, but also of teachers who did not necessarily succeed in producing alternative quality materials. So, it appears that more quality written materials are required as facilitators of teachers' day-to-day work.

Consequences for Teacher Training

From the literature reviewed in chapter four, it can be inferred that future teachers are perhaps being advised not to rely strongly on textbooks. Based on the results of this research, it seems necessary to express reservations about this course of action, as even experienced teachers clearly demonstrated that they could benefit from the support of quality materials. It

seems that the decision of successfully moving away from printed materials in general (and textbooks in particular) requires experience, good content knowledge and plenty of time to plan in advance at the very least.

It can be inferred from this investigation that teachers in training should be prepared to start their teaching career by adopting printed materials, even if these materials are not to be given to the students. Comparison among different sources should be encouraged and research results taken into account. The first personal adaptations should be attempted only when the teacher becomes confident in his/her content knowledge. An extensive background of published materials should be encouraged, as a powerful tool for good classroom practice.

11.4. Limitations of the Present Research

The main factors of limitation of the present research are: (1) the size of the sample of teachers - it was only possible to observe eight teachers - and (2) the fact that long term issues were not considered.

The size of the sample of teachers can be linked to the following constraints:

- Half of the sample used the same textbook series. As it was decided that two teachers in the sample should be non-textbook users, this research was constrained to two teachers who used different textbooks.
- The final categorisation obtained for the present research can be defined as a 3x3 matrix, in which several cells ended up empty. It is pointed out in chapter eight that, at least in particular cases of empty cells, a larger sample of teachers should be expected to provide instances of behaviour associated with them.
- Several issues raised when designing this research (such as changing materials due to readability or teaching the students how to use reference materials) were not observed. Nevertheless, due to the size of the sample, the present research can only hypothesise that secondary mathematics teachers are not necessarily worried about these issues.
- Due to the size of the sample and to the methodology of testing students' outcomes, it was not possible to be conclusive about links between observed decision orientations in using written materials and achievement of teacher's aims. The present research has to be considered as an 'early stage' in research on use of written materials, in the sense recommended by Kilpatrick (1977) and Good and Biddle (1988).

As far as information on school policies, it should be pointed out that:

- The analysis of the school scheme of work for each school was not an issue in this research, except for its recommendations on written materials. So, the curricula of these schools were not deeply analysed in the present work (in fact, some schools did not have written schemes). As a consequence, there was lack of information when evaluating the curricula and corresponding strategies.
- Also, the school results on mathematics national tests were not collected, and there was no attempt to add any information on how effective the schools were, and no inference on these students' final achievement was included.

11.5. Future Research

The constraints on the methodology in the present research lead to the conclusion that two possible pieces of research could add to the findings discussed here: (1) A study using similar methodology, involving a larger sample of teachers, and (2) An in-depth longitudinal research, using a smaller sample but including long term variables, such as overall curriculum development, children's achievement in national tests, and other teaching variables.

In the first case, the categorisation presented here could be either validated, by the inclusion of other teachers who would fit the categorisation and even fill in some of the empty categories, or refuted by the conclusion that the teachers in the present sample are not representative of what is actually happening in classroom situations. It would also allow the validation of hypotheses raised in the present work. Another possible consequence of a similar research with a larger sample could be the inclusion of other teaching aspects in the categorisation of teachers. As for the second research suggestion, a different 'effective teaching' definition could be used. It has been stressed in this research that the use of teacher's short term aims as the main effectiveness indicator did not take into account the overall performance of these students on teacher's or school's long term aims.

Another interesting piece of research could focus the attention on one of the four types of student groups described in this research. Since this research produced a categorisation focused on differences between strategies in the use of written materials with different groups, the final categorisation did not take into account the particularities of the strategies directed to each kind of group. For example, an attempt to categorise teachers as users of textbooks when teaching medium groups would certainly produce different information, adding to the findings of the present research.

However, in all these cases, classroom observation should be an essential tool for data collection. Throughout the present research, it has been argued that a teacher opinion and his/her practice do not necessarily agree. This point of view was reinforced by the data collected. As it was demonstrated in chapter eight, the overall idea expressed by a teacher about his/her work on the preliminary interview had to be revised in informal interviews during the data collection, either because the classroom practice took a completely different turn from what had been previously intended, or because real constraints of classroom situations had not been taken into account, and the practice differed from what was initially intended.

Finally, in the personal case of the researcher, two main directions can be pursued: (1) To develop a similar study in Brazil, where the education system is organised in a way that those students who 'fail' are kept behind. This organisation implies that textbooks have a completely different structure, as all groups are 'mixed-ability', although not in the broad sense the term acquires in England. (2) The comparison of the effectiveness between the set of strategies adopted here and other obtained from data generated in Brazil.

BIBLIOGRAPHY

Assessment of Performance Unit [A.P.U] (1982). *A Report of Monitoring in Mathematics - Report from the National Foundation for Educational Research in England and Wales to the Department of Education for Northern Ireland and the Welsh Office Education Department.* United Kingdom: D.E.S.

The Association of Teachers of Mathematics [A.T.M.] (1990). *Using and Applying Mathematics.* United Kingdom: A.T.M.

Askew, M.; Brown, M.; Johnson, D.; Millet, A.; Prestage, S. and Walsh, A. (1993). *Evaluation of the Implementation of National Curriculum Mathematics at Key Stages 1, 2 and 3. Volumes 1 and 2.* London: School Curriculum and Assessment Authority.

Askew, M.; Brown, M.; Rhodes, V.; William, D.; Johnson, D. (1997): Effective Teachers of Numeracy in U.K. Primary Schools: Teachers' Beliefs, Practices and Pupils' Learning. In Pehkonen, E. (ed.) *Proceedings of the 21st. Conference of The International Group of Psychology of Mathematics Education,* vol. 2, pp 25-32. Lahti: P.M.E.

Ausubel, D. (1963): Some Psychological and Educational Limitations of Learning by Discovery. In McIntosh, J. A. (ed.) (1971): *Perspectives on Secondary Education*, Prentice Hall Inc., New Jersey, reprinted under authorisation of *New York State Mathematics Teachers Journal*, XIII (June, 1963) pp 90-108.

Ausubel, D. (1968): *Educational Psychology: a Cognitive View.* New York, N. Y. : Holt, Rinehart and Winston.

Ball, D.L. and Feiman-Nemser, S. (1988). Using Textbooks and Teachers' Guides: A Dilemma for Beginning Teachers and Teacher Educators. *Curriculum Inquiry, vol. 18 (4),* pp 401-423.

Bauersfeld, H. (1988): Interaction, Construction and Knowledge: Alternative Perspectives for Mathematics Education. In Grouws, D. and Cooney, T. (eds.) *Perspectives on Effective Mathematics Teaching.* pp 27-46. Reston: N. C. T. M..

Beeby, T., Burkhardt, H., and Fraser, R. (1979). *Systematic Classroom Analysis Notation [SCAN] for Mathematics Lessons.* Nottingham: Shell Centre for Mathematical Education.

Berlyne, D. E. (1957): Recent Developments in Piaget's Work. In *British Journal of Educational Psychology*, vol. 27, pp 37-49.

Bigge, L. M. (1982): *Learning Theories for Teachers* (4th Edition). New York, N. Y.: Harper and Row.

Bodin, A. ad Capponi, B. (1996): Junior Secondary School Practices. In Bishop, A.; Clements, K.; Keitel, C.; Kilpatrick, J. and Laborde, C. (eds.) *International Handbook of Mathematics Education*. pp. 565-614. Dordrecht: Kluwer Academic Publishers.

Bolister, M.A. (1977). [Review of] SMP 7-13: Units 1 and 2. *Mathematics Teaching. Dec. 77.* pp 51-55.

Brophy, J. and Good, T.L. (1983): Teacher Behavior and Student Achievement. In Wittrock, M. (ed.) *Handbook of Research on Teaching*. pp. 328-375. New York: Macmillan.

Clarke, B.; Clarke, D and Sullivan, P. (1996): The Mathematics Teacher and Curriculum Development. In Bishop, A.; Clements, K.; Keitel, C.; Kilpatrick, J. and Laborde, C. (eds.) *International Handbook of Mathematics Education*. pp. 1207-1233. Dordrecht: Kluwer Academic Publishers.

Cobb, P. (1986): Making Mathematics: Children's Learning and the Constructivist Tradition [Review of "Young Children Reinvent Arithmetic" and "Learning from Children"]. In *Harvard Educational Review*, 56, pp 301-306.

Cockcroft, W.H. (ed.) (1980). *Mathematics Counts - Report of the Committee of Inquiry into the Teaching of Mathematics in Schools*. London: Her Majesty's Stationery Office [H.M.S.O.].

Confrey, J. (1990): What Constructivism Implies for Teaching. In Davis, R. B.; Maher, C. A. and Noddings, N. (eds.): *Constructivist Views on the Teaching and Learning of Mathematics*. J. R. M. E., monograph n. 4, pp 107-122. Washington, DC: N.C.T.M.

Cooney, T.J. (1988): Teacher's Decision Making. In Pimm, D. (ed.) *Mathematics, Teachers and Children*. pp 273-286. London: The Open University.

Costello, J. (1991). *Teaching and Learning Mathematics 11-16*. London: Routledge.

Davey, B. (1988). How do Classroom Teachers use their Textbooks?. *Journal of Reading, vol. 31 (4),* pp 340-345.

Davis, P. J. and Mason, J. H. (1986): *Notes on a Radical Constructivist Epistemethodology Applied to Didactic Situations*. Unpublished manuscript, Open University, Centre for Mathematics Education.

DeCoste, D. (1991). [Review of] Discrete Mathematics Across de Curriculum, 1991 year book. *Mathematics Teacher. vol. 84 (9), dec. 91.* pg. 437.

Department of Education and Science [DES]. (1991). *Mathematics in the National Curriculum*. United Kingdom: H.M.S.O.

Denvir, B. and Brown, M. (1986). Understanding of Number Concepts in Low Attaining 7-9 Years Old: Part II. The Teaching Studies. *Educational Studies in Mathematics 17*, pp. 143-164.

Doyle, W. (1983). Academic Work. *Review of Educational Research, vol. 53, No. 2,* pp 159 - 199.

Eggeston, J. F., Galton, M., and Jones, M. E. (1975). *A Science Teaching Observation Schedule.* London: Macmillan Education

Eggleston, J.F., Galton, M.J. and Jones, M.E. (1976). *Processes and Products of Science Teaching.* London: Macmillan Education

Ernest, P. (1991): *Philosophy of Mathematics Education.* Basingstoke: Falmer.

Fauvel, J. (1991). Tone and the Teacher: Instruction and Complicity in Mathematics Textbooks. In: Pimm, D. and Love, E. (eds.). *Teaching and Learning School Mathematics.* London: Open University.

Fennema, E.; Carpenter, T. and Peterson, P. (1989a): Teachers' Decision Making and Cognitive Guided Instruction: A New Paradigm for Curriculum Development. In Ellerton, N.F. and Clements, M.A. (eds.) *School Mathematics: The Challenge to Change.* pp. 174-187. Victoria: Deakin University Press.

Fennema, E.; Carpenter, T. and Peterson, P. (1989b): Learning Mathematics with Understanding: Cognitively Guided Instruction. In Brophy, J. (ed.), *Advances in Research on Teaching.* pp. 195-221. Greenwich, KT: JAI Press.

Gagné, R. M. (1970): *The Conditions of Learning* (2nd Edition). New York: Holt, Rinehart and Winston,

Gagné, R. M. (1977): *The Conditions of Learning* (3rd Edition). New York, N. Y.: Holt, Rinehart and Winston,

Gelman, R (1972): Logical Capacity of Very Young Children: Number Invariance Rules, in *Child Development,* 43, pp 77-90.

Ginsburg, H. and Opper, S. (1969): *Piaget's Theory of Intellectual Development: an Introduction.* Englewood Cliffs, N. J.: Prentice Hall Inc.

Good, T. L. and Biddle, B. J. (1988): Research and Improvement of Mathematics Instruction: The Need for Observational Resources. In Grouws, D. and Cooney, T. (eds.) *Perspectives on Research on Effective Mathematics Teaching.* pp. 114-142. Virginia: N.C.T.M.

Good, T.L. and Brophy, J.E. (1983): School Effects. In Wittrock, M. (ed.) *Handbook of Research on Teaching.* pp. 570-602. New York: Macmillan.

Good, T. L. and Brophy, J. E. (1997): *Looking in Classrooms.* 7th. Edition. New York: Longman.

Good, T.L., Grouws, D.A. and Ebmeier, H. (1983): *Active Mathematics Teaching.* New York: Longman.

Gowin, D. B. and Novak, J. D. (1984): *Learning How to Learn.* Cambridge: Cambridge University Press.

Gray, E. (1991). The Primary Mathematics Textbook: Intermediary in a Cycle of Change. In: Pimm, D. and Love, E. (eds.). *Teaching and Learning School Mathematics.* London: Open University.

Hart, K. (ed.) (1981): *Children's Understanding of Mathematics: 11- 16. London:* John Murray.

Hart, K. (1992). Measuring the Emperor's New Clothes. *Times Educational Supplement. Oct. 9, 1992*, pg. IV.

Hewins, R.W. (1972). [Review of] New Oxford Junior Mathematics Books 1-4 (2nd. Edition), Metric and Work Units. *Mathematics Teaching. Dec. 92,n 61.* pp 58-59.

Her Majesty's Inspectorate. (1982). *Mathematics in the Sixth Form. H.M.I. series- Matters of Discussion.* United Kingdom: H.M.S.O.

Her Majesty's Inspectorate. (1987). *Curriculum Matters 3: Mathematics from 5 to 16.* United Kingdom: H.M.S.O.

Howson ,G., Keitel, C. and Kilpatrick, J. (1981). *Curriculum Development in Mathematics.* Bath: Cambridge University Press.

Hoyles, C. (1988): From Fragmentation to Synthesis: An Integrated Approach to Research on the Teaching of Mathematics. In Grouws, D. and Cooney, T. (eds.) *Perspectives on Research on Effective Mathematics Teaching*. pp. 143- 168. Virginia: N.C.T.M.

Inhelder, B. (1954): Les attitudes experimentales de l'enfant et de l'adolescent. In *Bulletin de Psychologie*, 7, pp 272-282.

Johnson, D.A. and Rising, G.R. (2nd edition -1972). *Guidelines for Teaching Mathematics.* Belmont, Cardiff: Wadsworth Pb.

Johnson, D.C. (ed.) (1989). *Children's Mathematical Framework 8-13: A Study of Classroom Teaching*. Windsor: NFER-Nelson.

Johnson, D.C. and Millett, A. (1996). The Research Programme 1991-93: Data, Data Analyses and Selected Results. In Johnson, D.C. and Millett, A. (eds.) *Implementing the Mathematics National Curriculum: Policy, Politics and Practice*, pp 29-53. London: New Bera Dialogues.

Kang, W. and Kilpatrick, J. (1992): Didactic Transposition in Mathematics Textbooks. In *For the Learning of Mathematics* 12(1), pp 2-7.

Kilpatrick, J. (1977): Research on Teaching Mathematics to the Elementary Pupil. In *Proceedings of the Research on Teaching Mathematics Conference*. East Lansing, MI: Institute for Research on Teaching.

Kilpatrick, J. (1987): What Constructivism Might be in Mathematics Education. In *Proceedings of the 11th. International Conference of P.M.E.*, pp 4-25, P.M.E., Montreal.

Kim, H. (1993). A Comparative Study Between an American and a Republic of Korean Textbook Series' Coverage of Measurement and Geometry Content in First Through Eighth Grades. *School Science and Mathematics. Vol. 93(3), March 93.* pp 123-126.

King, H. (1992). [Review of] Basic Algebra. *Mathematics Teacher. vol. 85 (5), may 92.* p. 394.

Koehler, M.S. and Grouws, D.A. (1992): Mathematics Teaching Practices and Their Effects. In Grouws, D.A. (ed.) *Handbook of Research on Mathematics Teaching and Learning*. pp 127-146. New York: Macmillan.

Komoski, P.K. (1985). Instructional Materials Will Not Improve Until We Change the System. *Educational Leadership. April 85.* pp 31-37.

Laborde, C. (1996). Introduction to Section 2: Teaching and Learning Mathematics. In Bishop, A.; Clements, K.; Keitel, C.; Kilpatrick, J. and Laborde, C. (eds.) *International Handbook of Mathematics Education*. pp. 505-509. Dordrecht: Kluwer Academic Publishers.

Lee, L. (1991). Radical Practice [A review of "Relearning Mathematics" by Marilyn Frankenstein]. *Mathematics Teaching. Dec. 1990.* pp 55-58.

Leinhardt, G. (1989). Maths Lessons: A Contrast of Novice and Expert Competence. *Journal for Research in Mathematics Education,* 20(1), pp. 52-75.

Livingston, C. and Borko, H. (1990): High School Mathematics Review Lessons: Expert-Novice Distinctions. *Journal for Research in Mathematics Education,* 21(5), pp. 372-387.

Love, E. and Pimm, D. (1996): 'This is so': a text on texts. In Bishop, A.; Clements, K.; Keitel, C.; Kilpatrick, J. and Laborde, C. (eds.) *International Handbook of Mathematics Education*. pp. 371-409. Dordrecht: Kluwer Academic Publishers.

Marchand, D.R. (1992). [Review of] Fundamentals with Elements of Algebra. *Mathematics Teacher. vol. 85 (5), may 92.* pg. 394.

Mathematics Centre, West Sussex Institute of Higher Education. (1987). *Better Mathematics: A Curriculum Development Study based on The Low Attainers in Mathematics Project.* London: H.M.S.O.

Miles, M. B. and Huberman, G. (1994): *Qualitative Data Analysis (An expanded sourcebook)*, second edition, London: Sage Pub.

Millett, A. and Johnson, D.C. (1996). Solving Teachers' Problems? The Role of the Commercial Mathematics Scheme. In Johnson, D.C. and Millett, A. (eds.) *Implementing the Mathematics National Curriculum: Policy, Politics and Practice*, pp 54-74. London: New Bera Dialogues.

National Council of Teachers of Mathematics [N.C.T.M.]. (1973).*Instructional Aids in Mathematics [34th. year book].* U.S.A.: N.C.T.M..

National Council of Teachers of Mathematics [N.C.T.M.]. (1982, second print: 1987). *How to Evaluate Mathematics Textbooks.* U.S.A.: N.C.T.M..

National Council of Teachers of Mathematics [N.C.T.M.]. (1989). *Curriculum Evaluation - Standards for School Mathematics.* U.S.A.: N.C.T.M..

National Curriculum Council [N.C.C.]. (1989). *Mathematics Non-Statutory Guidance.* York: N.C.C..

N.C.C. (1991). *Mathematics Non-Statutory Guidance.* United Kingdom: H.M.S.O.

Nibbelink, W.H. Stockdale, S.R. Hoover, H.D. and Mangru, M (1987): Problem Solving in the Elementary Grades: Textbook Practices and Achievement Trends Over the Past Thirty Years. *Arithmetic Teacher. September 87.* pp 34-37.

Novak, J. D. (1977): *A Theory of Education.* London: Cornell University Press.

Orton, A. (1992): *Learning Mathematics Issues: Theory and Classroom Practice* (2nd Edition). Cassell, London.

Piaget, J. (1961): The Genetic Approach to the Psychology of Thought. In *Journal of Educational Studies*, vol. 52, n 6, pp 275-281.

Piaget, J. (1964): Development and Learning. In *Journal of Research in Science Teaching*, vol. 2, pp 176-186.

Piaget, J. and Inhelder, B. (1969): *The Psychology of the Child.* New York: Basic Books.

Pimm, D. and Love, E. (eds.) (1991). *Teaching and Learning School Mathematics.* London: Open University and Hodder & Stoughton.

Philippou, G. N. and Christou, C. (1997): A Study of Teachers' Conceptions About Mathematics. In Pehkonen, E. (ed.) *Proceedings of the 21st. Conference of The International Group of Psychology of Mathematics Education,* vol. 4, pp 09-16. Lahti: P.M.E.

Reynolds, P. (1972). [Review of] SMP Books A-H; SMP Books A-G Teacher's Guides & Transparencies for Books A-D. *Mathematics in School. vol. 1(5).* pp 30-31.

Robitaille, D.F. and Garden, R.A.(eds.) (1989): *The I.E.A. Study of Mathematics II: Contexts and Outcomes of School Mathematics.* New York: Pergamon Press.

Robitaille, D. F., Schmidt, W. H., Raizen, S. (1993): *TIMSS Third International Mathematics and Science Study, Monography n^{c} 1, Curriculum Frameworks for Mathematics and Science.* Vancouver: Pacific Educational Press, UBC.

Romberg, T. A. and Carpenter, T. P. (1983): Research on Teaching and Learning Mathematics: Two Disciplines of Scientific Inquiry. In Wittrock, M. (ed.) *Handbook of Research on Teaching.* pp. 850-873. New York: Macmillan.

Sanders, D. (1991). [Review of] Curriculum and Evaluation Standards for School Mathematics: Developing Number Sense. *Mathematics Teacher. vol. 84 (9), dec. 91.* pg. 437.

Sinclair, B. (1968). Piaget's Theory of Development: the Main Stages. In N.C.T.M. (ed.) *Piagetian Cognitive-Development Research and Mathematical Education.* Washington, D.C., N.C.T.M., pp 1-11.

Shores, L. (1960). *Instructional Materials: An Introduction for Teachers.* New York: The Ronald Press Company.

Shuard, H. and Rothery, A. (1984). *Children Reading Mathematics.* London: John Murray.

Skinner, B. F. (1938): *The behavior of organisms.* New York: Appleton.

Steffe, L. P. (1991): The Constructivism Teacher Experiment: Illustrations and Implications. In von Glasersfeld, E. (ed.): *Radical Constructivism in Mathematics Education.* pp 177-194. Netherlands: Kluwer Academic Publishers.

Sturgess, D. (1971). [Review of] Schools Mathematics Project: Books A to H, Cambridge University Press. *Mathematics Teaching. Autumn 71, 56.* pp 63-64.

Thompson, A.G. (1992): Teacher's Beliefs and Conceptions: A Synthesis of the Research. In Grouws, D.A. (ed.) *Handbook of Research on Mathematics Teaching and Learning.* pp 127-146. New York: Macmillan.

Toumin, S. (1972): *Human Understanding Vol. 1: the Collective Use and Evolution of Concepts.* Princeton: Princeton University Press.

von Glasersfeld, E. (1983): Learning as a Constructive Activity. In *Proceedings of the 5th. North American Meeting of P.M.E.,* vol. 1 pp 41-69. P.M.E., Montreal.

von Glasersfeld, E. (1985): Reconstructing the Concept of Knowledge. In *Archives de Psychologie,* 53, pp 91-101.

von Glasersfeld, E. (1990): An Exposition of Constructivism: Why Some Like it Radical. In Davis, R. B.; Maher, C. A. and Noddings, N. (eds.): *Constructivist Views on the Teaching and Learning of Mathematics.* J. R. M. E., monograph n. 4, Washington, D.C.:N. C. T. M.

von Glasersfeld, E. (ed.) (1991): *Radical Constructivism in Mathematics Education.* Netherlands: Kluwer Academic Publishers.

Vygotsky, L. S. (1978): *Mind and Society.* Cambridge, MA.: Harvard University Press.

Vygotsky, L. S. (1986): *Thought and Language.* Cambridge, MA.: Harvard University Press

Waters, L. (1992). Level Structure [A Review of Mathematics: Level 3 & 4, by David Alcorn and Mathematics: level 5, Mathematics: level 6 and Mathematics: level 7, by Jean Holderness]. *Times Educational Supplement. May 22, 1992.* pg. 23.

Watson, J. B. (1919): *Psychology from the Stand Point of a Behaviorist.* Philadelphia: Lippncott,

Werry, B. (1989). The Teachers of Mathematics. In Robitaille, D.F. and Garden, R.A. (eds.): *The I.E.A. Study of Mathematics II: Contexts and Outcomes of School Mathematics* (pp. 49-62). New York: Pergamon Press.

Williams, S. (1991). [Review of] Mathematics in Action -Further Questions Book 1. *Mathematics in School. Nov. 1991.* pg. 46.

Woodward, A. (1987). Textbooks: Less than meets the eye. *Journal of Curriculum Studies, 19,* pp 511-526.

APPENDIX 1 - A

GLOSSARY OF TERMS USED IN THIS RESEARCH

Answer Book is defined as being the guide of a printed material containing the answers to its exercises and problems.

Case Matrix is defined as the matrix display where headings are matched against the groups of students, having the codes from the meta-categorisation as data in the cells.

Conceptual Matrix is defined as the matrix display where headings are matched against the meta-categories, having the groups of students as data in the cells.

Crisis is defined as an unexpected situation in classroom, motivated by the students' needs and requiring a change in the way these students have been working. The most common examples of 'crisis' are: (1) having students having difficulties in solving the proposed exercises and (2) students having already finished the proposed task

Developmental Exercises: a series of exercises carefully planned and organised in sequence given to the students in order to allow them the opportunity to acquire a new concept based on the existing ones. The students are required to apply their former skills and concepts together with the results of the previous exercises in the series in order to solve the next one, building up towards a new concept.

Differentiation is defined as the planned introduction by the teacher of different activities simultaneously, in order to respond to students' different needs.

Examples can be defined as illustrations of a topic (definition, concept, rules, etc.) through its applications in exercises and/or problem situations.

Exercises Given for Practice: exercises given to students in order to allow them the opportunity to practice a new skill. They are usually given to the students after the introduction of a new concept and are design to help students to master the skills related to the new acquisition.

Feedback is defined as the set of responses given by the teacher to students' actions.

Graph of the Conceptual Matrix is defined as a display of data, where the rows of the conceptual matrix are presented under the form of a bar chart and the groups are differentiated.

Headings can be defined as the set of identified teacher's behaviours and decisions influenced by the use of written materials.

'Keep Going' is defined for this research as the teacher's behaviour (strategy) characterised by giving the material to the students with no further introduction nor illustration, and asking the students to follow the activities proposed there.

'Match' (or 'Matching') Questions (Answers) are defined as those comparing how close the other features in the lesson are to the main activity developed in it. They are related with the 'how close' questions presented in chapter 5.

Own Produced Written Materials: is used to designate unpublished materials developed by the teacher or by the school.

Printed Materials: written materials that have been published, usually as small books, booklets or series of worksheets. Some of these materials are complemented by teacher's guides, with suggestions on how they should be used, but do not necessarily provide a complete set for instruction nor a suggestion for the progression of the content.

Problems: exercises given to students in order to allow them the opportunity to expand their previous knowledge by applying it to different situations, including real life ones. Problems are also a good way to integrate several pieces of knowledge, as they may be needed to obtain the solution.

Remedial Activities (Materials) are defined as activities (materials) introduced by the teacher in response to unplanned students' needs.

Revision Exercises: exercises given to students in order to allow them the opportunity to revise concepts that have not been used for a certain period of time.

Strategy (in using written materials) is defined as the way a teacher chooses and uses written materials for classroom work. See for example 'keep going'.

Teacher-led lesson is defined as the lesson in which the strategy adopted by the teacher in using written materials has the following characteristics: (1) the teacher introduces the lesson to all students before asking them to use the written material and (2) the students are asked to work through the same tasks.

Textbooks: printed materials that form a complete set for instruction purposes. Usually a textbook is designed to provide a complete basis for classroom work, including explanations, examples and exercises and sometimes suggestions for assessment. It is also common for a textbook to be complemented by an answer book and/or teacher's guide, offering suggestions on how the textbook should be used. They can be presented as a series of books, booklets or worksheets. For the purposes of this research, what characterises printed matter as a textbook is that it carries in itself a suggestion for the progression of the content.

Written Materials: any sort of materials produced on paper to be used by the students for classroom work (or sometimes as homework). The term **'own produced written materials'** is used to designate unpublished materials developed by the teacher or by the school.

APPENDIX 1 - B

DESCRIPTION OF CODES USED IN ANALYSIS

1B.1 Sets of Headings:

.Set I : Choices of Materials made by the Teacher for the School Year.

- **I - 1 :** Main Source of Materials:
- **I - 2 :** Progression of the Content:
- **I - 3 :** Materials given to the Students for the purpose of reference at home.

.Set II: Choices of materials: and Set III: Use of the chosen material:

- **II - 1 / III - 1 :** source of activities.
- **II - 2 / III - 2 :** to introduce a new topic.
- **II - 3 / III - 3 :**to introduce a lesson.
- **II - 4 / III - 4 :** reference material during the lesson.
- **II - 5 / III - 5 :** source of exemplification.
- **II - 6 / III - 6 :** conclude a topic.
- **II -7 / III - 7 :** establish links between related topics or topics in progression.
- **II - 8 / III - 8 :** promote differentiation within class.
- **II - 9 / III - 9 :** remedial material.
- **II - 10 / III - 10 :** source of homework.
- **II - 11 / III - 11 :** provide feedback on exercises:

1B.2. Meta Categories applied to Sets I and II of Headings (concerning Choice of Materials):

@ - Teacher does not present the behaviour being observed.

1 - Teacher's observed behaviour shows independence from any written material.

2 - Teacher's observed behaviour shows independence from printed materials.

3 - Teacher's observed behaviour shows that several printed materials are being adapted.

4 - Teacher's observed behaviour shows that printed materials from different sources are being used, without adaptations.

5 - Teacher's observed behaviour shows that the main source of printed material is being used as resource for free adaptations.

6 - Teacher's observed behaviour shows that main source of printed material is being used as base for instruction, with evidence of complementation.

7 - Teacher's observed behaviour shows that the main source of printed material is being used, but the teacher is not considering the suggestions of the teacher's guides.

8 - Teacher's observed behaviour shows that the main source of printed materials is being used, and the teacher is considering the suggestions of the teacher's guides.

1B.3. Meta Categories applied to Set III of Headings (concerning Use of Chosen Materials):

@ - Teacher does not present the behaviour being observed.

1 - Teacher does not use written materials,(even when there is a chosen one)

2 -Teacher is adding completely new features to the chosen material(s), modifying it(them).

3 - Teacher is doing several modifications to the chosen material(s), without adding new features to it(them).

4 - Teacher is basing the teaching on the chosen material(s), but he/she neither refers to it(them) nor gives a copy to the students.

5 -Teacher is basing the teaching on the chosen material(s), complementing it(them) without modifying its(their) basic structure. A copy of the material(s) is given to the students.

6 -Teacher is basing the teaching on the chosen material(s),doing small modifications, which do not alter its(their) basic structure. A copy of the material(s) is given to the students.

7 -Teacher is actually following the chosen material(s) (and nothing else), without considering the suggestions in the guide(s). A copy of the material(s) is given to the students.

8 -Teacher is actually following the chosen material(s), and considering the suggestions in the guide(s). A copy of the material(s) is given to the students.

1B.4. Sub Categories in the Meta-Categorisation:

sub - category (a) - Teacher's observed behaviour shows that the written material(s) is(are) being used in a way that is incompatible with the guidelines suggested by the teacher's guides.

sub - category (b) - Teacher's observed behaviour shows that some decisions about the use of written material(s) are given to the students.

(ab) is used whenever the 'keep going' behaviour is characterised.

1B.5. Codification of the Answers for 'Matching' Questions

The code to be used in answers for 'matching' questions will be a (*) put aside behaviours that <u>do not match</u> with the main activity developed during the lesson.

1B.6. Key for the Code Name of the Groups of Students in the Sample:

The format of the names given to the group of students is:

(letter)(letter)(number)(letter).

The first letter represents the school's code name, the second letter represents the teacher's code name, the number represents the school year and the third letter is taken from the set: {T, M, L, X}. T is used if the group is considered a 'top-group'; M, if the group is considered as being of medium ability level, L, if the group is formed by students considered low attainers and finally X, if the group is considered mixed ability.

1B.7. Levels of Performance for the Analysis of Each Group Results of the Tests:

- **L1:** At least three quarters of the students (75%) were able to achieve 70% grade or more in the test (in all those cases, at least 83% of the students tested were able to correctly complete more than half of the test items).
- **L2:** At least three quarters (74% or more) of the students were able to correctly complete more than half of the test items.
- **L3:** At least half (55% up to 66%) of the students were able to correctly complete more than half of the test items.
- **L4:** Less than half (46% or less) of the students were able to correctly complete more than half of the test items.

1B.8. Models for Orientations in Teacher's Decisions towards Written Materials\

	Worksheet oriented		Textbook oriented	
	all students completed the task (Co)	not all students completed the task (In)	all students completed the task (Co)	not all students completed the task (In)
same material for all students(Sa)	**WSaCo**	**WSaIn**	**TSaCo**	**TSaIn**
different material for different students(Di)	**WDiCo**	**WDiIn**	**TDiCo**	**TDiIn**

1B.9. Teacher's Expectations on Student's Performance

(H) - High: The teacher showed his/her belief that all (or almost all) students in the group are able to achieve the teacher's aims, and these aims should be ambitious.

(R) - Regular: The teacher showed his/her belief that all (or almost all) students in the group are able to achieve the teacher's aims, whenever these aims are not too ambitious.

(L) - Low: The teacher showed his/her belief that the group of students as a whole usually have problems in achieving teacher's aims, even if these aims are not too ambitious.

(I) - Individualised: The teacher showed his/her belief that some students in the group are able to go further then others where achievement of teacher's aims is considered.

APPENDIX 2A

SAMPLE OF COMPLETED OBSERVATION SCHEDULE

① 19/01 (ultima ora)

School: B

Teacher: B

Class: 11 M.

Material used: Red 3 for 4 students calculators Blue 5 for the rest of the groups

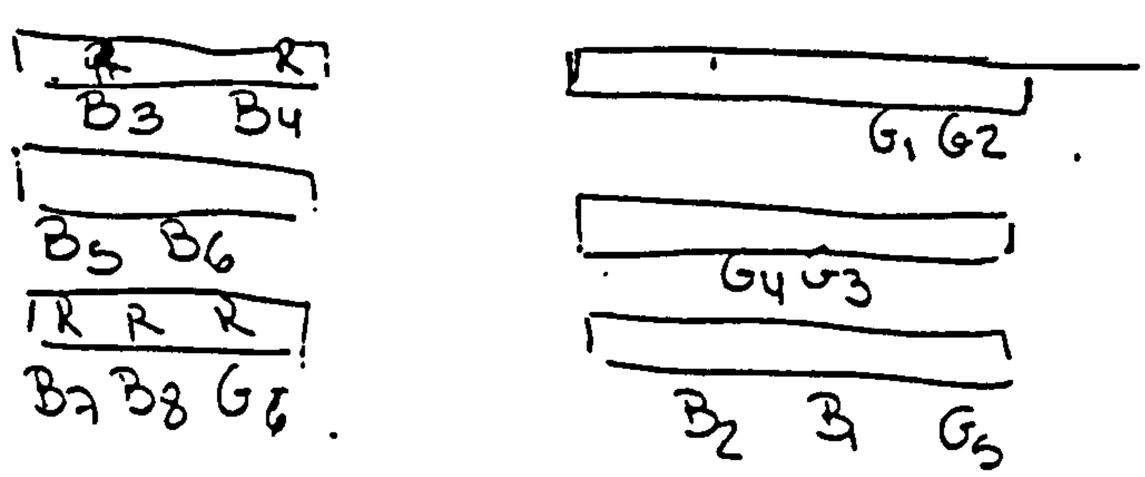
Arrows books

pl B_7. SMP book B pg 150 e seguenti.

DIAGRAM OF THE CLASSROOM:

BB

T. Table



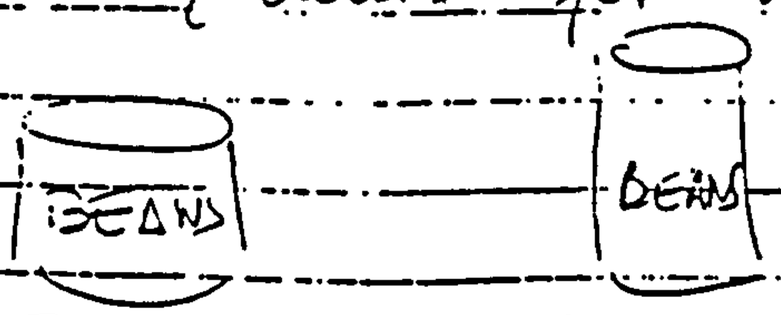
Observations: Red pg 32

Blue pg. 12. 13 - end of unit

START:

Teacher call "Blues" and write in the bb: (value for money)

(pg 15 of book



CLASS. ORGAN.	TEACHER MM – BB ACTIVITIES	TEACHER'S MATERIAL	STUDENTS (O) 19/01 ACTIVITIES	STUDENTS MATERIALS
	220 : 23 = 9.56 g. grams for 1 penny longest size: 11.48 g/penny She asks the students what (Blues) some finished this probability/odds	(calculator)	boys doing the sums. they have been doing section. and started	
	T. Reds + Bz help him and go to other to the extra comfacor		Bz having problems with fractions 'sums'	
	Teacher leaves the room (problems - noise outside)		Few students keep on task while she's out (Bz, G6, B4)	
	T. Reds (Bz) revise a somar + propi alguns problemas em fracções (Revision work on R_3)		B1 : pg 32, 33 (G6) B3 : pg 30 B4 : pg 18	Reds
	T. Bz helps him to find the right factor helps him work a irredondamento.		Blues G1 G7 - prob. G3 G4 .. B5, B7 value. for money - B2 value for money ∴ beginning	
	Teacher sends Bz and G6 to speak with the head about Teacher verifies individually the work of the blues.		A Level uses the answer book	
	T - G1, G2 correct the work, verifica enunciado e B1 e B2 - ask them to do better			
	End of the lesson/			

APPENDIX 2B

SAMPLE OF A REPORT OF THE LESSONS: GROUP AA9L

School : A Teacher : A Group : year 9
Perceived attainment level: low ability group (third out of three)

2b.1. First Lesson Observed (55 minutes), Wednesday, third lesson (out of five) of the day. Second half of the autumn term, 20 students present.

Printed Material Used by the Teacher: Worksheet "SNOOK" , from 'Problems with Patterns and Numbers', published by The Shell Centre, Nottingham. (not given to the students). There is a copy in the appendix I.2(a)
Other Materials used by the Teacher: Blackboard, coloured chalk.

Material Used by the Students: Squared Paper, coloured pens (all available from the teacher's desk).

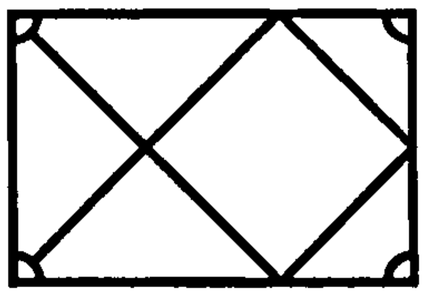
Comment 1: Although similar printed materials are available from other sources, the teacher said that this investigation was based on the Shell Centre Material.

Comment 2: The teacher did not bring the worksheet to the classroom and proposed the problem orally, using the blackboard as a visual aid.
Comment 3: The teacher modified the original worksheet because when proposing the problem he did not explicitly include the initial and final 'hits' when counting the number of 'rebounds' .
Comment 4: The teacher proposed the activity as an investigation. He left it more open ended than initially proposed by the worksheet, as some students could be investigating a pattern for the number of rebounds in $m \times n$ 'snook tables' while others could be investigating the patterns to predict where the ball would end.
Comment 5: The activity was proposed as a MA 1 one (integrated with Algebra) in the Mathematics School Scheme of Work for the second part of the Autumn term for students in year 9.

Comment 6: This lesson is the second of a series of three lessons on this particular piece of work. In fact, in informal interview after the lesson, the teacher told the researcher that the first lesson was introduced in a similar way as the second, only using several examples instead of one. The children have in their notebooks the examples they copied from the blackboard in the previous lesson, but do not have a copy of the worksheet.

Introduction to the lesson: The lesson was introduced by the teacher for all students. He was standing in front of the blackboard and gave an example on the blackboard as a recall for the activity developed in the previous lesson, allowing time for the students to copy it in their notebooks.

The teacher drew and wrote at the blackboard:



size of the table: 2 x 3

3 rebounds
1 hit
1 fall

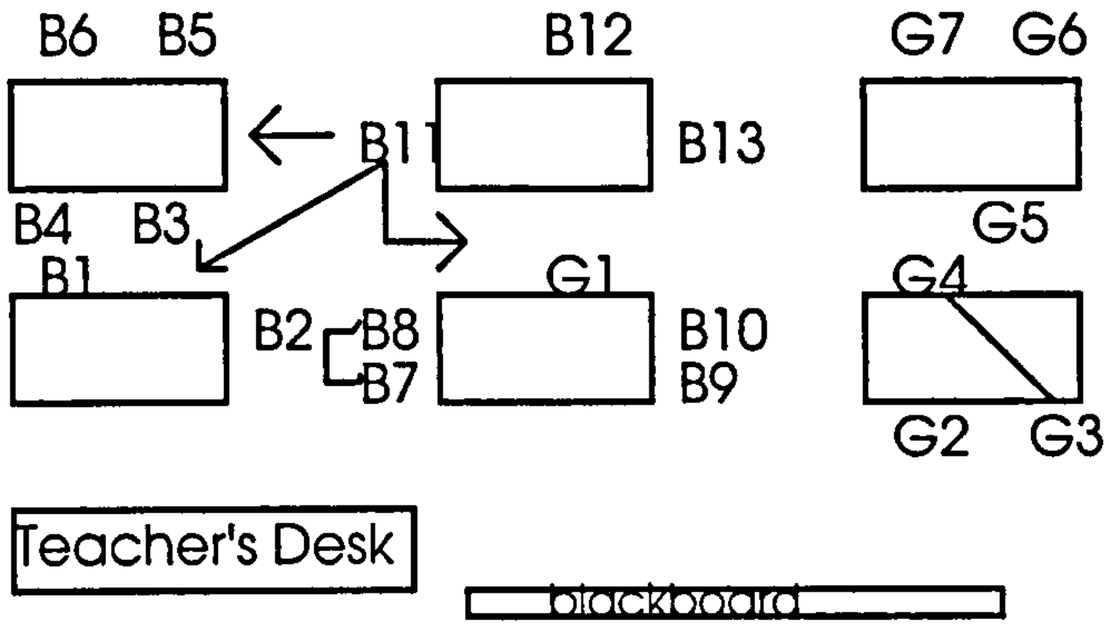
While he was drawing, he explained the situation using simple and visual terms. For example: instead of emphasising that the angle or the 'shot' was 45 degrees, causing the 'rebound' to be also of 45 degrees, he emphasised that the ball would go in the 'diagonal direction' and would 'rebound' in the 'diagonal direction' as well.

He suggested that the students should try different sizes of 'snook tables' and they should obtain several results. The students were attentive but did not ask any questions while the teacher was developing the example.

Comment 7: He did not speak of patterns or organisation of results in tables at this time.

Development of the lesson: As soon as the teacher finished the introduction, some students asked for help, raising their hands.

The diagram below shows the organisation of the classroom:



As we can see in the diagram , there were six large tables in the classroom and the children were sitting around them.

Comment 8: The teacher allowed students sitting close to each other to discuss their work or even work together, but did not explicitly encourage this behaviour. The children linked by a curve in the diagram were working together througout the lesson.

During the lesson, the teacher walked around the tables. He stop to look at children's pieces of work and offered suggestions on continuity. We present here some examples of these suggestions in chronological order:

To B9: "Try 'snook tables' sizes 2 x 4 and 2 x 5"

To B10: The teacher explained once more the rules of rebounding "45 degrees, rebounding at 45 degrees, this means taking the diagonals of the squares". He suggested B10 to organise his work "fix one dimension of the snook table and change the other"

To B13: (the pupil has done several cases and asked the teacher if it was enough). Teacher A asked him if he could predict what would happen if they were using a snook table size 2 x 20. B13 answered negatively and the teacher suggested he should try to organise his results in tables, working systematically. The teacher suggested that he should start with the table (He drew for B13):

size: 2 x	1	2	3	4	5	6
number of rebounds						

To G2, G3 and G4: (they have found a pattern for 2 x n tables) The teacher asked them if they could use the pattern they found to predict a result. He also suggested them that they should verify their prediction.

Comment 9: I was told by the teacher that B8, G1 and B10 are children with special needs. He told me that B8 had been consistently helped by B7 and was usually able to develop some of the proposed work in this way. He also told me that he sometimes has another teacher in class to help him with these children, but she was not there during this lesson. During the lesson, the teacher helped these children several times, although he asked them to do simplified pieces of work related to the general task proposed:

To G1 he proposed that she try to draw several sizes of snook tables and to reproduce the trajectory of the ball for each one of them. By the end of the lesson, she has completed 4 complete pages of such drawings, although some of the trajectories were wrong and there were sizes of snook tables repeated. The teacher helped her to correct the wrong trajectories. The teacher did not ask her to count the number of rebounds or to look for patterns.

To B9 the teacher proposed that he tried to draw snook tables of different sizes, which he did while the teacher was there with him, but stopped immediately after the teacher left him.

B8 was receiving help from B7 and the teacher supervised the 2 x n tables they were doing and let them carry on with this work throughout the lesson. The teacher suggested that B7 should prepare a table of their results by the end of the lesson.

Comment 10: The majority of students worked throughout the lesson, albeit not systematically (meaning that the changes made to the size of the snook tables did not follow

any pattern) , making it difficult to obtain tables of results and to recognise patterns. None of the students counting number of rebounds included the initial and/or final 'hits' in their counting, probably because of the way the example was developed at the beginning of the lesson. (This decision make it more difficult to find the pattern suggested by the teacher's guide). The teacher had the opportunity to verify these students' work at least once during the lesson and suggested that they try to work systematically.

The observed students that differ from this general behaviour were:

G1 and B9: already discussed in comment 9.

B2 remained quiet throughout the lesson but did not try any size of table, although he had his material in front of him throughout. (B1 and B2 were the only students whose work was not checked by the teacher).

B11 changed places several times during the lesson and did only one case (the 2x4 table)

B6 worked on square snook tables (1x1 up to 8x8) throughout the lesson. The teacher saw this when he was doing the size 6x6 and told him that he had found a pattern. He also said that B6 should write it down and move on to different snook tables.

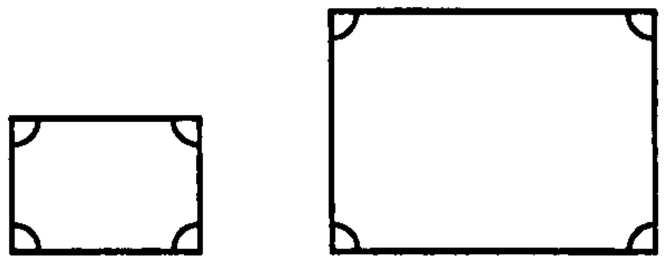
G3 and G4 working together found the pattern for 2 x n tables (shared afterwards with G2). By the end of the lesson they were verifying their predictions for 2 x 12 and 2 x 13 tables.

Conclusion of the lesson: The teacher told the students that he was happy with their effort and with the amount of work most of them had done during the lesson. He suggested that they should use the five minutes remaining to 'try a bit more' and also suggested that they should 'carry on the work at home, in order to put the remaining lessons on this subject to better use'.

2b.2. Second Lesson Observed (55 minutes), Friday, third lesson (out of five) of the day. Second half of the autumn term, 18 students present.

Material used by teacher and students: The same as for the previous lesson (this was the third and final lesson dedicated to the MA 1 task SNOOK).

Introduction of the lesson: The teacher drew on the blackboard rectangles to represent snook tables of different sizes, while mentioning what the proposed task was about:



He said 'by now you should have tried several cases'. He reinforced the importance of working systematically and of recording their results in tables. He suggested two different ones, explaining and drawing on the blackboard:

size: 2 x	1	2	3	4	5	6
number of rebounds						

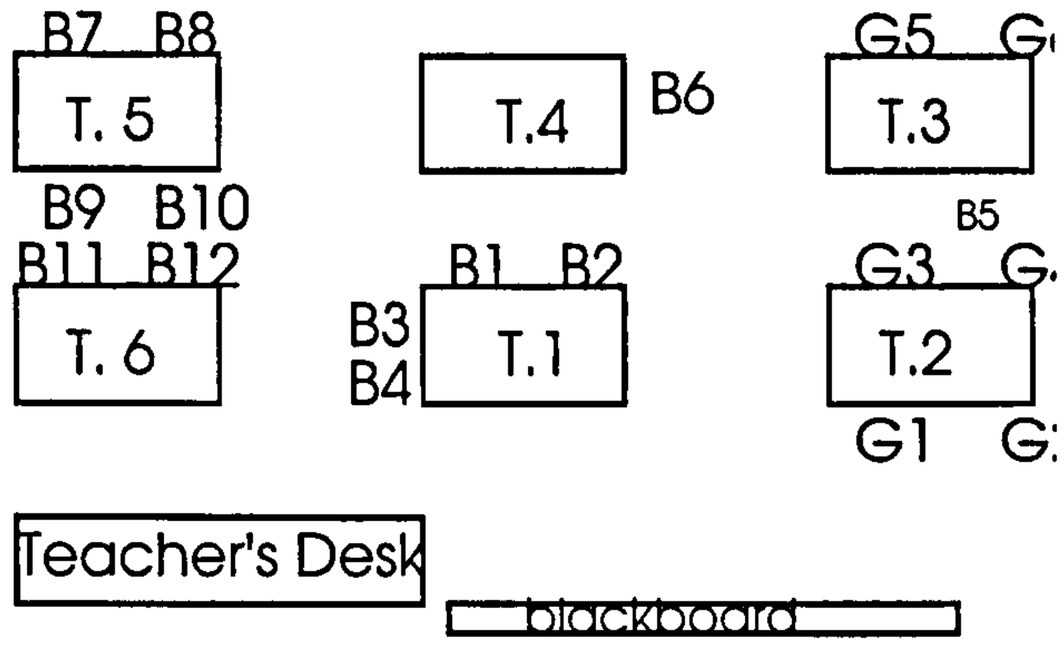
and

	1	2	3	...
1	-	-	-	-
2	-	-	-	-
3	-	-	-	-
...	-	-	-	-

Comment 1: The teacher did not write results on the tables in the blackboard and while he was explaining he made clear that these results could be from different types: some students could be investigating number of rebounds while others could be investigating the final hole for different table sizes.

Development of the lesson: As in the previous one, the students started to work individually or in small groups, and the teacher walked around the tables checking their work and offering suggestions.

The diagram below shows the organisation of the classroom.



Some examples of the teacher's comments are, in chronological order:

Comment 1: B1 was one of the students whose work had not been checked during the previous lesson. B2 was preparing a table of results based on an unsystematic set of attempts (for example, his first four attempts were, as checked by the teacher, sizes: 2 x 3, 1 x 3, 4 x 5 and 3 x 7, and were listed one after the other in his table of results).

To B1: The teacher tried to convince the boy that he was not working systematically and that would be 'very hard' to identify a pattern from his table of results. He suggested that B2 should try to fix one dimension (2) and vary the other, trying to produce a table of results similar to the one suggested on the blackboard.

To G1, G2, G3 and G4: The girls were working together (two of them were the girls who obtained the pattern for 2 x n tables in the previous lesson). The teacher asked: 'what are you doing?' and the girls said that they were checking their predictions for 2 x 13 tables. The teacher suggested that after they finish checking they should try to explain their pattern.

Comment 2: the teacher did not explain what he meant by explaining (it could be either 'describe' or 'justify'). By the girls' behaviour, it seems that they understood it to mean 'describe', as they wrote down a description of the pattern, but did not try to justify it. It is also important to note that, as the teacher's guide comments, justification or proof of results would be rather difficult for secondary students.

To B5: B5, working individually in 2 x n tables, concluded that 'if n is even, the ball finishes in one of the holes at the other end of the table. If n is odd, the ball returns to the initial end of the table, finishing in the 'top' hole.' The teacher reinforced his work ('good') and asked him if he could predict when the ball would finish in the 'top' or in the 'bottom' for even sizes. B5 could not, so the teacher asked him to explain the result he has obtained so far.

Comment 3: B5 was the only student in the class who was not counting number of rebounds but working on finding a pattern for the final hole instead. The teacher did not offer any further suggestions or explanations to the boy to answer his question about the 'top' or 'bottom' hole for even numbers.

To G6: G6, working individually, had done the 1 x n tables (but she did a 2 x 2 instead of a 1 x 4 in the sequencing). The teacher pointed to the 2 x 2 table and asked her what size it was.

The girl perceived what she had done and replaced the table. The teacher suggested that she should carry on the work doing the 2 x n tables.

To B7, B8, B9 and B10: These boys, working together, had done several 2 x n tables. The teacher suggested that they record their results on a table and predict what would happen for bigger 2 x n tables (he suggested 2 x 13 and 2 x 16), verifying their predictions.

To B3: B3 is one of the students with special needs, in fact the one that is regularly helped by another pupil (B4). Together, they had finished the 2 x n tables and were trying some bigger tables (3 x n and 4 x n). The teacher suggested that he try to find out 'what are the sizes of tables whose lines look like a fish?'. B3 looked for them and found the 2 x 3 and the 4 x 6 tables. The teacher said that he should write down this conclusion.

Comment 4: The other special needs students were not in class today. It is interesting to notice that the teacher gave B3 a 'simpler' task. (evidence of differentiation)

To B4: The boy had found the pattern for 2 x n tables and the teacher suggested that he try some prediction for 2 x 12 and 2 x 13 tables, checking them afterwards.

To B6: B6 had been working on the final version of his introduction and showed it to the teacher. The teacher said it was well organised, and that he should carry on with his work.

Comment 5: The teacher did not seem pleased with B6. In fact, he commented with me that B6 should be working on finding the patterns and should leave the final version to be written at home.

To B5, G5 and G6: These students were still doing the 2 x 6 table. The teacher asked them why they were still working in the same example they were working on fifteen minutes ago, and told them to go back to work. They replied 'yes, sir'.

To B11 and B12: The teacher checked that they have found the pattern for 2 x n tables and suggested that they try some predictions for 2 x 14 and 2 x 15 checking them afterwards.

Comment 6: Time on task (summary from observation schedule) During the first 20 minutes of the lesson, almost all the students were working. After that, eight of them only carried out work when the teacher was close to their table.

Comment 7: Although a comment about time on task was not made on the other lessons. it is possible to take this information from the observation schedules, if we decide it is useful.

Conclusion of the lesson: The teacher said that this had been the final lesson on the task and that they would have one week more to finish their report. He reinforced that this task was important because it was training for the GCSE course work they would have in years 10 and 11.

2b.3. Third Lesson Observed (55 minutes), Tuesday, third lesson (out of five) of the day. Second half of the autumn term, 20 students present.

Material used by the teacher: Worksheet on factor facts from "What if" pack, by M. Mclachlan, 1988. (copy in annex II.2(a)).

Comment 1: The material was suggested in the school scheme of work for thesecond half of the spring term. The teacher said that he was expecting some of the students to have their conclusions in the SNOOK task linked to factors and he had decided to anticipate this task. It is interesting to note that none of the students had discussed factors in previous lessons. Most of them were talking about 'odd' and 'even' and very few had distinguished between 'multiple of four' and other even numbers when looking for patterns in 2 x n tables. None of the students had identified a pattern in class for 3 x n tables or bigger ones.
Comment 2: (evidence of timing the material) The teacher did not give the worksheet to students at the beginning of the lesson. As we will see in the lesson description, he proposed the first worksheet exercise orally and gave the sheet to the students when they finished this first task.

Material used by students: Their notebooks and, after finishing the first task, the worksheet given by the teacher.

Special feature: Another teacher was in class today. She sat at table 1 and offered support for G1 and B1, two of the students with special needs. She also offered some support for B2, although most of the time he was carrying on his work together with B3.

Introduction to the Lesson: The teacher introduced the lesson saying that they would have no other homework than the report of their investigation task. He also told them to not forget to take squared paper home with them.
The teacher asked them to open their notebooks 'front of the book, fresh page' and to write down FACTORS (copied from the blackboard).
The teacher asked 'what is a factor?'.
Three children raised their hands and B4 answered: 'they are numbers that go to another number' .
The teacher accepted the 'definition' and gave an example. He wrote on the blackboard:

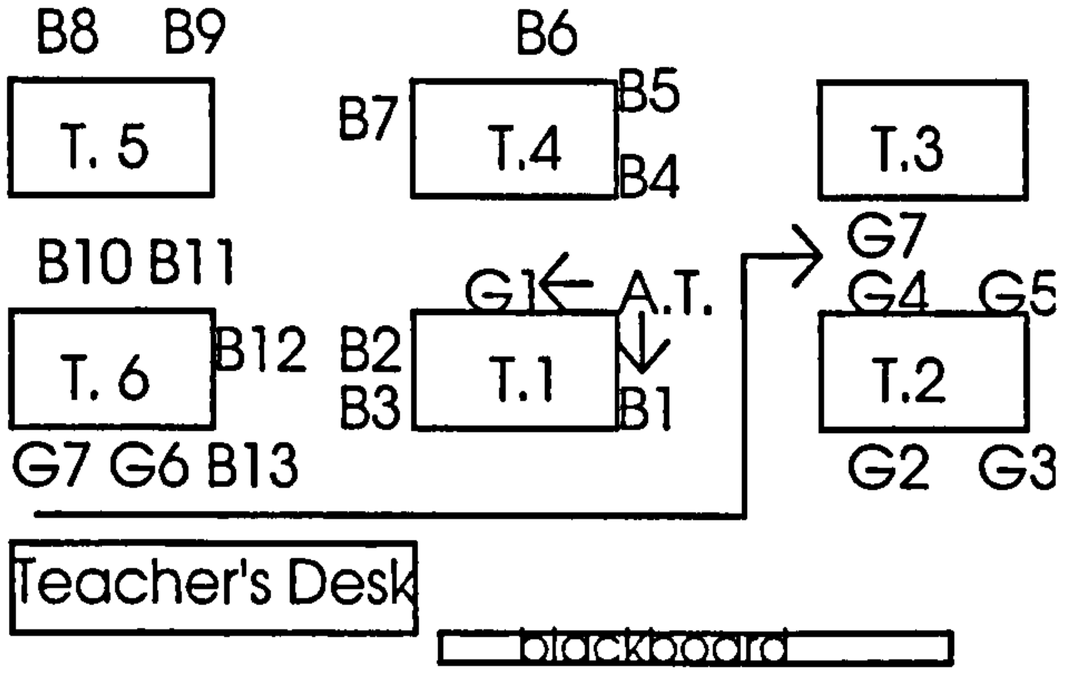
6 : 1, 2, 3, 6.

The teacher proposed that they should do a table. He wrote on the blackboard:

number	factors
1	
2	
3	1, 3
4	
5	
6	1, 2, 3, 6
...	
25	

The teacher asked the students to carry on with this work.

Development of the lesson: The diagram below shows the organisation of the classroom.



The teacher started by doing the registration, using the opportunity to ask students whether or not they had brought their notebooks (two students had not). After finishing registration, the teacher started to walk around the tables checking students' work and offering suggestions. The teacher did not go to table 1 during this lesson, as the other teacher was there helping the students with special needs. She started by helping them to organise the table suggested by the teacher in their notebooks.

The teacher was checking the students' work. In most cases he needed to tell them to add factors that were missing to the list. Some of these comments, in chronological order:

To B11: the teacher looked at the factors of 12 and said 'four must be into it'. B11 added it to his list.

Comment 3: This was the first time that I saw the teacher offer a direct answer.

To G5: (G5 was working individually, but checking her answers afterwards with G4). The teacher looked at her table and commented that there were factors missing in the 12's and 24's list.

To G2: (G2 and G3 were also checking their results). The teacher commented that there were factors missing in the 24's list

To G7: The teacher asked her if she was sure that 12 divided 18.

Comment 4: This was the only case where the student found an extra factor. The common case was that they forgot some of them.

To B6: (B6 was consistently forgetting n as a factor of n) and the teacher showed him the examples developed on the blackboard and asked him if he was not missing some factors. B6 added the number as a factor of itself in all cases.

To B5: B5 was only considering 1 and n as factors of n. The teacher showed him the example developed in the blackboard and said 'most of these numbers will have other factors than 1 and itself, you should look for them.'

Comment 5: None of the students found all the factors by him(/her)self. The teacher had the opportunity to check the work of all students (except those at table 1) and, for those who finished the first task proposed, he gave the worksheet and asked them to do exercises 1 to 5 in section 1. By the end of lesson B5, G7, B11 and B12 were the only ones (except for those in table 1) who had not received the worksheet.

Comment 6: B1 and G1, helped by the extra teacher were, by the end of the lesson, half way throughout their factor table. This teacher also helped B2 and B3, who were finishing the table by the end of the lesson.

Conclusion of the lesson: The teacher told them that they would have one more lesson to conclude this work. He also told them to work hard in their reports on the investigation at home and not to forget to take the material they would need home.

Comment 7: The teacher told me by the end of the lesson that he was not expecting them to work in the second half of the worksheet (except for G4 and G5, who he believed would finish the first part early enough in the next lesson to have a try in the second part). He also said that he was not going to have a third lesson working with these worksheets.

2b.4. Features on the use of Printed Materials Observed during these Lessons:

A) About the SNOOK task.

- The SNOOK investigation was done without using the worksheet with the students. The teacher counted the 'rebounds' and both the initial and final 'hits' separately. This could explain why most of the students were only counting rebounds without including the initial and final 'hits', making the pattern suggested in the teacher guide more obscure.

- The teacher's guide for the SNOOK task suggested that if the material were to be used with low attaining students without the resource of a computer program, some problems were to be expected with ball trajectories (this problem was observed even in the second lesson - In fact the teacher said that he spent a great amount of time in the first lesson helping students to draw correct trajectories in different tables). The teacher's guide suggested that it would be better to use the software with low attainers, in order to allow the students to generate hypothesis and to verify them without the interference of inaccuracy in drawing. We can conclude that the teacher was using the material in a different way than suggested in the teacher's guide. In fact, the teacher did not use any copy of the guide in applying the material.

- The teacher's guide highlighted a series of features that a teacher could expect a year 7 mixed ability group to achieve during the task (see page 19). We numbered them, and we can conclude that there was evidence in the classroom work that:
  - features 1 and 2 were achieved by almost all the students in the classroom (except for two of the students with special needs).
  - features 4 and 5 were achieved by at least 70% of the students.
  - features 6 and 8 were achieved by at least 50% of the students.
  - features 3 and 7 were not observed in the classroom.

- The final report on the students showed a great improvement in their final classroom work. This will be discussed when considering the results of the students, but it is important to note that the teacher thought that most parents were aware of the importance of the course work and that they cared about their children's final report.

B) About the Factors Worksheet:

- Although this worksheet could be related to the previous work (common factors, coprime numbers and similar rectangles could be explored using the SNOOK investigation as

suggested in the teacher's guide), this was not the case. I believe that most of the students saw no relation between this task and the previous one. The topics were not linked by the teacher during the lessons, although in an informal interview, the teacher said he decided to anticipate this worksheet because none of the students had spotted any patterns in the SNOOK investigation related with factors.

- The teacher did not have a copy of the teacher's guide for this material . He also said that he was not planing to complete it with this group of students. We can conclude that he was adapting the worksheet in a different way from that suggested by the authors.
- The school scheme of work does not plan for students within different attainment levels. The teacher said in an informal interview that the usual way he adapted the school scheme for low attainers was to cut off some of the work (sometimes a complete series of task, sometimes giving a simplified version of a common task, as observed). This behaviour allowed these students more time to do the proposed work. There was no special material planned for low attainers groups.

2b.5. Materials Used in Class:

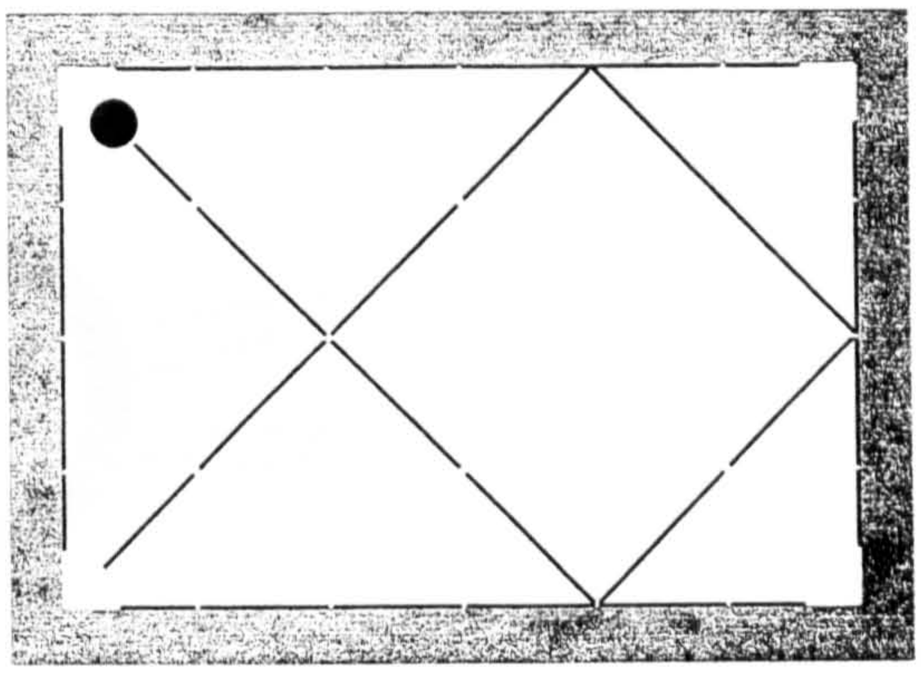
. Snook Worksheet:

SNOOK

The snooker table illustrated has four pockets, one at each corner. A ball is placed at one corner, and is then hit away from the corner at an angle of 45° to the sides of the table. It rebounds from each side at an angle of 45° and eventually falls into the top left hand pocket. Altogether 5 "hits" are made. (These "hits" are made up of the initial strike, the three "bounces" and the final "pot").

How can you predict the number of "hits" that will be made by the ball, when it is struck in a similar way, on rectangular tables with other dimensions?

Which pocket will the ball fall into?

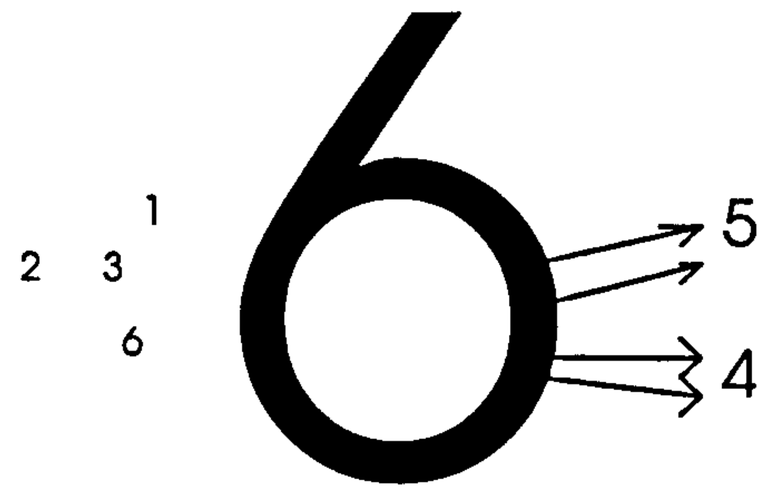


. *Factors Facts*

Factor Facts

You will need: Squared paper, calculator

A factor is a number which will divide into another number without a remainder



6 has four factors 1, 2, 3, 6

9 has three factors 1, 3, 9

1. Write down all the factors for the numbers 1 to 25.
   1. Which numbers have exactly two factors?
   2. Which numbers have exactly three factors?
   3. Which numbers have exactly four factors?
   4. Which numbers have exactly five factors?
   5. Which numbers have exactly six factors?

   Have you missed any number between 1 and 25?
   If so which ones and why?

2. 32 is called a strange number because if you add its factors apart from 32, you get one less than 32.

   $1 + 2 + 4 + 8 + 16 = 31 = 32 - 1$

   Can you find other strange numbers less than 25?

What If ?

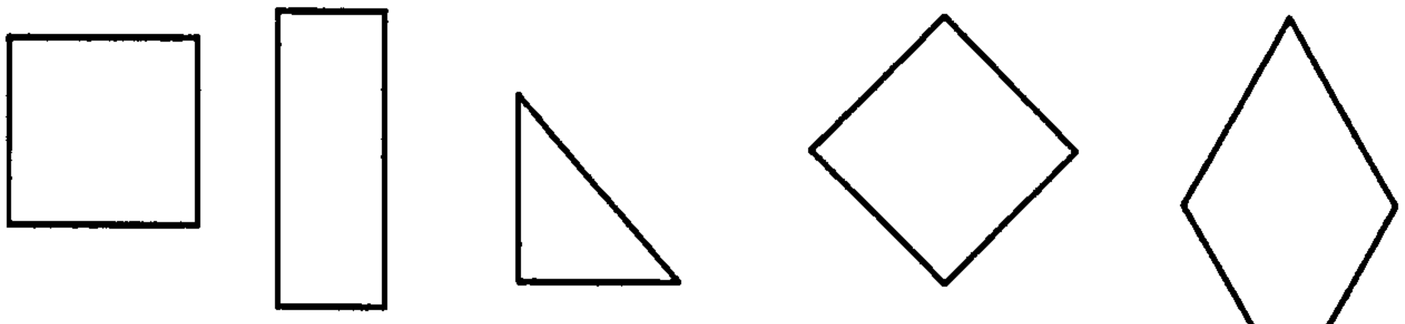
1. What if you look at the sets of numbers with just THREE or FIVE factors?
   Write down what is special about this set. Look at the other set of factors you have found. Write down any patterns you see.
2. What if you try to predict which numbers under 100 will have 7, 8 or 9 factors etc.
   To help you, make a list of the factors of the numbers up to 50. What patterns can you see? Use your list to answer the 'What If' question. 2. How good were your predictions?
3. What if you looked for all the strange numbers less than 100? Can you spot a pattern?
   Explain your pattern to your friend or teacher. Now write down your explanation.

APPENDIX 2C

SAMPLE OF TESTS

Test given to CC8L:
SCHOOL C YEAR 8 .....
NAME: ..... date:.....

Shade
1) Shadow the squares:

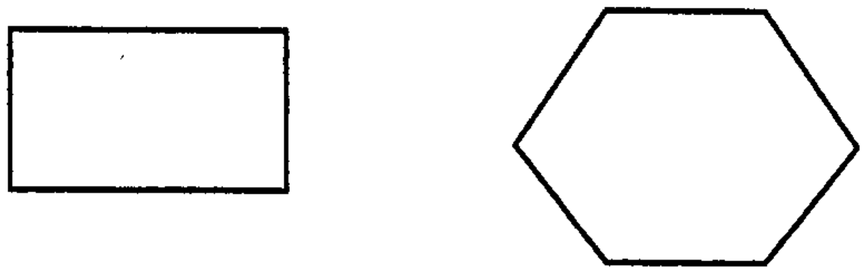


2) Draw:

a) a triangle

b) a pentagon

3) Write the name of the shapes:



....

4)Write the missing numbers:

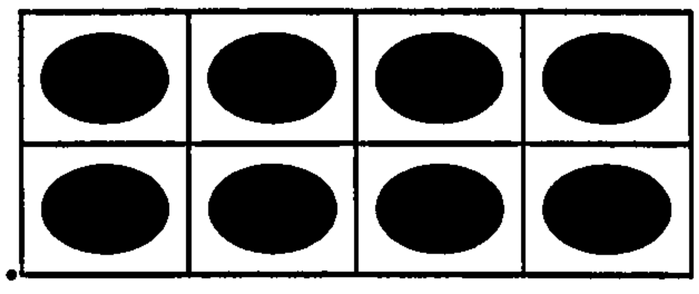
11 + 7 = ...	18 - 7 = ...	28 ÷ 7 = ...
16 + ... = 21	18 - ..... = 6	40 ÷ ..... = 8
12 + ... = 28	65 - 23 = ...	132 ÷ 11 = ...
72 + ... = 91	87 - ..... = 19	..... ÷ 5 = 6

Test given to DD7X (common part):
SCHOOL D YEAR 7 .....
Name:..... date:.....

1 a)Write down the ratio of white to black beads on this string:

1b) What is the ratio of black to white?
2) Write some ratios which mean the same as 6 to 2.

3 a) Use two different colours to divide the whole bar of chocolate in the ratio 3 milk to 1 plain.



3 b) Draw another bar of chocolate and divide it in the ratio 5 milk to 2 plain.

4) The ratio of green to red beads in a necklace is 5 to 2. How many green and how many red beads would there be in a necklace of 35 beads? (Draw it if you find it difficult)

(these questions were followed by four other individualised questions)

Test given to DD9T:

SCHOOL D YEAR 9 .....

Name: .....

1) Two dice are thrown.
a) What is the probability that the total score is 10?
b) What the probability that the total score is greater than 9?

2) Three coins are thrown.
a) Make a list of all the different equally likely outcomes.
b) What is the probability of getting three heads?

3) The four volumes of a encyclopaedia are taken at random and put on a shelf. What is the probability that:
a) volume 4 is in the right position (from left to right)?
b) volume 3 is in the right position (from left to right)?
c) volume 1 come before volume 2 (from left to right)?
d) every volume is in its correct position (from left to right)?

4) A train travels between two stations. It starts from rest (speed zero). Its speed increases until it reaches a maximum. The train travels at this maximum speed for a while and then it slows down to a stop. The time taken at the beginning to reach the maximum speed is greater than the time taken to slow down to a stop. Sketch a graph of (time, speed).

Please show your work in all the questions.

Test given to EE7X:

Name:.....Form:.....

1) After the calculations were done, the brackets were erased from the expressions bellow. Put them back, if you think they are necessary.

a) $4 : 2 - 1 = 1$ c) $4 \times 3 + 2 = 14$ e) $3 + 1 \times 4 = 7$

b) 4 : 2 - 1 = 4 d) 4 x 3 + 2 = 20 f) 3 + 1 x 4 = 16

2)Using the rules for the 1,2,3,4 investigation, the number 55 can be obtained as

55 = 12 + 43

Can you find other ways to get the number 55 using the rules for 1,2,3,4 investigation?
List as many as you can:

3) Using the rules for 1,2,3,4 investigation, can you get:

a) a number smaller than 5 ?

b) a number between 20 and 30?

c) a number bigger than 300?

Test given to GG7M:

(a) Common Part:

Name: _____ 7___

1) Mark those exercises. Put a (√) in the correct ones and a (X) in the incorrect.

( ) There are right angles, but not left angles.

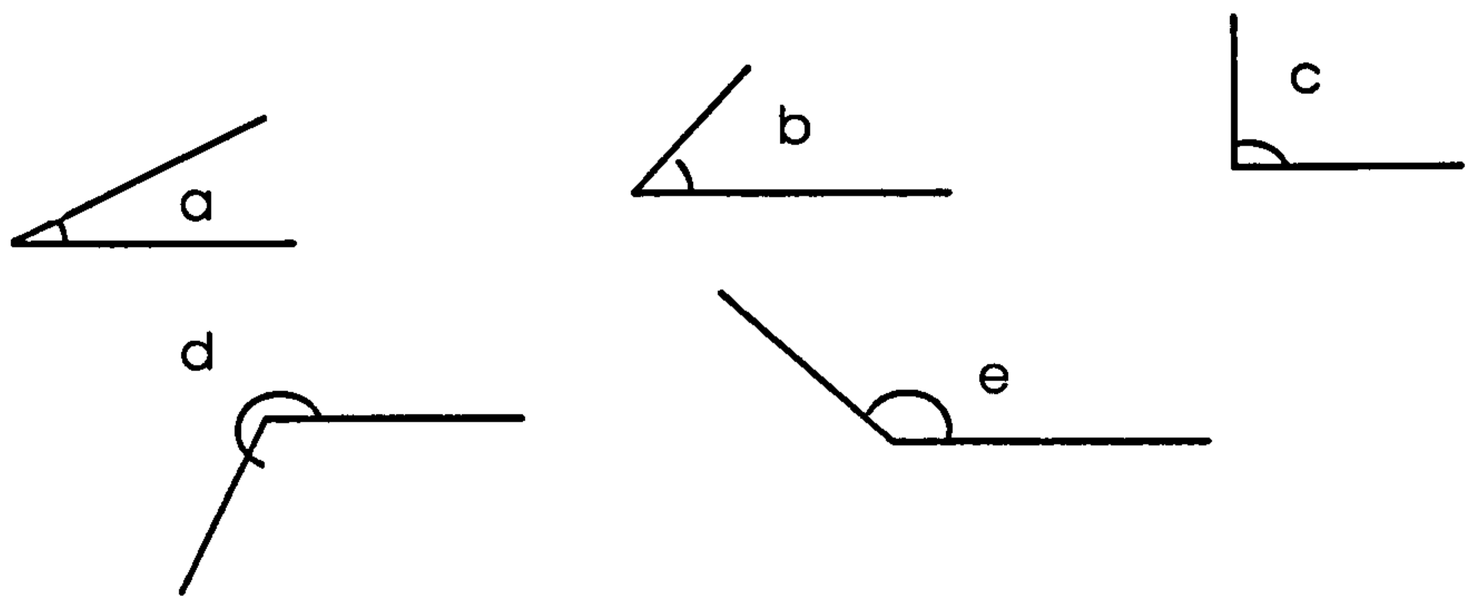
( ) A right angle is 90%.

( ) Every triangle has 3 angles of 180 degrees.

( ) A square has 4 right angles.

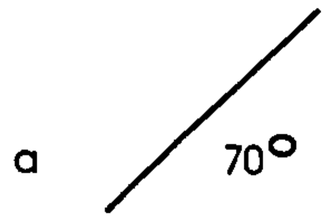
( ) An angle in a square has four sides.

2) Estimate the angles:

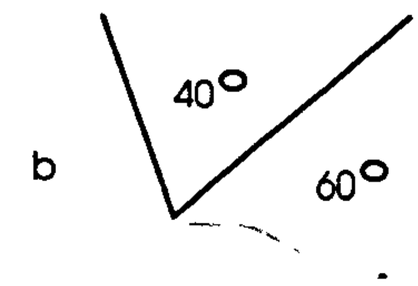


(b) Second part for the book sub-group:

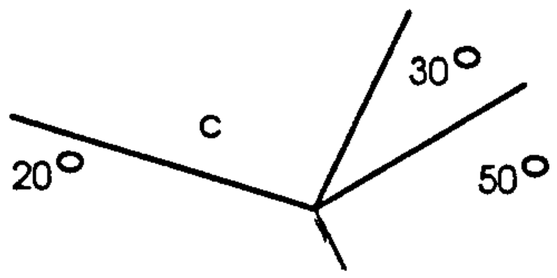
3) What angles fit in the gaps?



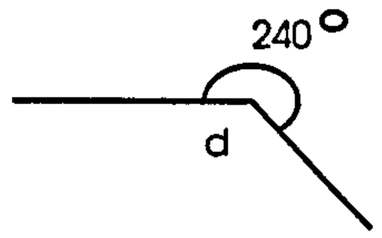
a = _____



b = _____

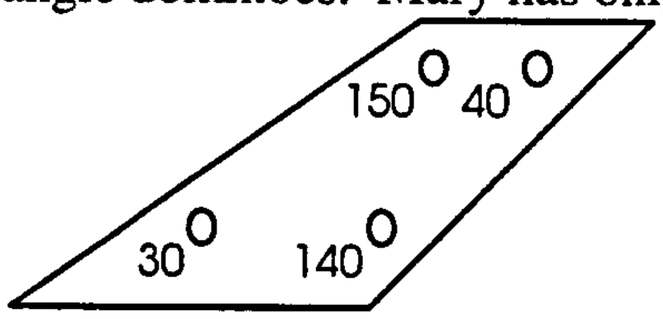


c = _____

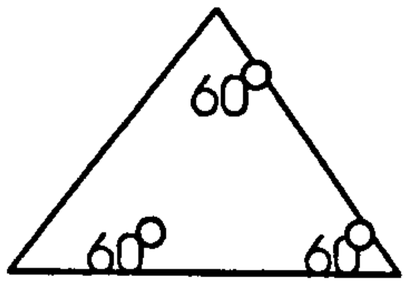


d = _____

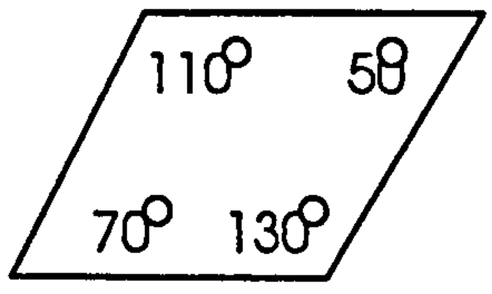
4) Mary and Ann are playing angle dominoes. Mary has only one shape:



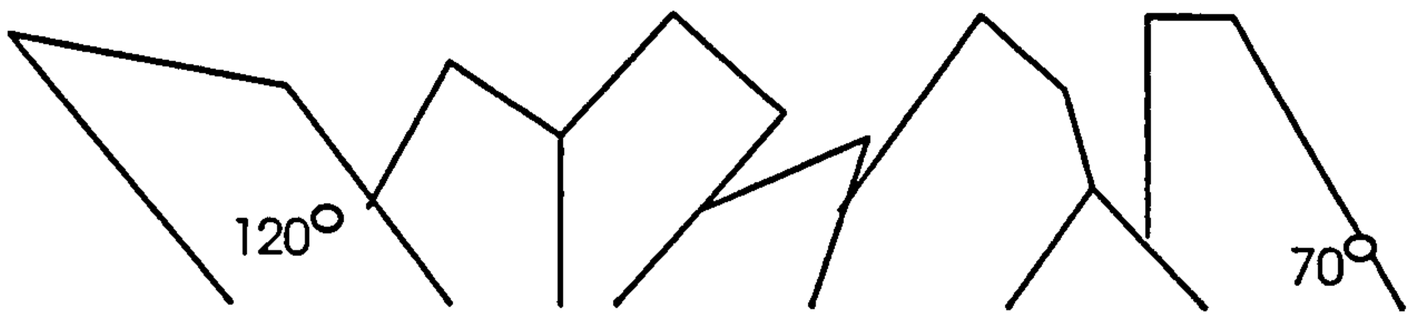
and Ann has two shapes:



and



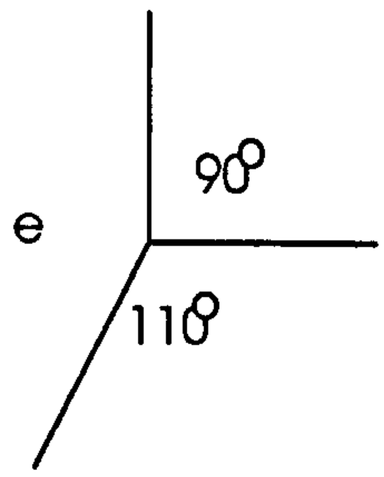
The ruler looks like this:



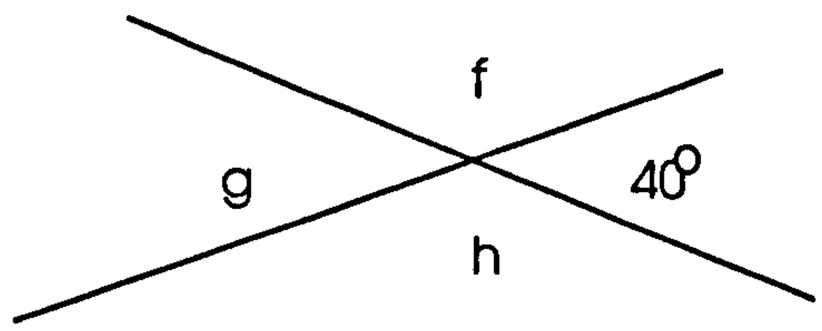
Who won the game ? _____

Why? _____

5) What angles fit the gaps?



e = _____



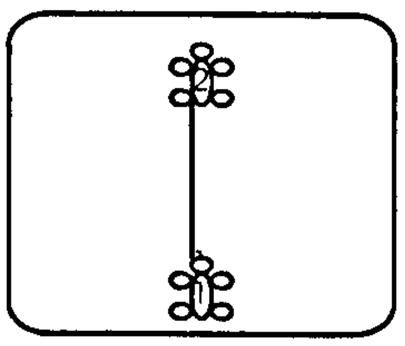
f = _____

g = _____

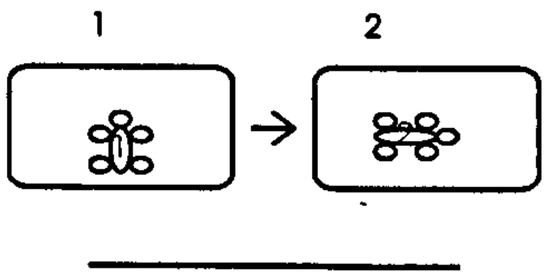
h = _____

(c) second part for the logo sub-group.

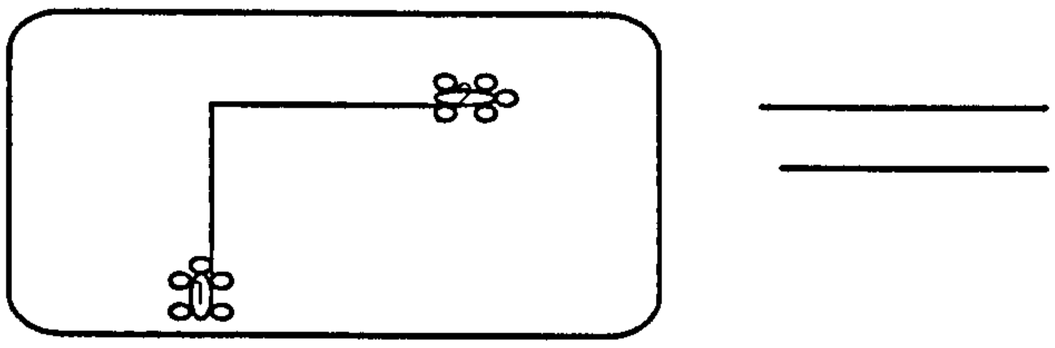
3) What are the commands to the movement of the turtle:



_____

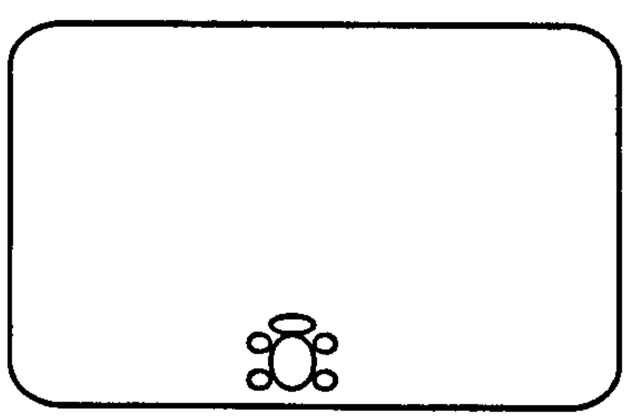


_____

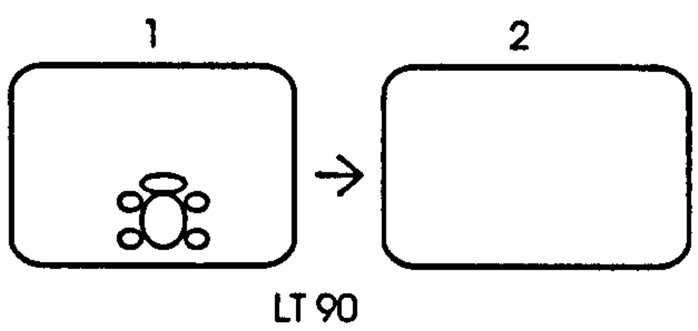


_____

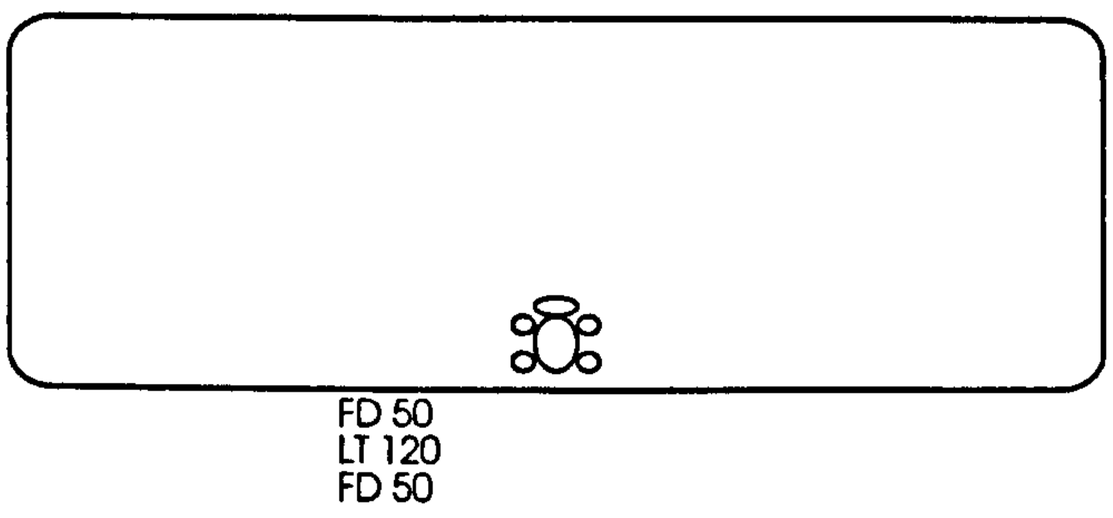
4) Show what the turtle will do. Draw the lines and the final position of the turtle:



FD 100



LT 90



FD 50
LT 120
FD 50

4) Using LOGO I drew an equilateral triangle with sizes 100 'turtle steps'.

Can you make the turtle draw an equilateral triangle bigger than mine? _____
What are the commands you will give to the turtle? _____

5) The following commands make the turtle draw a square:

FD 100
LT 90
FD 100
LT 90
FD 100
LT 90
FD 100

Use the command **REPEAT** to make this list of commands shorter.

Test given to GG8T:

Name: _____ 8_____

1) Give an example of a number between:
a) 0.7 and 0.8 _____
b) 2 and 2.1 _____

2) Which is the nearest tenth to:
a) 0.53 _____
b) 0.92 _____
c) 0.18 _____

3) Write these numbers in order of size, smallest first:
0.5 0.09 0.85 0.07 0.6 : _____

4) Find the total length of each pair of strips:

1 m 68 cm

1 m 03 cm

1 m 58 cm

1 m 48 cm

_____

5) John and Liz measured the length of the blackboard in their classroom. Their measurements were:

John: **2.5**
Liz: **250**

Can you explain what happened?

_____

Test given to HH9M:

Name_____ Group 9____

1) On the squared paper, draw the graph of the straight line which has the points in the following table:

x	0	1	2	3	4
y	0	2	4	6	8

use scale 1 = 2 cm in the x-axis and 1 = 1/2 cm in the y-axis.

2) Plot the graph of the straight line which has the points in the following table:

x	0	2	4	6	8
y	0	3	6	9	1 2

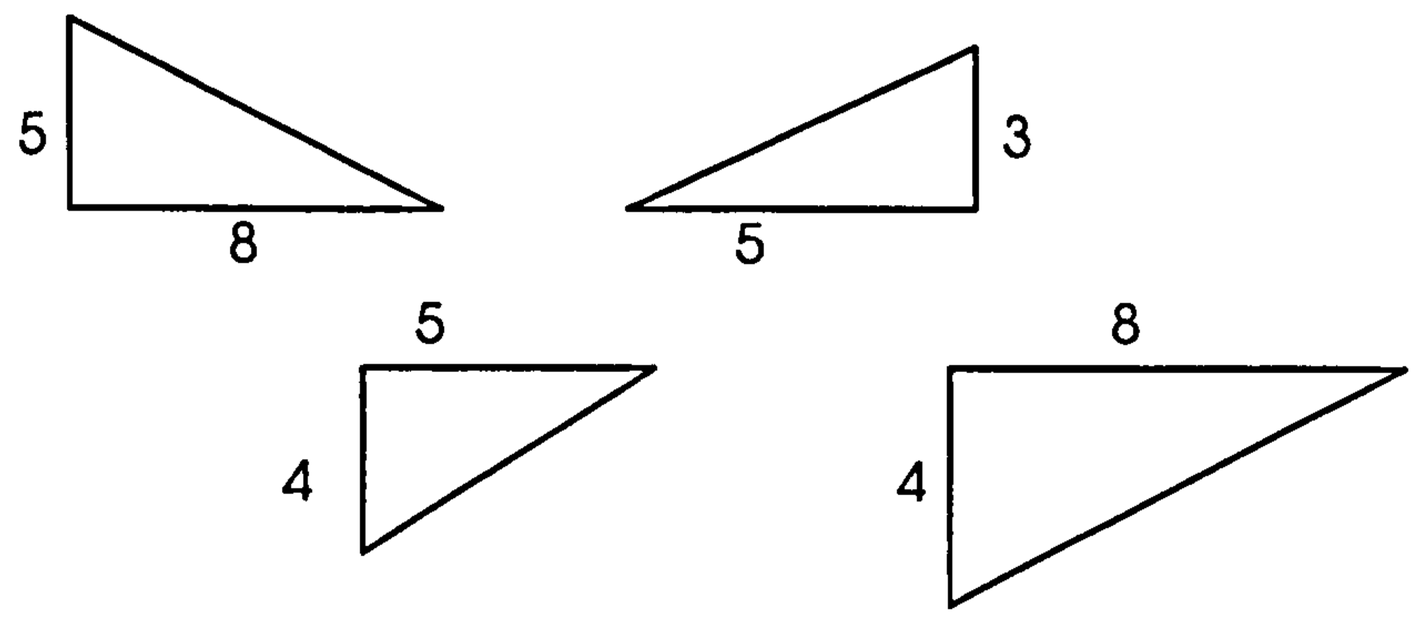
3) Given the equation $y = x + 3$:
(a) make a table for the values of x between zero and 5.
(b) plot the graph

4) Work out five ordered pairs from the line $y = 2x - 1$. Plot the graph.

5) Plot the graph of the only one of the following which is not a straight line:

$y = -3x + 4$ $y = 2x - 8$ $y = 5x$ $y = x^2 - 3$

6) Calculate the gradient of the following slopes:



7) Calculate the gradient of the following graph: (see beginning of the squared paper)

APPENDIX 2D

SAMPLE OF A TEST REPORT: GROUP AA9L

2d.1. The test:

This group of students spent the major part of the observed lessons doing an investigation. The teacher asked them to do a report on it, as part of their assessment for year 9. It would be rather difficult to do a test on an investigative work, so it was decided that the students' report would be used as the test. The researcher had the opportunity to look at these reports and create a series of questions from it, according to what the teacher suggested that they do in class. These activities are:

- Activity 1: Start the work in a sub-set of the possible snook tables, in a systematic way (the teacher suggested tables 2 x n, increasing n by one unit at a time).
- Activity 2: Create a table of results for the observations done in Activity 1.
- Activity 3: Find some patterns that explain the table of results in Activity 2.
- Activity 4: Make some predictions using the pattern in order to check whether if it is working. Test such predictions.
- Activity 5: Express the pattern as a rule.
- Activity 6: Start the work again, using another set of possible snook tables, in a systematic way and creating tables of results. (expand the investigation) - an ideal situation would be to expand for several sets of data.
- Activity 7: Repeat Activity 3 for this new set of data. (find a pattern)
- Activity 8: Repeat Activity 4 for this new set of data (Make some predictions and test them)
- Activity 9: Repeat Activity 5 for this new set of data (express the pattern as a rule).

2d.2. Test correction criteria:

A table was produced in order to register the students results. In this table, the number one (1) was used to represent a correct answer, zero (0) was used to represent an answer incorrect and a blank space ( ) was used to indicate that the student did not carry out the activity. The criteria used for each one of the activities is described below:

- Correction criteria for Activity 1 [Start the work in a sub set of the possible snook tables, in a systematic way (the teacher suggested tables 2 x n, increasing n by one unit at a time)]:

Most of the students followed the teacher's suggestion, and started their work by analysing 2 x n snook tables. One student was not able to draw the correct rebounds and presented a report with errors in the first activity (it was the only case of someone carrying out this activity incorrectly).

Other students adapted the teacher's idea and started their investigation using 1 x n snook tables, and others started their investigation using one very particular case: squared snook tables n x n, that produced very simple results.

In order to represent all these cases, the activity was divided into two items: The students using snook tables 1 x n or 2 x n received full marks in both items; the student using squares snook tables received marks in the first item and zero in the second item, and the student that made mistakes in rebounding received zero in both items.

- Correction criteria for Activity 2 [Create a table of results for observations done in 1]: This activity was also divided into two items to explain all the possible cases:

  The students using snook tables 1 x n or 2 x n that produced tables for number of rebounds or number of 1x1 squares cut by the ball's trajectory received full marks (full mark were also given to those students presenting more than one table of results); the students that produced tables only for the 'final pot' received one in the first item and zero in the second.
- Correction criteria for Activity 3 [Find a pattern that explains the table of results in activity 2]: Students that gave incomplete explanations for the table of results in activity 2, together with the ones that only explained the squared snook tables received marks in the first item and zero in the second. Students that were able to explain the all results in their table for activity 2 received full marks.

  It is important to note here that it was not necessary to express the pattern as rules to receive a mark in this activity (this is going to be assessed in activity five). If the student indicated the pattern in his table of results or in any other form, it was considered correct.
- Correction criteria for Activity 4 [Make some predictions using the pattern in order to verify whether it is working] : The activity was divided into two items: 4a: the student made a prediction, and 4b: the student tested his/her prediction.
- Correction criteria for Activity 5 [Express the pattern as a rule]: Same criteria as that one applied to activity three, considering the rules instead of the patterns. The idea was to add an (*) to answers that expressed the rule using algebraic language, but none of the students gave such answers.
- Correction criteria for Activity 6 [Start the work again, using another set of possible snook tables] : The activity was divided into two items: 6a- the student starts with another set of data, systematically; and 6b- the student organises the results in a table.
- Correction criteria for Activity 7 [Repeat activity 3 for this new set of data. (find a pattern)] : The same criteria used for activity three.
- Correction criteria for Activity 8 [Repeat activity 4 for this new set of data (Make predictions and test them)] : The same criteria used for activity four.

- Correction criteria for Activity 9 [Repeat activity 5 for this new set of data (express the pattern as a rule)] : The same criteria used for activity five.

2d.3. The Table of the Results for the Test - Group AA9L:

We present below the table of results for the test (table 2c.1), obtained using the above criteria. Of the 23 students in the registration list, only 18 submitted the final report of the investigation used as test for the present research. The table below list these 18 students, and their results in the nine activities, divided into items (a) and (b). The table presents all the features described in chapter seven (section 7.1)

Stud/Qst .	1a*	1b*	2a*	2b*	3a	3b	4a**	4b**	5a	5b	6a*	6b*	7a	7b	8a**	8b**	9a	9b	%p1	%p2	% gr.
1	1	1	1	1	1	0			1	0	1								60	17	50
2	1	0			1	0			1	0									30	0	21
3	1	1	1	1	1	0			1	0	1	1							60	33	57
4	0	0	1	1							1								20	17	21
5	1	0	1	0	1	0			1	0	1	1	1	0			1	0	40	67	57
6	1	1			1	0	1	0			1								40	17	36
7	1	0	1	1	1	0			1	0									50	0	36
8	1	1	1	1							1	1							40	33	43
9	1	1			1	0													30	0	21
10	1	1	1	1	1	1			1	0	1	1	1	0			1	0	70	67	79
11	1	1	1	1	1	1	1	1	1	1	1	1	1	0					100	50	93
12	1	1	1	1	1	0					1	1							50	33	50
13	1	1	1	0							1	1							30	33	36
14	1	1	1	1	1	0			1	0	1	1							60	33	57
15	1	1	1	1	1	0			1	0	1	1							60	33	57
16	1	1	1	1	1	1	1	1	1	1	1	1							100	33	86
17			1	0								1							10	17	14
18	1	1	1	1	1	0	1	1			1		1	1	1	1			70	83	86
N correct	16	13	15	12	14	3	4	3	10	2	14	11	4	1	1	1	2	0			
N try	17	17	15	15	14	14	4	4	10	10	14	11	4	4	1	1	2	2			
% correct	89	72	83	67	78	17	22	17	56	11	78	61	22	6	6	6	11	0			
% try	94	94	83	83	78	78	22	22	56	56	78	61	22	22	6	6	11	11			
N. C. Level	5	5	3	3	5	5	6	3	6	6	6	6	4	4	6	3	5	6			
N. C. ATs.	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1			

table 2d.1: group AA9L test results.

APPENDIX 3

THE META-CATEGORIES APPLIED TO THE THREE SETS OF HEADINGS

Observation on the set of tables presented in this appendix: In the tables below the 'meanings' of the meta-categories are specified under each of the three sets' headings. This description is made by corresponding each code (column one) with the description (column two) of an actual behaviour observed under the heading considered (enunciated as general as possible to include possible similar behaviours). Some 'blanks' are observed in some of these tables, meaning that none of the observed behaviours matched the meta-categorisation code 'independent from written materials → dependent on printed materials'. In these cases, an 'interpretation' is given in the second column, filling the space of an observed behaviour, with the objective of describing other possible attitudes teachers could present when using written materials. The problem of deciding whether one of the descriptions of behaviours presented in column two is an 'example' or an 'interpretation' is easily solved by looking to column three, which presents the list of code names of all groups of students in which the described behaviour was observed.

Appendix 3A : Tables of the meta-categories applied to headings in Set I:

Meta-Category	Descriptions of Observed Behaviours Under the Heading Considered	)bserved in
1	School Scheme of Work that does not determine which material should be used.	FF7L
2	School Scheme of Work based on written materials invented by the teachers	
3	School Scheme of Work based on written materials adapted by the teachers from printed materials.	AA8X, AA9L
4	School Scheme of Work that determines which printed material should be used, selecting from a large range of such materials	AA10M, AA10T, CC8L, EE7X, EE10L, EE9M
5	School Scheme of Work that proposes adaptation mainly from one series of textbooks.	FF7X, HH7X
6	School Scheme of Work based on one series of textbooks, but suggesting regular complementation.	CC11T, DD10L, FF8M, GG7M, GG8T
7	Textbook Series, with modifications from the suggestions in the teacher's guides	BB10X, BB9L, BB11M, BB9T, CC8X, CC10M, DD7X, DD9M, DD9T, EE11T, FF8T, HH10L, HH9M, HH10T
8	Textbook Series and Teachers Guides	

table 3a.1: Meta-categories applied to the choice of main source of materials.

Meta-Category	Descriptions of Observed Behaviours Under the Heading Considered	Observed ir
1	Decided by the teacher, without take into consideration the school scheme.	
2	Decided by the teacher, adapting from the school scheme	AA9L, FF7L
3	Decided mainly by the school scheme, with occasional modifications introduced by the teacher.	AA8X
4	Decided by the school scheme, which is based on a series of different printed materials.	AA10M, AA10T, CC8L, EE7X, EE10L, EE9M,
5	Based on a textbook series, with regular alterations (and eventual complementation) done by the school or by the teacher.	FF7X, FF8M, HH7X
6	Based on a textbook series, with evidence of regular complementation decided by the school or by the teacher.	CC11T, DD10L, GG7M, GG8T
7	Taken mainly from a textbook series, with no particular consideration for the suggestions made in the teacher's guides.	BB10X, BB9L, BB11M, BB9T, CC8X, CC10M, DD7X, DD9M, DD9T, EE11T, FF8T, HH10L, HH9M, HH10T
8	Taken mainly from a textbook series, with clear evidence of use of suggestions made by the teacher's guides.	

table 3a.2: Meta-categories applied to the behaviours related to <u>progression of the contents</u>.

Meta-Category	Descriptions of Observed Behaviours Under the Heading Considered	Observed ir
@	Teacher does not allow any reference material to be kept by the students.	BB9L,
1	Teacher only allows the notebook to be kept by the students, but it contains some explanations, definitions and examples given (or corrected) by the teacher.	AA9L, CC8L, DD10L, EE10L, FF7X, FF7L, GG7M, GG8T,
2	Teacher allows some reference material produced by the teacher (or by the school) to be kept by the students, together with the notebooks.	AA8X, CC8X, DD7X,
3	Teacher allows some adapted material produced based on printed materials to be kept by the students, together with the notebooks.	AA10M
4	Teacher allows some printed material (mainly worksheets) to be kept by the students, together with the notebooks.	AA10T, EE7X, EE9M, FF8M, FF8T,
5	Teacher allows some adaptations from the main source of materials, produced by the teacher (or by the school) to be kept by the students, together with the notebooks.	
6	Teacher allows some complementary material to be kept by the students, together with the notebooks. The teacher also allows the textbook to be taken home occasionally.	
7	Teacher allows the notebook to be kept by the students. Teacher also allows the textbook to be taken home occasionally.	
8	Teacher allows the textbook to be kept by the students, together with the notebooks.	BB10X, BB11M, BB9T, CC10M, CC11T, DD9M, DD9T EE11T, HH7X, HH10L, HH9M, HH10T

table 3a.3 : Meta-categories applied to decisions related to <u>material given to the students for reference.</u>

<u>Observation:</u> In order to match the particular observed behaviours presented in table 3a.3 with meta-categories, it was assumed that the less explanations, definitions and solved examples there are in such materials, the more the teacher has to rely in his/her own examples and explanations, and the more the students have to rely on their own notes to study, characterising more independence from printed materials.

Appendix 3B : Tables of the meta-categories applied to headings in Set II:

Observation concerning the below set of tables: In the below tables, headings addressing choices of material made by the teacher are analysed, so all of them fit the meta-codes as described for Set II. To make these tables lighter, some conventions are adopted here and used throughout the tables:

1. Whenever the term written material is used, it is considered that this material is not a printed material.
2. Whenever it is said that the material is created or adapted by the teacher, it includes cases in which the material was produced by the school's mathematics department.
3. Whenever the term printed material is used, it is considered that this material is not taken from the main source of material adopted by the school (school's adopted textbook), but is taken from a wide range of possible sources.
4. Whenever the term textbook is used, it is considered to be the main source of materials adopted by the school.
5. Whenever the term guide is used, it is considered to be the teacher's guide to the material being addressed.

Meta-Category	Descriptions of Observed Behaviours Under the Heading Considered	Observed ir
1	Teacher gives no such written materials during the lesson.	AA8X, AA10M, BB9L, FF7X, FF7L, GG7M, GG8T, HH7X
2	Teacher is using own produced written materials, not based in existing printed materials.	HH9M
3	Teacher is using own produced written materials, adapted from several printed materials.	AA8X, AA10T, BB9T, DD7X, EE9M
3ab	(ab) Teacher gives the students a series of worksheets adapted from printed materials and ask them to 'keep going'.	DD7X
4	Teacher gives the students printed material to be used during the lesson.	AA9L, AA10M, AA10T,EE7X, EE10L, EE9M, CC8L, FF8M
4a	(a) Teacher gives the students more than one printed material, asking them to use these materials for different activities than proposed by the material.	EE9M, GG7M
4b	(b) Teacher gives the students the choice of different printed materials.	AA10T, EE10L
4ab	(ab) Teacher gives the students the printed material, and ask them to 'keep going'.	HH7X
5	Teacher gives the student own produced material, adapted from the textbook.	CC11T
6	Teacher gives the student the textbook, complemented with other written materials or with activities proposed by the teacher.	BB9T,FF8M, HH7X
6ab	(ab) Teacher gives the students the material, and ask them to 'keep going'.	DD10L
7	Teacher gives the student the textbook, but uses it in a different way than suggested by the guides.	EE11T, FF8T, HH10T
7ab	(ab) Teacher gives the students the textbook, and ask them to 'keep going'.	BB10X, BB9L, BB11M, CC8X, CC10M, DD7X, DD9M, DD9T, EE11T, GG7M, GG8T, HH10L
8	Teacher gives the student the textbook, and follows the suggestions of use in the guides.	GG7M, GG8T,

table 3b.1: Meta-categories applied to the choice of materials to be used in class as source of activities

Meta-Category	Descriptions of Observed Behaviours Under the Heading Considered	Observed ir
@	The new topic is introduced neither by the teacher nor by the material used.	AA10M, EE10L, FF7L
1	Teacher does not use any written material to introduce the new topic.	AA8X, BB9L, GG7M, GG8T
2	Teacher bases the introduction on his/her own created material.	HH9M
3	Teacher uses own produced material, based on printed materials.	DD7X, EE9M
4 4b	Teacher bases the introduction on one printed material. Teacher bases the introduction on more than one printed material.	AA9L, AA10T, CC8L, EE7X, EE9M, GG7M
5	Teacher adapts the introduction done by the textbook, with possible complementation	BB9T, CC11T, FF7X, FF8M, HH7X, HH10T
6	Teacher complements the introduction offered by the textbook.	BB10X, BB9T
7 7ab	Teacher uses the textbook to introduce a new topic, not using the suggestions given by the guides. (ab) The students are supposed to read the introduction from the textbook, with no complementation offered by the teacher.	FF8T BB9L, BB11M, CC8X, CC10M, DD7X, DD10L, DD9T, EE11T, HH10L
8	Teacher uses the textbook to introduce a new topic, close to the suggested by the guides.	GG7M

table 3b.2: Meta-categories applied to choice of materials to introduce a new topic

Meta-Category	Descriptions of Observed Behaviours Under the Heading Considered	Observed ir
@	The lesson is introduced neither by the teacher nor by the material.	AA10M
1	No written material was used to introduce the lesson.	AA8X, FF7X, FF7L, GG7M, GG8T
2	Material created by the teacher was used to introduce the lesson.	HH9M
3	The material used to introduce the lesson was produced by the teacher, adapted from printed materials. (ab) the students were asked to 'keep going' with their activities taken from the selected adapted materials	AA10T, BB9T, DD7X, EE9M, AA8X, DD7X,
4 4b 4ab	Printed material was used to introduce the lesson. (b) The introduction was made by the teacher, based on more than one printed materials, and/or offering the students choice of materials. (ab) the students were asked to 'keep going' with their activities taken from the printed materials.	AA9L, AA10M, AA10T, CC8L, EE7X, EE10L, EE9M, FF8M, GG7M, EE10L, EE9M HH7X
5	The textbook was adapted to introduce the lesson.	BB9T, CC11T, FF7X, FF8M, FF8T, HH7X, HH10T
6	The textbook was complemented with other materials to introduce the lesson.	BB10X, BB9T,
7 7ab	The textbook was used to introduce the lesson, without taken into consideration the suggestions given by the guides. (ab) the students were asked to 'keep going' with their activities taken from the textbook.	BB11M, FF8T, BB10X, BB9L, BB11M, CC8X, CC10M, DD7X, DD10L, DD9M, DD9T, EE11T, GG7M, GG8T, HH10L
8	The textbook was used to introduce the lesson, close to the general guidelines given by the guides.	GG7M, GG8T

table 3b.3: Meta-categories applied to choice of materials used to introduce a lesson.

Meta-Category	Descriptions of Observed Behaviours Under the Heading Considered	Observed in
@	Teacher gives no reference materials during the lesson. Students have no notes.	AA10M, FF7X, FF7L, GG7M, GG8T
1	Teacher gives no reference materials during the lesson. Students use their own notes.	AA8X, AA9L, AA10M. AA10T EE7X, EE9M, FF7X, HH9M
2	Teacher gives own produced written materials for reference, not based on printed materials.	AA10M, HH9M
3	Teacher gives own produced written materials for reference, adapted from printed materials.	AA10T, DD7X, EE9M
4	Teacher gives the students one printed material to be used as reference during the lesson.	AA10T, CC8L, EE10L, FF8M, GG7M, HH7X
5	Teacher gives the student own produced material, adapted from the textbook to be used as reference.	
6	Teacher gives the student the textbook, complemented with other written materials to be used as reference.	BB9T
7	Teacher gives the student the textbooks, but uses it as reference material in a different way than suggested by the guides.	FF8M, FF8T, HH7X, HH10T
7ab	(ab) Teacher gives the students the textbook and asks them to 'keep going', reading the explanations and examples by themselves whenever they think it is the case.	BB10X, BB9L, BB11M, CC8X, CC10M, CC11T, DD7X, DD10L, DD9M, DD9T, EE11T, GG7M, GG8T, HH10L
8	Teacher gives the student the textbook, and follows the suggestions of use as reference in the guides.	GG7M, GG8T

table 3b.4: Meta-categories applied to the choice of reference materials for classroom work.

Meta-Category	Descriptions of Observed Behaviours Under the Heading Considered	Observed in
@	Examples were not given by the teacher or by the material.	AA8X, AA10M, EE9M, FF7L, FF8M, GG7M, GG8T
1	Examples were given by the teacher, without using written materials.	AA8X, BB9L, FF7X, FF7L, GG7M, GG8T, HH7X
2	Examples were provided by teacher's own invented materials.	HH9M
3	Examples were observed in material developed by the teacher based on existing printed materials.	AA8X, BB9T, DD7X,
4	Examples were observed in printed materials.	AA9L, AA10M, CC8L, EE7X, EE10L, EE9M, GG7M
4ab	(ab) Students were supposed to read the examples in the material by themselves.	AA10T, HH7X
5	Examples were selected to complement and adapt existing ones in the textbook.	BB10X, HH7X
6	Examples were selected to complement existing ones in the textbook.	BB9T, FF8M, FF8T
7	All examples given by the teacher were observed in the textbook, in a different way than suggested by the guides.	HH10T
7ab	(ab) Students were supposed to read the examples in the textbook by themselves.	BB10X, BB9L, BB11M, CC8X, CC10M, CC11T, DD7X, DD10L, DD9M, DD9T, EE11T, HH10L
8	Examples observed in the textbook, used close to the suggested by the guides.	BB9T, GG8T,

table 3b.5: Meta-categories applied to choice of materials for exemplification.

Meta-Category	Descriptions of Observed Behaviours Under the Heading Considered	Observed ir
@	Neither the teacher nor the chosen material provide conclusions to the topic.	AA8X, AA9L, AA10M, CC8L, EE7X, EE10L, FF7X, FF7L, FF8M
1	Teacher chooses to establish the conclusions without basing them on any written material.	AA8X, FF7L, GG7M
2	Teacher chooses to establish conclusions based on the ones provided by own created material.	HH9M
3	Teacher chooses to establish conclusions using the ones provided by own produced material, based on existing printed materials.	EE9M, FF8T
3ab	(ab) Teacher does not reinforce the conclusions established by the material. Students are supposed to read them by themselves.	AA10T
4	Teacher chooses to establish conclusions based on the ones provided by the printed materials.	GG7M
4ab	(ab) Teacher does not reinforce the conclusions established by the printed material. The students are supposed to read them.	HH7X
5	Teacher chooses to establish conclusions by adapting the ones provided by the textbook.	BB10X, FF8T, GG7M, HH7X
6	Teacher chooses to establish conclusions by complementing the ones provided by the textbook.	BB9T, HH10T
7	Teacher chooses to establish conclusions based on the ones provided by the textbook, without following the guide's suggestions.	-
7ab	(ab) Teacher does not reinforce the conclusions established by the textbook. Students are supposed to read them by themselves.	BB10X, BB9L, BB11M, CC8X, CC10M, CC11T, DD7X, DD10L, DD9M, DD9T, EE11T, GG8T, HH10L
8	Teacher chooses to establish conclusions based on the ones provided by the textbook, following the guide's suggestions.	

table 3b.6: Meta-categories applied to <u>choice of materials to conclude a topic</u>.

Meta-Category	Descriptions of Observed Behaviours Under the Heading Considered	Observed ir
@	Teacher recognises the existence of links but does not establish them during the lesson. The material also fails to establish links.	AA9L
1	Teacher chooses to establish the links without basing them on any written material.	AA8X, EE7X, EE9M, FF7X, FF7L, GG8T,
2	Teacher chooses to establish links based on the ones provided by own created material.	HH9M
3	Teacher chooses to establish links using the ones provided by own produced material, based on existing printed materials.	EE9M,
4	Teacher chooses to establish links based on the ones provided by the printed materials.	AA10T, GG7M
4ab	(ab) Teacher does not reinforce the links established by the printed materials. Students are supposed to read them by themselves.	HH7X
5	Teacher chooses to establish links by adapting the ones provided by the textbook.	BB9T, GG7M, HH7X
6	Teacher chooses to establish links by complementing the ones provided by the textbook.	HH10T
7	Teacher chooses to establish links based on the ones provided by the textbook, without following the guide's suggestions.	-
7ab	(ab) Teacher does not reinforce the links established by the textbook. Students are supposed to read them by themselves.	BB11M, CC8X, CC10M, CC11T, DD7X, DD10L, DD9M DD9T, EE11T, HH10L
8	Teacher chooses to establish links based on the ones provided by the textbook, following the guide's suggestions.	

table 3b.7: Meta-categories applied to <u>choice of material to promote link between related topics or topics in progression.</u>

Meta-Category	Descriptions of Observed Behaviours Under the Heading Considered	Observed in
@	No differentiation provided by the teacher. All students expected to do the same activities.	AA8X, AA10M, AA10T, BB9T, DD7X, EE7X, EE10L, EE9M, FF7X, FF7L, FF8M, GG7M, GG8T, HH9M, HH10T
1	All students working from the same source of materials. Differentiation is provided by the teacher without using any extra material.	AA9L, BB9L, FF8M, HH7X, HH9M
2	Teacher provides differentiation by introducing extra own created materials.	
3	Teacher provides differentiation by introducing extra own produced materials, adapted from printed materials.	AA10T
3ab	(ab) Teacher provides differentiation by given the students the adapted material and allowing them to work at their own pace.	AA8X
4	Teacher provides differentiation by introducing extra printed materials.	GG7M
4b	(b) Teacher provides differentiation by offering the students options within a set of tasks and/or printed materials.	AA9L, AA10M, AA10T, EE10L
4ab	(ab) Teacher provides differentiation by given the students the printed material and allowing them to work at their own pace.	CC8L, HH7X
5	Teacher provides differentiation by introducing extra material adapted from the textbook.	CC11T
6	Teacher provides differentiation by introducing complementary material to the textbook.	DD10L
7	Teacher provides differentiation selecting from the textbook, without using suggestions given by the guides.	FF8T
7ab	(ab) Teacher provides differentiation by given the students the textbook and allowing them to work at their own pace.	BB10X, BB9L, BB11M, CC8X, CC10M, DD7X, DD9M, DD9T, EE11T, GG7M, GG8T, HH10L
8	Teacher provides differentiation using the textbook and the differentiation suggestions given by the guides.	GG8T

table 3b.8: Meta-categories applied to <u>choice of materials to promote differentiation within class</u>.

Meta-Category	Descriptions of Observed Behaviours Under the Heading Considered	Observed in
@	No remedial situations were introduced, although some students needed them.	AA10T, BB10X, CC8X
1	All students working from the same source of materials. Remedial situations are provided by the teacher without using any extra material.	AA8X, AA9L, AA10M, AA10T, BB9L CC8L, DD7X, DD10L, EE10L, EE9M, FF7L, GG7M, HH10L
2	Teacher provides remedial situations by introducing extra own created materials.	
3	Teacher provides remedial situations by introducing extra own produced materials, adapted from printed materials.	AA10T
4	Teacher provides remedial situations by introducing extra printed materials.	BB11M, GG7M
4b	(b) Teacher provides remedial material, giving the students the choice as to whether they change materials.	AA10T, EE10L
5	Teacher provides remedial situations by introducing extra material adapted from the textbook.	
6	Teacher provides remedial situations by introducing complementary material to the textbook.	
7	Teacher provides remedial situations selecting from the textbook, without using the suggestions given by the guides.	BB11M
8	Teacher provides remedial situations using the textbook and the suggestions of activities proposed by the guides.	

table 3b.9: Meta-categories applied to <u>choice of remedial material</u>.

Meta-Category	Descriptions of Observed Behaviours Under the Heading Considered	Observed ir
@	Teacher sets no homework.	AA8X, AA10T, BB10X, BB9L, BB11M, CC8X, CC8L, CC10M, DD7X, DD10L, DD9M, DD9T, EE7X, EE10L, EE9M, FF7L, FF8T, GG7M, GG8T, HH7X, HH10L, HH9M
1	Teacher sets no written materials as source of homework.	FF7X, FF8T, HH7X, HH10T
2	Teacher sets own created written materials as source of homework,	HH9M
3	Teacher sets homework using own produced written materials, adapted from several printed materials.	AA8X, BB9T, EE9M
4	Teacher sets homework from one (or more) printed material.	AA9L, AA10M, AA10T, EE7X. FF7X, FF8M
5	Teacher sets own produced material as source of homework, adapted from the textbook.	CC8X, CC8T, CC10M, CC11T, DD7X, DD10L, DD9M
6	Teacher sets homework from material complementary to the textbook (or complementing the homework set from the book).	
7	Teacher sets homework from the textbook used in classroom, without considering the suggestions of the guides.	BB9T, CC11T, DD9T, ET11T, HH10L, HH10T
7ab	(ab) Teacher sets homework by asking the students to 'keep going', without setting target or time limit.	BB11M
8	Teacher sets homework from the textbook following the suggestions of use in the guides.	AA8M, BB10X, BB11M, GG8T

table 3b.10: Meta-categories applied to the choice of materials for homework.

Meta-Category	Descriptions of Observed Behaviours Under the Heading Considered	Observed ir
1	Teacher used no material to provide feedback.	AA8X, AA9L, AA10M, AA10T, CC8L, DD10L, EE7X, EE10L, EE9M, EE11T, FF7X, FF7L, FF8M, FF8T, GG7M, GG8T, HH7X, HH10L, HH9M, HH10T
2	Teacher produces own materials to provide feedback.	
3	Teacher adapts answer books from printed materials to provide feedback.	DD7X
4	Teacher uses answer book from printed materials to provide feedback.	FF8M
4ab	(ab) Teacher allows students to consult the answer book of the material whenever they feel it is necessary.	HH7X
5	Teacher uses own adaptation of answer books from the textbook to provide feedback to students	CC11T
6	Teacher complements feedback provided by the answer book of the textbook with his/her own feedback or complementary material	BB9T, CC8X, CC10M, DD9T, FF8T
7	Teacher mainly checks students answers using the answer book of the textbook. Very few comments are added.	BB10X, BB9L, BB11M, BB9T, DD10L, DD9M, DD9T,
7a	(a) Teacher reads the answers from answer book of the textbook loud for the whole class.	DD9T
7b	(b) Teacher allows the students to consult the answer book of the textbook and discusses their performance with them.	EE11T, GG8T, HH10L, HH10T
7ab	(ab) Teacher allows the students to consult the answer book of the textbook whenever they feel it is necessary.	DD7X, GG8T
8	The teacher uses the answer book of the textbook to provide feedback, following the general guidelines offered by the teacher's guides on exercise feedback.	BB9T

table 3b.11: Meta-categories applied to choice of materials for feedback on exercises.

Appendix 3C : Tables of meta-categories applied to headings in Set III:

Observation concerning the below set of tables: In the below tables, headings addressing the use of the previously chosen material by teacher are analysed, so all of them fit the meta-codes as described for Set III. To make these tables lighter, some conventions are adopted here and used throughout the tables:

1. Whenever the term material is used, it is considered that this is the material previously chosen by the teacher.
2. Whenever the term guide is used, it is considered to be the teacher's guide from the chosen material.
3. Whenever the teacher is using the guide only as a source of answers to exercises and problems, it will be refered as to the 'answer book'.

Meta-Category	Descriptions of Observed Behaviours Under the Heading Considered	Observed in
1	Teacher proposes activities without using any source of written materials.	AA8X, AA10M, BB9L, FF7X, FF7L, GG7M, GG8T, HH7X
2	Teacher proposes activities, complementing and modifying the material.	AA10M, FF7X, HH7X
2a	Teacher proposes activities modifying the aims of the material.	EE9M
3	Teacher proposes activities selecting from those proposed in the material.	EE9M
4	Teacher proposes activities based on the material, without giving the students a copy.	AA9L, AA10M, AA10T, EE7X, EE9M
5	Teacher mainly proposes activities from the material, complementing it with a few activities from other sources.	AA10T, FF8M
6	Teacher proposes activities from the material, with evidence of simple selection.	AA9L, EE7X, HH9M
7	Teacher proposes activities from the material.	AA10T, BB9T, EE10L, EE11T, FF8T, GG7M, HH10T
7ab	(ab) The students are supposed to gather the activities in the material by themselves.	AA8X, BB10X, BB9L, BB11M, CC8X, CC8L, CC10M, CC11T, DD7X, DD10L, DD9M, DD9T, EE11T, GG7M, GG8T, HH7X, HH10L
8	Teacher proposes activities from the material, in accordance with the guide's suggestions.	BB9T, DD7X, FF8M, GG7M, GG8T

table 3c.1: Meta-categories applied to use of the chosen material as source of activities.

Meta-Category	Descriptions of Observed Behaviours Under the Heading Considered	Observed ir
@	Neither the teacher nor the material introduces a new topic. Students are supposed to work through the exercises straight away.	EE10L
1	Teacher does not use written materials to introduce a topic, doing it in his/her own way.	AA8X, AA10M, BB9L, FF7L, GG7M, GG8T
2	Teacher introduces a new topic adding new aspects to the introduction presented in the material.	BB9T, FF8T, HH7X
3	Teacher introduces a new topic by selecting from the material, modifying its aims.	EE9M, FF7X, FF8M
4	Teacher introduces a new topic based on written materials, but does not give a copy of these materials for the students, doing the introduction him(her)self.	AA9L, EE7X, EE9M
5	Teacher introduces a new topic based on the material, with clear evidence of minor complementation.	BB10X, BB9T, GG7M, HH9M, HH10T
6	Teacher introduces a new topic based on the material, with evidence of minor modification.	AA10T
7	Teacher introduces a new topic essentially in the same way done in the material.	FF8T
7ab	(ab) Teacher does not highlight the introduction of a new topic, students are supposed to read the introduction to the material by themselves.	BB9L, BB11M, CC8X, CC8L, CC10M, CC11T, DD7X, DD10L, DD9M, DD9T, EE11T, HH10L
8	Teacher introduces a new topic close to the material and follos the guide's suggestions.	DD7X, GG7M

table 3c.2: Meta-categories applied to use of the chosen material to introduce a new topic.

Meta-Category	Descriptions of Observed Behaviours Under the Heading Considered	Observed ir
@	Neither the teacher nor the material introduce the lesson. Students are supposed to work through the exercises straight away.	
1	Teacher does not use written materials to introduce the lesson, doing it in his/her own way.	AA8X, AA10M, FF7X, FF7L, GG7M, GG8T, HH7X
2	Teacher introduces the lesson by adding new features to the material	BB10X, BB9T, CC8L, FF7X, FF8M, FF8T
3	Teacher introduces the lesson by modifying, by selection, the introduction presented in the material.	EE9M, FF7X
4	Teacher introduces the lesson based on written materials, but does not give a copy to the students, doing the introduction him(her)self.	AA9L, AA10M, AA10T, EE7X, EE9M
5	Teacher introduces the lesson based on the material, with clear evidence of minor complementation.	AA10T, BB10X, BB9T, FF8T, HH9M, HH10T
6	Teacher introduces the lesson based on the material, with clear evidence of minor modifications by selection.	
7	Teacher introduces the lesson essentially using what is presented in the material.	BB11M, EE7X, FF8T, GG7M
7ab	(ab) Teacher introduces the lesson by telling the students to reassume their previous work by themselves, reading whatever introduction is necessary from the material.	AA8X, AA10T, BB10X, BB9L, BB11M, CC8X, CC10M, CC11T, DD7X, DD10L, DD9M, DD9T, EE10L, EE11T, GG7M, GG8T, HH7X, HH10L
8	Teacher introduces the lesson essentially using what is presented in the material and follos the guide's suggestions.	BB9T, DD7X, FF8M, GG7M, GG8T

table 3c.3: Meta-categories applied to use of the chosen material to introduce a lesson.

Meta-Category	Descriptions of Observed Behaviours Under the Heading Considered	Observed ir
@	No material is given as reference, not even notes in students' notebooks.	AA10M, FF7L, GG7M, GG8T
1	Teacher provides the reference, without using materials, (usually these references take the form of notes written by students under teacher's guidance in their notebooks).	AA8X, AA9L, AA10M, AA10T EE7X, EE9M, FF7X
2	Teacher provides the reference by modifying, through complementation, the source(s) of references chosen.	AA10M, FF8M, HH7X, HH9M
3	Teacher provides the reference by modifying, through selection, the source(s) of references chosen.	AA10T, EE9M
4	Teacher offers guidance to the students in writing reference notes in their notebooks. These references are based on the material, which is not given to the students, and revised by the teacher.	
5	Teacher offers minor complementation to the material to be used as reference by the students, usually in the notebooks.	AA8X, HH10T
6	Teacher makes minor modifications, usually trough selection, to the material to be used as reference.	
7	Teacher highlights the reference in the material	FF8T, GG7M
7ab	(ab) Students have to look for the references in the material by themselves. Teacher does not highlight them.	AA10T, BB10X, BB9L, BB11M, BB9T, CC8X, CC8L, CC10M, CC11T, DD7X, DD10L, DD9M, DD9T, EE10L, EE11T, GG7M, GG8T, HH7X, HH10L
8	Teacher uses the suggestions in the guides as reference material.	DD7X, GG7M, GG8T

table 3c.4: Meta-categories applied to <u>use of the chosen reference material during the lesson</u>.

Meta-Category	Descriptions of Observed Behaviours Under the Heading Considered	Observed ir
@	Neither the teacher nor the material provides examples or solved exercises.	AA8X,AA10M,AA10T, EE9M, FF7L, FF8M, GG7M, GG8T
1	Teacher provides examples, without using any written material as resource.	AA8X, BB9L, FF7X, FF7L, GG7M, GG8T, HH7X
2	Teacher provides examples adding to the ones given in the material, including new situations.	AA9L, BB9T, FF7X, FF8M, FF8T, GG7M, HH7X
3	Teacher provides examples, using features from the material, but modifying its objectives.	EE9M
4	Teacher provides examples based on the material, without giving the students a copy.	AA10M,AA10T, EE7X, EE9M
5	Teacher provides examples complementing the ones given in the material, with similar ones.	AA8X, BB10X, BB9T, EE7X, FF8M
6	Teacher provides examples taken from the material, with evidence of simple selection done by the teacher.	GG7M, HH9M, HH10T
7	Teacher provides examples taken from the material.	
7ab	(ab) Students are supposed to read the examples in the material by themselves.	AA10T, BB10X, BB9L, BB11M, CC8X, CC8L, CC10M, CC11T, DD7X, DD10L, DD9M, DD9T, EE10L, EE11T, HH7X, HH10L
8	Teacher provides examples taken from the material, and used as suggested in the guide.	BB9T, DD7X, GG8T

table 3c.5: Meta-categories applied to <u>use of the chosen material as source of examples</u>.

Meta-Category	Descriptions of Observed Behaviours Under the Heading Considered	Observed ir
@	Neither the teacher nor the material presents a conclusion to the topic.	AA8X, AA9L, AA10M, CC8L, EE7X, EE10L, FF7X, FF7L, FF8M
1	Teacher provides conclusion without using written materials.	AA8X, FF7L, GG7M
2	Teacher provides conclusion modifying, through complementation the ones offered by the material.	BB9T, FF8T, GG7M, HH7X
3	Teacher provides conclusion, modifying through selection the ones offered by the material.	BB10X, GG7M
4	Teacher provides conclusion, based on the ones of the material, without given the students a copy of the material.	
5	Teacher provides conclusion based on the material, with evidence of simple complementation.	BB9T, HH9M, HH10T
6	Teacher provides conclusion based on the material, with evidence of small modifications.	EE9M
7	Teacher provides conclusion by highlighting the ones given by the material.	
7ab	(ab) Students are supposed to read the conclusions offered by the material by themselves.	AA10T, BB10X, BB9L, BB11M, CC8X, CC10M, CC11T, DD7X, DD10L, DD9M DD9T, EE11T, GG8T, HH7X, HH10L
8	Teacher provides conclusion taken from the material and presented as suggested in the guide.	FF8T

table 3c.6: Meta-categories applied to use of the chosen material to conclude a topic.

Meta-Category	Descriptions of Observed Behaviours Under the Heading Considered	Observed ir
@	Neither the teacher nor the material provides links between related topics.	AA9L
1	Teacher provides links between related topics, without using written materials.	AA8X, EE7X, EE9M, FF7L, GG8T
2	Teacher provides links between related topics, modifying the material through complementation.	AA8X, BB9T, FF7X, GG7M, HH7X
3	Teacher provides links between related topics, modifying, through selection the ones offered by the material.	
4	Teacher provides links between related topics, based on links made by the material, without giving a copy to the students.	
5	Teacher provides links between related topics based on the material, with evidence of simple complementation.	AA10T, EE9M, HH9M, HH10T
6	Teacher provides links between related topics based on the material, with small modifications introduced.	
7	Teacher provides links between related topics by highlighting the ones made by the material.	
7ab	(ab) Students are supposed to gather the links between related topics covered by the material by themselves.	BB11M, CC8X, CC10M, CC11T, DD7X, DD10L, DD9M DD9T, EE11T, HH7X, HH10L
8	Teacher provides links between related topics taken from the material and presented as suggested in the guide.	

table 3c.7: Meta-categories applied to use of chosen material to promote links between related topics to topics in progression.

Meta-Category	Descriptions of Observed Behaviours Under the Heading Considered	Observed ir
@	No differentiation took place during the lesson. All students working on the same set of activities.	AA8X, AA10M, BB9T, EE7X, EE10L, EE9M, FF7X, FF7L, FF8M, GG7M, GG8T, HH9M, HH10T
1	Teacher proposes differentiated activities without using any written materials for differentiation.	AA9L, BB9L, HH7X, HH9M
2	Teacher proposes differentiated activities modifying, by complementation the material used for differentiation.	AA9L
3	Teacher proposes differentiated activities modifying by selection the material used for differentiation.	
4	Teacher proposes differentiated activities based on the material used for differentiation, without giving students a copy. (b) Teacher proposes differentiated activities by giving the students choice of different activity options based on the material.	AA10M
5	Teacher proposes differentiated activities based on the material used for differentiation, with evidence of slight complementation.	AA10T
6	Teacher proposes differentiated activities based on the material used for differentiation, with evidence of simple selection done by the teacher.	FF8T
7	Teacher proposes differentiated activities from the material used for differentiation.	AA10T, GG7M
7ab	(ab)Students are supposed to gather by themselves the differentiated activities in the material used for differentiation	AA8X, BB10X, BB9L, BB11M, CC8X, CC8L. CC10M, CC11T, DD7X, DD10L, DD9M, DD9T, EE10L, EE11T, GG7M, GG8T, HH7X, HH10L
8	Teacher proposes differentiated activities following the guides' suggestions on the material used for differentiation.	FF8M, GG8T,

table 3c.8: Meta-categories applied to use of the chosen material to promote differentiation.

Meta-Category	Descriptions of Observed Behaviours Under the Heading Considered	Observed ir
@	No remedial situation takes place during the lesson, although 'crisis' situations are observed. All students are working on sets of activities planned by the teacher.	AA10T, BB10X, CC8X
1	Teacher proposes remedial activities without using any written remedial material.	AA8X, AA9L, AA10M, AA10T, BB9L, BB11M, CC8L, CC10M, CC11T, DD7X, DD10L, EE10L, EE9M, FF7L, GG7M, HH10L
2	Teacher proposes remedial activities modifying, by complementation the remedial material.	
3	Teacher proposes remedial activities modifying by selection the remedial material.	
4	Teacher proposes remedial activities based on the remedial material, without giving the students a copy.	
5	Teacher proposes remedial activities based on the remedial material, with evidence of slight complementation.	
6	Teacher proposes remedial activities based on the remedial material, with evidence of simple selection done by the teacher.	
7	Teacher proposes remedial activities from the remedial material.	GG7M
7ab	(ab) The students are supposed to gather by themselves the remedial activities in the remedial material	BB11M, EE10L
8	Teacher proposes remedial activities following the suggestions in the remedial materials' guide(s). (ab) The students are supposed to gather the remedial activities in the remedial material by themselves.	AA10T

table 3c.9: Meta-categories applied to use of the chosen remedial material.

Meta-Category	Descriptions of Observed Behaviours Under the Heading Considered	Observed in
@	Teacher sets no homework.	AA8X, AA10T, BB10X, BB9L, BB11M, CC8X, CC8L, CC10M, DD7X, DD10L, DD9M, DD9T, EE7X, EE10L, EE9M, FF7L, FF8T, GG7M, GG8T, HH7X, HH10L, HH9M
1	Teacher sets homework without using any written material.	AA10M, FF7X, FF8T, GG7M, HH7X, HH10T
2	Teacher sets homework adding new tasks to the chosen homework material, modifying it.	
3	Teacher sets homework modifying by selection the tasks from the chosen homework material.	
4	Teacher sets homework based on the chosen homework material, without giving a copy to the students.	EE7X, EE9M
5	Teacher sets homework proposing slight complementation to the chosen homework material.	
6	Teacher sets homework by making a simple selection on the chosen homework material.	AA9L, AA10M, BB9T, DD9T, HH9M
7	Teacher sets the chosen material as homework.	AA10T, CC11T, EE11T, FF7X, FF8M, HH10L, HH10T
7ab	(ab) Teacher sets as homework: 'keep going' with their class work, without setting either targets or time limit.	BB11M
8	Teacher sets homework as suggested by the guides to the chosen material.	BB10X, BB11M, CC8X, CC8L, CC10M, DD7X, DD10L, DD9M, GG8T

table 3c.10: Meta-categories applied to use of the chosen material as source of homework.

Meta-Category	Descriptions of Observed Behaviours Under the Heading Considered	Observed in
1	Teacher gives feedback without using any written material.	AA8X, AA9L, AA10M, AA10T, CC8L, DD10L, EE7X, EE10L, EE9M, EE11T, FF7X, FF7L, FF8M, FF8T, HH7X, HH10L, HH9M, HH10T
2	Teacher modifies feedback given with the chosen feedback material by providing other explanations and suggestions.	AA9L, AA10M, GG7M, GG8T
3	Teacher modifies the feedback given with the chosen feedback material by selecting some explanations and suggestions.	
4	Teacher gives feedback based on the chosen feedback material, without given the students access to it.	
5	Teacher gives feedback based on the chosen feedback material, adding small explanations and suggestions.	BB9L, BB9T, CC8X, CC10M, CC11T, DD10L, DD9M, DD9T, FF8T
6	Teacher gives feedback based on the chosen feedback material, making simple selections.	
7	Teacher mainly checks students answers using the answer book of the textbook. Very few comments are added.	BB10X, BB11M, BB9T, DD9T,GG8T
7a	(a) Teacher reads the answers from the answer book of the textbook aloud to the whole class.	DD9T
7ab	(ab) Teacher allows students to consult textbook's answer book whenever they feel it necessary, and no comments are added.	DD7X, EE11T, GG8T, HH7X, HH10L, HH10T
8	Teacher gives feedback based on the chosen feedback material and on suggestions given by the guide.	BB9T, DD7X, FF8M,

table 3c.11: Meta-categories applied to use of the chosen material for feedback on exercises.

APPENDIX 4

SUMMARY OF CODIFICATION OF DATA CONCERNING THE FIRST RESEARCH QUESTION

Appendix 4a: Teacher A, School A

4a.1: Analysis of headings in Set I for teacher A's four groups.

Set I - Headings	Meta-Categories	Sources of evidence	Summary of evidence
		Interview	'we don't use textbooks, we base our lessons on the school scheme' 'I will certainly not present the Pythagoras' theorem for my students in year nine.'
Main Source	**8X: 3**	School Scheme	Mainly a list of proposed printed materials, some adapted by the school. A few topics have no suggestions and are left to the teacher to decide. The School has no adopted textbook.
of materials for Classroom Work	**9L: 3** **10M: 4** **10T: 4**	Reports of the Lessons	**8X**: L1: material adapted from 'Journey into Maths' series. No material used for the other lessons. **9L**: L1 and L2: investigation material from the 'Shell Centre', not given to the students. L3: Worksheet from 'What If' series in factors used. In both cases, the teacher decided where to stop. **10M**: investigation material from the 'Shell Centre' for the first three lessons. No written material for the last one. **10T**: Worksheets in Logo from the SMP series for the three lessons observed. Not given to the students in the first one, and complemented by the teacher own material in the third.
		Interview	'The school scheme is set by half -terms. It's up to us to decide what we are going to do first.
Progression of the	**8X: 3** **9L: 2**	Scheme	Into the same attainment target, the school scheme suggests some progression. With **9L**, several decisions are left to the teacher.
Contents	**10M: 4** **10T: 4**	Report of the Lessons	All topics presented listed in the school scheme for the actual half-term. Order decided by the teacher. The only exceptions were the anticipation of one worksheet for **9L** group and the decisions on development in three lessons without using material for group **8X**.
Materials given to the students for the purpose of reference	**8X: 2** **9L: 1** **10M: 3** **10T: 4**	Interview	'the students take the notebook home, yes...'
		Reports of the Lessons	The materials kept by the children are: their notebooks and some (not all) used worksheets. **10M** also kept a sheet containing instructions on what to look for when doing an investigation and how to prepare an organised report for the course work. **10T** were allowed to take the SMP worksheet home as source of examples, but not to keep them.
		Notebooks	Marked by the teacher on regular basis, although he does not complement students' own definitions and explanations. No worksheets observed kept together with notebooks for **9L. 10 T** was the group with the greatest number of worksheets added to their notebooks.

Table 4a.1: School A, Teacher A - analysis of the first set of headings, concerning general decisions on materials taken by the teacher for the school year.

4a.2: Analysis of headings in Sets II and III for AA8X group.

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities.	3ab 1 1 1	7ab 1 1 1	**L1** - students are asked to 'keep going' with the exercises they were doing in previous lessons, using a set of adapted worksheets (strongly based on 'Journey into Maths' - a textbook series) **L2, L3** and **L4** - all the exercises proposed by the teacher. No written material used. **L4** - mental test designed by the teacher given to the students, matched with the other activities developed in Lessons 2, 3 and 4.
Introduce a new topic.	- 1 - -	- 1 - -	**L2** - Teacher introduced the topic by writing its title on the board and asking questions about 'factors'. **L1, L3** and **L4** - No topics were introduced.
Introduce a lesson.	3ab 1 1 1	7ab 1 1 1	**L1**- the teacher asked the students to continue the work from the previous lesson. No other introduction was offered. **L2** - see above - new topic. **L3** and **L4** - the teacher introduced these lessons by proposing own designed activities for the whole group.
Reference During the Lesson	1 1 1 1	5 1 1 1	**L1** - worksheets used have neither explanations nor examples. Teacher gives examples by solving some exercises (or helping the students individually to do so). **L2, L3** and **L4** - students have written in their own words the definitions in their notebooks, following instructions given by the teacher. These definitions were not yet reviewed by the teacher.
Exemplification.	3 1 @ @	5 1 @ @	**L1** - Examples given individually by solving proposed exercises, giving concrete materials (cubes) or recalling previous exercises. **L2**- teacher wrote down examples on the blackboard. **L3** and **L4** - no examples given.
Conclude a topic.	@ - - 1	@ - - 1	**L1** - the topic was not concluded by the teacher or by the material. **L2** and **L3** - no conclusion of topic. **L4** - conclusion made by the teacher, using exercises and poster activity.
Promote links between related topics.	1 1 1 1	2 1 1 1	**L1** -The series of worksheets provided no links. Teacher used students' previous work and support from plastic cubes in order to create links between the different sheets. **L2, L3** and **L4** - Some links were provided by the teacher, who did not use materials to promote them.
Promote differentiation within class.	3ab @ @ @	7ab @ @ @	**L1** - teacher is using a sequenced series of worksheets. At least 30% of the students did not finish the last one. **L2, L3** and **L4** - no differentiation noticed.
Remedial Material.	1 - - -	1 - - -	**L1** - teacher used previous worksheets' exercises and concrete material (cubes) to help students. **L2, L3** and **L4** - teacher followed his plans for the lessons and no student asked for extra support.
Source of Homework.	3(*) @ @ @	8(*) @ @ @	**L1** - worksheet given as homework produced by the school, based on printed material. No connection with classroom work. **L2, L3** and **L4** - no homework given.
Provide Feedback on Exercises.	1 1 1 1	1 1 1 1	**L1** - feedback given by the teacher by using concrete materials (cubes) and answers to the previous exercises. No written materials used. **L2, L3** and **L4** - feedback given by orally discussing students' answers, with the whole group at the same time.

Table 4a.2: AA8X Group - analysis of the second and third set of headings over four lessons.

4a.3: Analysis of headings in Sets II and III for AA9L group.

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities.	4 4 4	4 4 6	**L1** and **L2** - Investigation based on material from 'Shell Centre', not given to the students. Teacher interpreted the exercises proposed by the material to the students. **L3** - Worksheet from 'What if' series: after the students had finished the first exercise (from the sheet, but proposed by the teacher), the material was given to the students and they were asked to do selected exercises.
Introduce a New Topic.	- - 4	- - 4	**L1** - Teacher introduced the lesson by summarising the introduction of the investigation done in the previous lesson using the chosen material. **L2** - No topic was introduced. **L3** - Introduced by the teacher, using no material.
Introduce a Lesson.	4 4 4	4 4 4	**L1** and **L2**- Teacher introduced based on the material (not given to the students). **L3** - Teacher introduced the topic and the first exercise from the worksheet. The material was given to the students later, after they had finished the first exercise proposed by the teacher.
Reference in Classroom.	1 1 1	1 1 1	**ALL** - No such materials given to the students. They had some examples given by the teacher in their notebooks.
Exemplification.	4 4 4	2 2 2	**L1** and **L2**- Examples given by the teacher, some based on the examples from the material and others caring on the proposed task (not taken from the material). **L3** - teacher partially solved the first proposed exercise as exemplification.
Conclude a Topic.	- @ -	- @ -	**L1** and **L3** -no topic concluded during these lessons **L2** - Last lesson on the investigation. No summary or other conclusion offered by the teacher.
Promote Links Between Related Topics.	- - @	- - @	**L1** and **L2**- teacher did not aim to establish links. **L3** - Although the material was selected because the students did not spot 'factors' in the investigation, the teacher did not emphasise links between this topic and the previous investigation.
Promote Differentiation within Class.	4b/1 4b/1 1	2 2 1	**ALL** -Same material used with all students. Teacher promoted differentiation by proposing different tasks based on the common material. In the first two lessons, some of the differentiated tasks proposed by the teacher were suggested in the material.
Remedial Material.	1 1 1	1 1 1	**ALL** - Teacher did not change material, but proposed (own designed) simpler tasks using the same material for those students facing difficulties.
Source of Homework.	4 4 4	6 6 6	**ALL** - The homework given was to work on the investigation report, with target given by the teacher. Based on the material used for class work.
Provide Feedback on Exercises.	1 1 1	2 2 1	**ALL** - No support material used to provide feedback. Teacher provided feedback individually, looking at each student's work. Some of the suggestions offered by the teacher during the first two lessons matched the material.

Table 4a.3: AA9L Group - analysis of the second and third set of headings over three lessons.

4a.4: Analysis of headings in Sets II and III for AA10M group.

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities.	4 4 4 1	4 2 2 1	**L1, L2** and **L3** - Investigation worksheet by 'Shell Centre'. Exercises proposed close to suggested by the material, but no copy given to the students. Further development of the investigation suggested by the teacher. **L4**- Tasks suggested by the teacher, no material used.
Introduce a New Topic.	- - - 1	- - - 1	**L1, L2** and **L3**- No topic was introduced. **L4** - teacher introduces the new topic without using any written materials.
Introduce a Lesson.	4 4 4 1	4 4 4 1	**L1, L2** and **L3**- introduced by the teacher by reminding the students what they have done and offering suggestions of continuity. **L4** - introduced by the teacher by proposing a practical measurement task.
Reference in Classroom.	1 2 2 @	1 8/1 8/1 @	**L1** no reference material given, but students have some examples in their notebooks, based on the material. **L2** and **L3** - adding to the students' previous notes, the teacher gave a school's produced sheet with general guidance on reports of investigations. **L4** - no reference material offered, not even notes or examples in the notebooks.
Exemplification.	4 4 4 @	5 5 5 @	**L1, L2** and **L3** - examples given by the teacher. Some close to the chosen material, others own developed examples. **L4** - no examples given.
Conclude a Topic.	- - @ -	- - @ -	**L1, L2** and **L4** - there were no conclusion of topics. **L3** - The teacher offered no summary nor other kind of conclusion for the investigation.
Promote Links Between Related Topics.	- - - -	- - - -	**ALL** - teacher did not aim to establish links.
Promote Differentiation within Class.	4b 4b 4b @	4b 4b 4b @	**L1, L2** and **L3** - Same common introduction given suggesting an investigation. Afterwards, the teacher offered individual suggestions of continuity and accepted different lines of investigation, in some cases modifying the guide's suggestions. **L4** - No differentiation occurred.
Remedial Material.	1 1 1 -	1 1 1 -	**L1** - No change of materials. Support offered by the teacher close to the guides of the chosen material. **L2** and **L3** - no change of materials. Teacher offered different support for different students, not always close to the guides. **L4** - No change of material, teacher's suggestions directed all students to the same task. No crisis situations occurred.
Source of Homework.	4 4 4/1 4	6 6 6/1 6	**ALL:** The homework given was to work on the investigation report, with target set by the teacher. Based on the material used and related with class work. **L3**- teacher proposed the students collect round objects, to be used in the next lesson.
Provide Feedback on Exercises.	1 1 1 1	2 2 2 1	**L1, L2.** and **L3** - General outline of the guidance offered individually by the teacher matches the chosen material. No material used to give feedback. **L4** - No material used. Feedback given to small groups by the teacher.

Table 4a.4: AA10M Group - analysis of the second and third set of headings over four lessons.

4a.5: Analysis of headings in Sets II and III for AA10T group.

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities.	4 4b 4b/3	4 7 5	**L1** - teacher introduces a series of SMP worksheets with the objective of programming the computer in LOGO to draw regular polygons. The first lesson was dedicated to prepare the programs in classroom to be implemented during the consecutive lessons in the computer room. The examples given and the exercises proposed during this lesson were based on the sheets, but the material was not given to the students. Introduced by recalling the previous lesson. **L2** - At the computer room- teacher offers four different SMP worksheets (one was review of LOGO, another was extension work and the other two could be used as starting point, depending on students' work in the previous lesson) and suggested that two of them could be used as starting point. No other introduction was offered by the teacher. **L3** - The teacher realised that some students were facing problems and produced an extra worksheet (strongly based on the chosen material). The lesson was introduced by presenting this material and all the students received a copy of it. This sheet was useful for the students with difficulties, but was not useful for some students who had finished the proposed task and extensions by the beginning of the third lesson. Teacher A admitted that he had no other activity to give them.
Introduce a new topic.	4 - -	6 - -	
Introduce a lesson.	4 4 3	4 7ab 5	
Reference in Classroom.	1 4 4/3	1 7ab 3	
Exemplification.	4 4ab 4ab	4 7ab 7ab	
Conclude a topic.	- - 3ab	- - 7ab	**L1** and **L2** - no topic was concluded during these lessons. **L3** - the material produced by the teacher summarised the topic, but the teacher did not highlight it.
Promote links between related topics.	4 - -	5 - -	**L1** - teacher emphasised and complemented the link between external angles of regular polygons and rotational angle in LOGO. **L2** and **L3** -teacher did not aim to establish links.
Promote differentiation within class.	@ 4b 4b/3	@ 7 6	**L1** - same task proposed to all students. **L2** and **L3** - students had a choice from a series of sheets. Teacher added own produced material.
Remedial Material.	- @ 4b/3/ @	- @ 8ab/ @	**L1** - Help given by the teacher, but no changes in the material. **L2** - students were offered a choice of materials. No remedial situation. **L3** - a teacher's own produced sheet was added to the choice given during the previous lessons. See description of **L3** above for other comments.
Source of Homework.	4(*) @ @	7(*) @ @	**L1** - homework not related with class work. Taken from short investigation produced by 'Shell Centre'. **L2** and **L3** - no homework given.
Provide Feedback on Exercises.	1 1 1	1 1 1	**ALL** - teacher does not use any material to provide feedback on exercises. Guidance given by the teacher directing the students towards the material's goals.

Table 4a.5: AA10T Group - analysis of the second and third set of headings over three lessons.

Appendix 4b: Teacher B, School B

4b.1: Analysis of headings in Set I for teacher B's four groups.

Set I - Headings	Meta-Categories	Sources of evidence	Summary of evidence
Main Source of materials for Classroom Work	10X: 7 9L: 7 11M: 7 9T: 7	Interview	'as we use the books for year 9 upwards the school scheme only complements it...with investigations and other activities... for years seven and eight the school scheme is complete...'
		School Scheme	School scheme of work presenting general objectives, composition of the groups, and list of materials to be used. For years 7 and 8, the school scheme is 'topic based', suggesting the material for each topic, mainly the 'Journey into Maths' textbook, with occasional complementation using the SMP booklets. For year 9 upwards, only the investigations are written in the school scheme (title and material to be used).
		Reports of the Lessons	All lessons observed based on the SMP series. Teacher used some complementary material, which was essentially based on the SMP series as well.
		Notebooks	Students notebooks contain mainly answers to SMP book exercises, complemented by investigations, usually taken from other materials. Few complementary worksheets observed.
Progression of the Contents	10X: 7 9L: 7 11M: 7 9T: 7	Interview	'the book provides the progression... and actually, it works very well... related topics are integrated... and the 'language' is the same.'
		Scheme	the school scheme does not present comments on progression, which is left to the books adopted in each year.
		Report of the Lessons	SMP used in a 'page by page' way with most groups. Few alterations promoted by the teacher.
Materials given to the students for the purpose of reference	10X: 8 9L: @ 11M: 8 9T: 8	Interview	we do not allow the low attainers to take the book home... our previous experience shows that they do not take good care of the books... even loosing their copies. All other groups take their books home.
		Reports of the Lessons	During the observed lessons the teacher never asked the students to take notes in their notebooks. Students used the textbook as reference.
		Notebooks	Students notebooks have no explanations or examples copied, only solved exercises, corrected on an individual basis by the teacher, usually during the lessons.

Table 4.b.1: School B , TeacherB - analysis of the first set of headings, concerning general decisions on materials taken by the teacher for the school year.

4b.2: Analysis of the headings in Sets II and III for BB10X group.

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities	7ab 7ab 7ab	7ab 7ab 7ab	General Description: This group is divided in two subgroups: the 'yellows' working through the 'higher' SMP track (about 40% of the students), and the 'reds' working through the second of four possible SMP tracks (about 60% of the students). The 'yellows' are using book Y3 and the 'reds' are using book R2. They follow the sequencing of the book, but the teacher
Introduce a new topic.	6(*) 7ab 7ab	5(*) 7ab 7ab	intervenes to keep each sub-group working at the same topic, although with the 'reds' it seems more difficult to achieve. **L1:** Teacher introduced the lesson to the 'yellows' discussing some ideas of proportionality on the board. 'Reds' were asked to 'keep going' with their work. 'Yellows' start working through the proportionality chapter in book Y3. Teacher provided
Introduce a lesson.	6/7ab 6/7ab 7ab	5/7ab 2/7ab 7ab	individual support for the 'reds'. As some students in the 'red' group have finished the 'angles' chapter, starting the proportionality one, teacher B concluded the lesson for both groups discussing some proportional and non-proportional quantities. (note: at least half of the 'reds' were still working on angles, so they could not benefit from this discussion). **L2:**
Reference in Classroom.	7ab 7ab 7ab	7ab 7ab 7ab	Teacher introduced the lesson for the 'yellows' by discussing their results in a test. She reminded them of gradient and ask them to repeat some of the 'unsuccessful test questions'. 'Reds' were asked to keep going, and after the introduction the 'yellows' were also asked to carry on with their
Exemplification.	5/7ab 7ab 7ab	5/7ab 7ab 7ab	work. By the end of the lesson, the 'reds' who had not yet finished the 'angle' chapter were asked to finish it at home. **L3:** Teacher B asked both groups to 'keep going' with their previous work. She also says that this is the last lesson the 'yellows' will be working on proportionality, and all 'reds' should be working on proportionality.
Conclude a topic.	- 5/7ab 7ab/-	- 3/7ab 7ab/-	Conclusions were observed three times: One done by the teacher, commenting on the results of the 'yellows' on the test and two others done by the book, as the teacher asked the students to do the revision section as homework, finishing the chapter first if that was the case.
Promote links between related topics.	- - -	- - -	**All**: Neither the teacher nor the book aim to establish links.
Promote differentiation within class.	7ab 7ab 7ab	7ab 7ab 7ab	Differentiation is promoted in two ways: by dividing the group in two sub-groups using different tracks of the same textbook and by allowing students to work through the chapters at their own pace.
Remedial Material.	@ @ -	@ @ -	A 'crisis' situation was observed during two of the observed lessons, concerning two 'reds' who were not progressing trough the 'angles' chapter. They did not ask for any extra support nor did the teacher notice their difficulties. She commented that they were too slow, and asked them to finish the chapter at home, without checking whether they did afterwards. No remedial material introduced.
Source of Homework.	@ @/8 8	@ @/8 8	All homework set during the observed lessons was taken from the book. Students were asked to finish the chapter if they had not already and to work through the revision section correspondent to the finished chapter.
Provide Feedback on Exercises.	7 7 7	7 7 7	Teacher provided feedback during the lessons by checking notebooks individually, with the aid of the answer book. The teacher corrected all exercises done by students since her last mark, writing the correct answer beside the wrong ones. She also asked students to review what they had done wrong.

note: in the table above, if a double situation is described, the first one refers to the 'yellows'.

Table 4b.2: BB10X Group - analysis of the second and third set of headings over three lessons.

4b.3: Analysis of headings in Sets II and III for BB9L group.

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities	7ab 7ab/1 7ab/1	7ab 7ab/1 7ab/1	**General Description:** This group has 12 students, working through SMP green series. At the moment, all of them are using book G2, but the teacher says they change at their own pace. Students work individually through the book. Teacher complements their book activities using some computer activities based on number skills or by working with them, one at a time, in activities related with money, different from what they were doing using the book. **L1**: Teacher proposes that one student do some work at the computer (this girl is the one who has gone further in the book) and asks the others to 'keep going' with their work. Teacher uses the lesson time to check what each one has been doing. **L2:** Teacher introduces the lesson as before and keeps the same girl at the computer to finish her task. Teacher starts an individualised task with one student at a time using coins and notes. **L3:** The girl using the computer is back to her book activities, all the others 'keep going'. One of them finishes a section and is given a SMP test. Teacher continues to call one student at a time for the money activity.
Introduce a new topic.	7ab 7ab/1 7ab/1	7ab 7ab/1 7ab/1	
Introduce a lesson.	7ab 7ab 7ab	7ab 7ab 7ab	
Reference in Classroom.	7ab 7ab 7ab	7ab 7ab 7ab	
Exemplification.	7ab 7ab/1 7ab/1	7ab 7ab/1 7ab/1	
Conclude a topic.	7ab 7ab 7ab	7ab 7ab 7ab	No conclusions provided by the teacher. Each student finish the sections of the books in their own time.
Promote links between related topics.	- - -	- - -	**All**: Neither the teacher nor the book aims to establish links.
Promote differentiation within class.	7ab 7ab/1 7ab/1	7ab 7ab/1 7ab/1	Differentiation is promoted by allowing students to work through the book at their own pace and by providing extra activities for those who are 'doing well'. The activities promoted by the teacher are also differentiated. All students were asked the same initial question but, depending on the answers given, the final point was different.
Remedial Material.	1 1 1	1 1 1	No remedial material introduced. Teacher solved 'crisis' situations observed by giving explanations to the students and helping them to read the explanations in the book.
Source of Homework.	@ @ @	@ @ @	No homework given
Provide Feedback on Exercises.	7 7/1 7/1	5 5/1 5/1	Teacher provided feedback individually, with the aid of the answer book. She corrected all exercises done since her previous marking, and commented on the results with the students, giving explanations. Feedback on the money activity was given without using material.

Table 4b.3: BB9L Group - analysis of the second and third set of headings over three lessons.

4b.4: Analysis of headings in Sets II and III for BB11M group.

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities	7ab 7ab 7ab	7ab 7ab 7ab	General Description: About 30% of the students in this group are 'reds' and the other 70% are 'blues'. This is a small class (15 students) and the 'reds' work in a individualised way. Although the 'blues' also work in a individualised scheme, the teacher keep them working within a certain range of the book. **L1:** Teacher asked the reds to 'keep going' and gave an example of possible ways to calculate 'value for money' to the 'blues'. After the example she asked the students to 'keep going' with their work. The introduction did not match the class activities because half of the 'blues' had already finished the 'value for money' chapter and moved to the 'probability' one, and the majority of the students still working at this chapter had already finished the commented section. It was also of little use to the two remaining blues, as they were far behind in the chapter. **L2:** Teacher asked the students to 'keep going' and used the time of the lesson to mark their previous work and offer suggestions. **L3:** Teacher asked the students to 'keep going' and used thelesson time to mark their work and offer suggestions on questions the students had not answered correctly.
Introduce a new topic.	7ab 7ab 7ab	7ab 7ab 7ab	
Introduce a lesson.	7ab 7ab/ 7(*) 7ab	7ab 7ab/ 7(*) 7ab	
Reference in Classroom.	7ab 7ab 7ab	7ab 7ab 7ab	
Exemplification.	7ab 7ab 7ab	7ab 7ab 7ab	
Conclude a topic.	7ab 7ab 7ab	7ab 7ab 7ab	The teacher offered no conclusions to the topics. Students are supposed to read the existing ones in the book, although the teacher did not ask them to do so.
Promote links between related topics.	7ab 7ab 7ab	7ab 7ab 7ab	No links promoted by the teacher. Students are supposed to read the existing ones in the book, although the teacher did not tell them to do so.
Promote differentiation within class.	7ab 7ab 7ab	7ab 7ab 7ab	Differentiation is promoted in two ways: by dividing the group in two sub-groups using different tracks of the same textbook and by allowing students to work through the chapters at their own pace.
Remedial Material.	- 4 7	- 7ab 1	Two crisis situations observed during the lessons: for one 'red' student the teacher proposed a review of fractions, changing the material, returning to the previous task after finishing it. To the pair of 'blues' facing difficulties in the 'value for money' chapter, teacher B offered explanations, but did not change the material.
Source of Homework.	@ 7ab 7ab/8	@ 7ab 7ab/8	Homework given in two of the observed lessons: to the 'reds': keep going' with no target or time limit set by the teacher; for the 'blues' the teacher proposed one 'keep going' once (as above) and revision of the 'value for money' chapter at the other time.
Provide Feedback on Exercises.	7 7 7	7 7 7	Teacher provided feedback during lessons by checking notebooks individually, with the aid of the answer book. The teacher corrected all exercises done by the student since her previous mark, writing correct answers beside the wrong ones. She also asked students to review what they have done wrong.

note: in the table above, if a double situation is described, the first one refers to the 'reds'.

Table 4b.4: BB11M Group - analysis of the second and third set of headings over three lessons.

4b.5: Analysis of headings in Sets II and III for BB9T group.

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities	6 3 6	7 8 7	**L1:** Teacher introduced the lesson by summarising the results concluded during the previous lesson (how to represent 'large numbers' in standard index form - S.I.F.). Teacher introduced the representation in S.I.F. for numbers close to zero. Teacher also promoted a discussion on two different ways of representing the same number and asked the students to work from an assigned section of the SMP R1 (red track) book. For those who finished the task, she assigned work in the S.I.F. chapter in the adopted textbook- SMP Y1. (yellow track). **L2:** Teacher introduced the lesson by discussing the proposed homework. After that she introduced a worksheet adapted from previous GCSE exams to the students on S.I.F. Teacher concluded the lesson by correcting the proposed exercises. **L3:** Teacher introduced the lesson by discussing the proposed homework. After that, she introduced a new topic: 'decimal search' using one example. After the introduction teacher B asked the students to solve assigned exercises from R1, and for those who finished she proposed exercises in Y1. **Comment:** The teacher said that she had tried before to start these activities straight from book Y1 and the students had experienced difficulties. For two years now she has introduced activities using the red track and changing to the yellow afterwards.
Introduce a new topic.	6 - 5	5 - 2	
Introduce a lesson.	6 3 5	5 8 2	
Reference in Classroom.	6 6 6	7ab 7ab 7ab	
Exemplification.	6 8 3	5 8 2	
Conclude a topic.	6 - 6	5 - 2	Teacher provided conclusions for each segment of the topic, complementing the ones presented in the book.
Promote links between related topics.	5 5 5	2 2 2	Teacher promoted links between different representations for the same number, between power of numbers and standard index form, and between representations of larger numbers and tiny numbers in S.I.F., complementing the conclusions presented by the book. She also discussed how to multiply powers on the same base.
Promote differentiation within class.	@ @ @	@ @ @	No differentiation was noticed. All students working through the same task, with the same aims. Teacher used the homework assigned to keep all the students on the same task.
Remedial Material.	- - -	- - -	No crisis situation observed. All students were able to work through the proposed tasks, and no students needed extension work.
Source of Homework.	7 3 7	6 6 6	Teacher assigned homework from the textbook in the first lesson, with aims established by the teacher. She used the worksheet to assign homework during the second lesson, with aims established, and used the textbook to assign homework in the third lesson, with aims established.
Provide Feedback on Exercises.	6/7 8/7 6/7	6/7 8/7 6/7	Feedback promoted by the teacher in two different ways: during the lesson the teacher marked the students notebook starting from the last marked exercise; and by the end of the lesson, the teacher asked the students their answers for the proposed exercises, discussing them.

Table 4b.5: BB9T Group - analysis of the second and third set of headings over three lessons.

Appendix 4c: Teacher C, School C

4c.1: Analysis of headings in Set I for teacher C's four groups.

Set I - Headings	Meta-Categories	Sources of evidence	Summary of the evidence
Main Source of materials for Classroom Work	**8X: 7**	Interview	'SMP is used with all groups... except for 'low atteiners' in year eight... these students can use neither the booklets not the green series...we give them the basic skills to use the green track of SMP after that... and SMP has no material to do this job'
	8L: 4	School Scheme	The school has no written scheme, but the interview with the teacher left it clear that they have a scheme for the low attainers in year eight, based on materials other than SMP, whilst using SMP as a source for all other groups.
	10M: 7 **11M: 6**	Reports of the Lessons	The lesson's reports show that SMP, complemented with homework material developed by the school has been used in all lessons with groups **8X** and **10M**, other series of books, designed for low attainers and/or lower grades has been used with group **8L**, and revision sheets prepared by the school are being used with group **11T**, as its students have already finished the SMP track they used during their secondary education.
Progression of the Contents	**8X: 7**	Interview	'we do not have a written scheme because the book is used page by page if you want... well, there are few exceptions but most of the time, it is the book that provide the progression of the contents...'
	8L: 4	Scheme	The progression of the contents is determined by the school for each group, and is determined by the textbook series in all other cases. There are no written schemes.
	10M: 7 **11T: 6**	Report of the Lessons	**8L**: Teacher is using two textbook series designed for low attainers with this group: 'Clearway Maths' for number work and 'A World of Mathematics' for shape and space work. **8X**: Teacher is using SMP booklets with these students, in a individualised way, each student progressing through the SMP booklets at their own pace. **10M:** Teacher is using the SMP 'red track' with these students. At the time of the observation they are all working through book R2 in an individualised way, but the teacher has scheduled a test and is proposing revision work. **11T:** Teacher had already finished the SMP books with these students (yellow track for about 60% of the students and red track for the remaining 40%). At the time of observation they are working through revision sheets produced by the school, based on the SMP series and on previous GCSE examinations.
Materials given to the students for the purpose of reference	**8X: 2**	Interview	'all students are allowed to keep the book... except for the low attainers, who take only the notebooks home'. Students in year eight do not take the booklets home as well'
	8L: 1 **10M: 8**	Reports of the Lessons	Students from both groups in year eight do not take written material being used home, they take their notebooks and the homework sheets. Students in year ten keep the book they are using all the time. Students in year eleven had already finished with all the books, but are allowed to take home any volume they feel necessary for their study programme.
	11T: 8	Notebooks	Notebooks from all groups contain mainly answers to proposed exercises taken from the selected material. For all groups, except **8L**, few worksheets are annexed to their notebooks.

Table 4c.1: School C, Teacher C - analysis of the first set of headings, concerning general decisions on materials taken by the teacher for the school year.

4c.2: Analysis of headings in Sets II and III for CC8X group.

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities	7ab 7ab 7ab	7ab 7ab 7ab	General Description: This group has been working in a completely individualised way since the beginning of the year. By the time of the observations, except for some students in the group that work in pairs, each one is using a different booklet, working at his/her own pace. Teacher C says that for some of the students (four of them) she has regularly given the SMP extension booklets, while the others work through the basic booklets only. She also says that by the end of the school year she expects that two or three more students will have the time to work through some extension material. During the observed lessons, only one student was working on an extension booklet. Teacher C started the lessons by giving the booklets the students are using back to them, and asking them to carry on with their work. During the lessons, she calls one student at a time to her table, and mark his/her work using the aid of the answer book. Usually incorrect exercises are re-done by the teacher in the student's notebook, together with oral explanations. Students facing difficulties also call at her desk If a student finish a booklet, he/she is given a test. Depending on his/her results, the student is either asked to do a complementary sheet on the same topic or moved on to the next booklet. The students are kept on task all the time, except for those who are queuing to speak to the teacher.
Introduce a new topic.	7ab 7ab 7ab	7ab 7ab 7ab	
Introduce a lesson.	7ab 7ab 7ab	7ab 7ab 7ab	
Reference in Classroom.	7ab 7ab 7ab	7ab 7ab 7ab	
Exemplification.	7ab 7ab 7ab	7ab 7ab 7ab	
Conclude a topic.	7ab 7ab 7ab	7ab 7ab 7ab	No conclusions were offered by the teacher. Students are supposed to read them from the book, without being asked by the teacher to do so.
Promote links between related topics.	7ab 7ab 7ab	7ab 7ab 7ab	No links between topics were offered by the teacher. Students are supposed to read the links established by the book, without being asked by the teacher to do so.
Promote differentiation within class.	7ab 7ab 7ab	7ab 7ab 7ab	Differentiation was promoted by allowing students to work through the booklets at their own pace. Differentiation is also promoted by offering the extension material proposed by the adopted textbook for those doing well in the 'core' booklets
Remedial Material.	@ @ @	@ @ @	During the observed lessons, it was observed that two students were not progressing in their booklets. As the teacher did not call these students to her desk during these lessons, and they did not ask for help, they had no solution for the 'crisis' situation.
Source of Homework.	@ 5* @	@ 8* @	Teacher set homework once a week using the worksheets prepared by the school, and based on the contents of the SMP booklets. As the same homework sheet was given to all students, and they were working through different topics, it can be said that homework does not match class activities.
Provide Feedback on Exercises.	6 6 6	5 5 5	Teacher provided feedback by marking students notebooks during the lesson. She uses the answer book as an aid to do so, but complements it, by providing the complete solution of selected exercises to students.

Table 4c.2: CC8X Group - analysis of the second and third set of headings over three lessons.

4c.3: Analysis of headings in Sets II and III for CC8L group.

Set II -Headings: 'Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities	4 4/1 4	7ab 7ab/1 7ab	General Description: The sixteen students in this group also work in a individualised way, the teacher not following textbooks. In fact, she uses three volumes of two textbooks simultaneously, all of them considered remedial textbooks. During the observed lessons, there was an opportunity to watching two of these books being used. Teacher C says that she keeps changing the textbook because these students do not respond well to do the same kind of work every lesson. **L1:** Teacher introduced the lesson by telling students that they were going back to number work. For each student she gave a different volume of the adopted material, depending on what they had been doing. During the lesson, teacher C behaved very much the same as observed in the previous group described. She marked exercises, calling one student at a time, unless there were students needing help. **L2:** Teacher introduces the lesson by giving the students the results of a test on the names of 2D shapes. She asked all the names again, commenting on them and asking students to write the correct answer in their test, so that they could refer to it later. After the introduction, the teacher asked the students to carry on with the work they were doing on 'numbers'. At the end of the lesson, teacher C gave games to the students involving number skills, giving 'one merit' to the winner of each round. **L3:** Teacher introduces the lesson by asking students to give her their homework and by giving them another worksheet for homework. After that, she said they could go back to some work on shapes, and gave them another textbook series to be used. Again, the students were working on an individualised basis, although this time they were all at the same section of the book. She had an auxiliary teacher for this lesson. The teacher went on marking students work and the auxiliary teacher helped those facing difficulties.
Introduce a new topic.	4 4 4	7ab 7ab 7ab	
Introduce a lesson.	4 4(*) 4	7ab 2(*) 7ab	
Reference in Classroom.	4 4 4	7ab 7ab 7ab	
Exemplification.	4 4 4	7ab 7ab 7ab	
Conclude a topic.	- @ -	- @ -	No conclusion of topics was given by the teacher. The students changed books, without any conclusion to what they have been doing.
Promote links between related topics.	- - -	- - -	Neither the teacher nor the adopted materials aim to establish links.
Promote differentiation within class.	4ab 4ab 4ab	7ab 7ab 7ab	Differentiation was promoted by using different volumes of the adopted number textbook and by allowing students to work at their own pace in all situations.
Remedial Material.	1 1 1	1 1 1	No remedial material was introduced. The teacher (and the auxiliary teacher) helped the students facing difficulties without changing the material. 'Crisis' situations were observed in two lessons (**L1** and **L3**).
Source of Homework.	@ @ 5(*)	@ @ 8(*)	Homework assigned weekly to the students, using adapted material developed by the school. Teacher C says that in the case of this group, homework is based on the textbooks used with the group It does not necessarily match class activities.
Provide Feedback on Exercises.	1 1 1	1 1 1	The teacher provided feedback during all observed lessons, by marking students' notebooks one at a time. She did not use answer books with this group. During the second lesson the teacher also provided feedback for the whole group, by commenting on their test results.

Table 4c.3: CC8L Group - analysis of the second and third set of headings over three lessons.

4c.4: Analysis of headings in Sets II and III for CC10M group.

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities	7ab 7ab 7ab	7ab 7ab 7ab	General Description: this is the second out of five groups in year ten. The students are working through the red track of the SMP textbook, and at the moment they are using book R2. Although these students work in an individualised way, they are kept together in the book, as the teacher regularly give them tests covering the content up to a certain point in the book. The students are told the day of the test and the chapters covered about one week before. Teacher C say that she expects students who are late in their work to compensate by doing extra work at home. During lessons, students are asked to keep going with their work and teacher C marks their previous exercises in a way similar to that described for the two groups above. She also helped students, always by solving with them the exercises they had problems with, or by giving a very similar example. Teacher C also gives feedback by asking students if they need any answers, and reading these answers out loud to the group.
Introduce a new topic.	7ab 7ab 7ab	7ab 7ab 7ab	
Introduce a lesson.	7ab 7ab 7ab	7ab 7ab 7ab	
Reference in Classroom.	7ab 7ab 7ab	7ab 7ab 7ab	
Exemplification.	7ab 7ab 7ab	7ab 7ab 7ab	
Conclude a topic.	7ab 7ab 7ab	7ab 7ab 7ab	No conclusion highlighted by the teacher. Students are supposed to read the conclusion offered by the book, without being asked to do so by the teacher.
Promote links between related topics.	7ab 7ab 7ab	7ab 7ab 7ab	No links between related topics highlighted by the teacher. Students are supposed to read the links established by the book, without being asked by the teacher to do so.
Promote differentiation within class.	7ab 7ab 7ab	7ab 7ab 7ab	Differentiation was promoted by allowing students to work through the textbook at their own pace. The teacher compensates for this differentiation by asking the students who are lagging to do extra work at home.
Remedial Material.	- - -	- - -	No 'crisis' situations were observed in this class. Students who faced difficulties asked the teacher for help and were able toprogress with their work.
Source of Homework.	@ 5(*) 7	@ 8(*) 7	Homework was assigned during the second lesson observed , using worksheets produced by the school based on the textbook. Homework does not necessarily match class work. In the third lesson observed, the teacher also assigned revision of the contents for the following week's test as homework.
Provide Feedback on Exercises.	6 6 6	5 5 5	Teacher provided feedback by calling students during the lesson and marking their work, using the answer book as an aid. She also used this opportunity to do corrections and to comment on student's work. Twice it was observed that students asked for answers to a section of the book, and the teacher read these answers aloud from the answer book.

Table 4c.4: CC10M Group - analysis of the second and third set of headings over three lessons.

4.c.5: Analysis of headings in Sets II and III for CC11T group.

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities	5ab 5ab 5ab	7ab 7ab 7ab	General Description: This group had already finished all volumes of the SMP track they were using (about 60% were using the yellow track and the remaining 40% were using the red). At the time of observation, students were working through revision sheets organised by the school based not only on the SMP series but also on previous GCSE exams. Teacher C says that more of these sheets are being developed, because there was not enough material for the brighter students in the group. Students work through the worksheets in a individualised way, each one at his/her own pace. The teacher provides support by correcting the students previous exercises and by answering students questions when facing difficulties. Teacher C also uses activities 'under test conditions' to prepare students for their exams. Two different tests are given to the students and commented on afterwards. Students do not have specific homework set, but the teacher recommends that they take the worksheet they are working on home and carry on with their work over a certain period of time. During the observed lessons there was an opportunity to observe one test being returned to students. Teacher C spent almost all lesson calling one student at a time and commenting on his/her mistakes, offering not only the correct answer but also some suggestion of revision of content each student should do. All volumes of both tracks of the SMP series are available in the classroom for reference and students can also take them home.
Introduce a new topic.	5ab 5ab 5ab	7ab 7ab 7ab	
Introduce a lesson.	5ab 5ab 5ab	7ab 7ab 7ab	
Reference in Classroom.	7ab 7ab 7ab	7ab 7ab 7ab	
Exemplification.	7ab 7ab 7ab	7ab 7ab 7ab	
Conclude a topic.	7ab 7ab 7ab	7ab 7ab 7ab	No conclusion highlighted by the teacher. Students are supposed to read the conclusion offered by the book if they want to revise the topic, without being asked to do so by the teacher.
Promote links between related topics.	7ab 7ab 7ab	7ab 7ab 7ab	No links between related topics highlighted by the teacher. Students are supposed to read the links established by the book if they want to revise the topic, without being asked by the teacher to do so.
Promote differentiation within class.	5ab 5ab 5ab	7ab 7ab 7ab	Differentiation was promoted by allowing students to work at their own pace through the worksheet. Differentiation was also observed on the series of worksheets. Two different ones were being used by the students.
Remedial Material.	- - -	- - -	No 'crisis' situations were observed in this class. Students who faced difficulties asked the teacher for help and were progress with their task.
Source of Homework.	5/7 5/7 5/7	7 7 7	Students are assigned the same worksheets they work on during the lesson as homework, with minimum time set by the teacher. These students were assigned homework in all observed lessons.
Provide Feedback on Exercises.	5 5 5	5 5 5	Teacher provided feedback by marking and commenting with each student on previous work. The teacher uses an answer book produced by the school and adds personal suggestions on what they should revise.

Table 4c.5: CC11T Group - analysis of the second and third set of headings over three lessons.

Appendix 4d: Teacher D, School D

4d.1: Analysis of headings in Set I for Teacher D's four groups.

Set I - Headings	Meta-Categories	Sources of evidence	Summary of evidence
Main Source of materials for Classroom Work	**7X: 7** **10L: 7** **9M: 7** **9T: 7**	Interview	'the school does not have a written scheme... we based our teaching on the SMP series'
		School Scheme	There is no school scheme of work. The textbook series is used as a basis, and is followed chapter by chapter, with some complementations, mainly for investigations.
		Reports of the Lessons	All lessons observed with teacher D were based on SMP books or booklets, except for the first two lessons with group **7X**, which were based on an own produced booklet on ratio. Teacher D says it was adapted from several printed materials.
		Notebooks	Students notebooks mainly contain answers to questions in SMP books or booklets. The investigations in year seven are also developed in the notebook, but for years nine and ten they are developed as projects, and not registered in the notebooks.
Progression of the Contents	**7X: 7** **10L: 7** **9M: 7** **9T: 7**	Interview	'we follow the progression suggested by the book... sometimes we skip a chapter... or complement it... but usually we follow the book'
		Scheme	There is no scheme of work. The book is responsible for the progression.
		Report of the Lessons	For groups in year nine and ten, the students finish one chapter and start the next without even asking the teacher. Students in year seven work throughout the given booklet.
Materials given to the students for the purpose of reference	**7X: 2** **10L: 1** **9M: 8** **9T: 8**	Interview	The students in year nine upwards keep the book, 'except for the bottom group... because they do not take care of them... In years seven and eight the students occasionally take the booklet home... but not always... they do not keep a copy.'
		Reports of the Lessons	Students in both groups observed in year nine kept their books and notebooks all the time, except when the teacher asked for notebooks to do corrections. Students in year seven kept their notebooks and the copy of the material produced by the teacher. Students in year ten only kept their notebooks.
		Notebooks	The notebooks mainly contained answers to exercises. No explanations or developed examples were found in the notebooks.

Table 4d.1: School D, Teacher D - analysis of the first set of headings, concerning general decisions on materials taken by the teacher for the school year.

4d.2: Analysis of headings in Sets II and III for DD7X group.

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities	3 3ab/7ab 3ab/7ab	8 8/7ab 8/7ab	**L1:** Teacher D introduced the lesson by saying that they were going to stop the individualised work they had been doing using the booklets to work together. She introduced an own produced booklet on ratio and discussed the concept using simple examples in ratio 2:1 and 3:1. **L2:** Teacher D introduced the lesson by commenting on homework. Merits were given for those who did well and she corrected some of the exercises on the board. After that, she gave a new homework sheet, and asked students to carry on with their work from the own produced booklets. During the lessons some students finished the booklets and the teacher asked them to revert to the individualised work they were doing using the SMP booklets. **L3:** Teacher D asked the students to carry on with their work. Students still working on the ratio booklet were told by the teacher that this was the last lesson in which to finish it. The other students are working through different booklets, at their own pace.
Introduce a new topic.	3 - 7ab	8 - 7ab	
Introduce a lesson.	3 3ab 7ab	8 8 7ab	
Reference in Classroom.	3 3/7ab 7ab	8 8/7ab 7ab	
Exemplification.	3 3ab 7ab	8 7ab 7ab	
Conclude a topic.	- 7ab 7ab	- 7ab 7ab	**L2** and **L3:** The teacher corrected the exercises of those who finished the booklets and offered no other conclusion.
Promote links between related topics.	- 7ab 7ab	- 7ab 7ab	**All:** The teacher did not aim to establish links. If a student moved to a booklet that established links with previous knowledge during the second or third lesson, it was not highlighted by the teacher.
Promote differentiation within class.	@ @/7ab 7ab	@ @/7ab 7ab	**L1:** All students were supposed to work through the same material, with the same final point. No differentiation noticed. **L2** and **L3:** Those students who finished the work proposed for the whole group went back to their individualised work through the SMP booklets, each using a different booklet and working at his/her own pace.
Remedial Material.	1 1 1	1 1 1	**All:** Some students were facing difficulties in solving the exercises proposed. The teacher offered suggestions and guidance, but did not change materials.
Source of Homework.	@ 5(*) @	@ 8(*) @	**L1** and **L3:** No homework assigned by the teacher. **L2:** Teacher assigned homework from a worksheet produced by the department. These worksheets follow a sequence and do not necessarily match the proposed class activities.
Provide Feedback on Exercises.	3 3ab/7ab 3ab/7ab	8 7ab 7ab	**L1:** Teacher provided feedback in two different ways: by marking students notebooks while they were working and by asking them their answers to the first two sections of the written material used at the end of the lesson, and commenting on these answers. **L2** and **L3:** teacher provided feedback in two ways: by marking students notebooks while they were working and by allowing those students who finished a booklet to access the answer book on their own.

Table 4d.2: DD7X Group - analysis of the second and third set of headings over three lessons.

4d.3: Analysis of headings in Sets II and III for DD10L group.

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities	6ab 6ab 6ab	7ab 7ab 7ab	**General Description:** This is a small group of 14 students. Teacher D had an auxiliary teacher helping during all observed lessons. Ten of these students are working through the SMP green track, and at the time of the observed lessons, were all using book G2, although each in a different section. The other four students were working through another series of books: Clearway Maths, designed to give support in 'numbers'. The auxiliary teacher helped these four students. **L1:** Teacher gave the students their notebooks, with the corresponding book and asked them to carry on with their work. During the lessons, she marked the students work, offering support for those facing difficulties. **L2:** The teacher started the lesson by assigning a homework sheet and commenting on the results of the previous homework. After that she asked the students to 'keep going' with their work. **L3:** Teacher D started the lesson saying that after that lesson the students that were doing number work would be doing shape work next lesson, so they should use this lesson well, and finish the section they have been working. She also gave an SMP test to one student who had finished the book G2 in the previous lesson.
Introduce a new topic.	6ab 6ab 6ab	7ab 7ab 7ab	
Introduce a lesson.	6ab 6ab 6ab	7ab 7ab 7ab	
Reference in Classroom.	6ab 6ab 6ab	7ab 7ab 7ab	
Exemplification.	6ab 6ab 6ab	7ab 7ab 7ab	
Conclude a topic.	6ab 6ab 6ab	7ab 7ab 7ab	**All:** The teacher did not highlight any conclusion taken from the book. The students were supposed to read them by themselves, in their own time.
Promote links between related topics.	6ab 6ab 6ab	7ab 7ab 7ab	**All:** The teacher did not highlight any links promoted by the book. The students are supposed to read them by themselves.
Promote differentiation within class.	6ab 6ab 6ab	7ab 7ab 7ba	**All:** Teacher promoted differentiation in two ways: By giving individualised material to those students having difficulties in following the adopted textbook, and by allowing each student to work at his/her own pace through the given material.
Remedial Material.	1 1 1	1 1 1	**All:** Teacher did not change the material for any student. She offered individualised help and explanations instead.
Source of Homework.	@ 5(*) @	@ 8(*) @	**L1** and **L3:** No homework assigned. **L2:** Teacher assigned a school produced worksheet as homework, the same for all students, which did not always match their class activities.
Provide Feedback on Exercises.	7 7/1 7	5 5/1 5	**All:** Teacher provided feed back by marking students work, using the answer book and complementing it with her own comments and suggestions. **L2:** The teacher corrected homework on the board, commenting on the students' answers.

Table 4d.3: DD10L Group - analysis of the second and third set of headings over three lessons.

4d.4: Analysis of headings in Sets II and III for DD9M group.

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities	7ab 7ab 7ab	7ab 7ab 7ab	General Description: This is the second out of five groups in year nine, was using the SMP R1 book at the time of the observed lessons. Teacher D says there was some movement of students at the end of the Easter term, and some students in this group had previously been in the 'top' group. This group also works in an individualised way, carrying on with their work in the book at their own pace. There was no opportunity to observe class teaching with this group, although there was an opportunity to observe the way teacher D sets the minimum to be done in a certain period of time: she tells the students the day they are supposed to do a test on the contents of the book, and what chapters they are supposed to have finished by the day of the test. **All:** the teacher asked the students to carry on with their work. While they were working, she them called one at a time and corrected their previous work, commenting on it. She interrupted this activity every time a student asked for help.
Introduce a new topic.	7ab 7ab 7ab	7ab 7ab 7ab	
Introduce a lesson.	7ab 7ab 7ab	7ab 7ab 7ab	
Reference in Classroom.	7ab 7ab 7ab	7ab 7ab 7ab	
Exemplification.	7ab 7ab 7ab	7ab 7ab 7ab	
Conclude a topic.	7ab 7ab 7ab	7ab 7ab 7ab	**All:** The teacher did not highlight the conclusions offered by the book. The students are supposed to read them by themselves, in their own time.
Promote links between related topics.	7ab 7ab 7ab	7ab 7ab 7ab	**All:** The teacher did not highlight any links between topics established by the book. The students were supposed to read them by themselves, in their own time.
Promote differentiation within class.	7ab 7ab 7ab	7ab 7ab 7ab	**All:** The teacher promoted differentiation by allowing each student to work through the book at his/her own pace. No differential material was introduced.
Remedial Material.	- - -	- - -	**All:** No crisis situations were observed. All the students were able to work through the book, with eventual help offered by the teacher.
Source of Homework.	@ @ 5	@ @ 8	**L1** and **L2:** No homework assigned. **L3:** Homework assigned was a school developed worksheet, revising six initial chapters from the book, which matched the revision students were asked to do for a test during the following week.
Provide Feedback on Exercises.	7 7 7	5 5 5	**All:** The teacher promoted feedback using the answer book and marking the students exercises herself. She also added some personal comments and offered help with questions the students did not answer correctly.

Table 4d.4: DD9M Group - analysis of the second and third set of headings over three lessons.

4d.5: Analysis of headings in Sets II and III for DD9T group.

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities	7ab 7ab 7ab	7ab 7ab 7ab	General Description: This is the 'top' group in year nine, with 32 students. This is the only group that uses the yellow track of SMP and the students were working through book Y2, except for two students, working as a pair, who were working through book Y1, and were asked to keep going during all lessons. In contrast to the other groups, all students are working through the same chapter of the book. The teacher keeps them together using homework to compensate for small differences in personal pace. **L1:** Teacher D asked students to keep going with their work in the probability chapter. She said that she expected all of them to finish section B by the end of the lesson. During the lesson,the teacher used the answer book to read answers to previous sections to the whole group. Homework was assigned at the end of the lesson. **L2:** Teacher D started the lesson by asking about homework. There were several students missing and several had not finished the chapter at home. Teacher D asked them to finish it during the lesson and asked those who had done homework to start the next charter, without introducing it. **L3:** Teacher started the lesson by giving the answers to the final sections of the probability chapter. She asked all the students to work through the following chapter (graphs) during this lesson. During the lesson, the teacher stopped the students to comment on answers to the first section exercises, using the board to complement the answer book.
Introduce a new topic.	- 7ab 7ab	- 7ab 7ab	
Introduce a lesson.	7ab 7ab 7ab	7ab 7ab 7ab	
Reference in Classroom.	7ab 7ab 7ab	7ab 7ab 7ab	
Exemplification.	7ab 7ab 7ab	7ab 7ab 7ab	
Conclude a topic.	7ab 7ab 7ab	7ab 7ab 7ab	**All:** The teacher did not highlight conclusions presented by the textbook. The students are supposed to read them by themselves, in their own time.
Promote links between related topics.	7ab 7ab 7ab	7ab 7ab 7ab	**All:** The teacher did not highlight the links established by the textbook. The students are supposed to read them by themselves, in their own time.
Promote differentiation within class.	7ab 7ab 7ab	7ab 7ab 7ab	There are two students working at a different pace to the others in the group. For the majority of the group no differentiation was noticed, the students were assigned the same task.
Remedial Material.	- - -	- - -	**All:** No 'crisis' situations were observed. All students were able to work through the proposed task, with eventual help provided by the teacher.
Source of Homework.	7 7/@ @	6 6/@ @	**L1:** The teacher assigned completion of the book's chapter as homework for all students, except the 'differentiated' two. These students had 'half hour of maths work' assigned as homework. **L2:** The teacher assigned the same homework to those who had not completed it, and none for the others. **L3:** No homework assigned.
Provide Feedback on Exercises.	7/7a 7/7a 7/6	7/7a 7/7a 7/5	**All:** The teacher provided feedback in two different ways; by marking students notebooks during the lesson and by giving answers to the whole group at the same time. She used the answer book for both activities. During **L3** the second method of providing feedback was complemented by the teacher, by adding her comments and discussing answers on the board.

Table 4d.5: DD9T Group - analysis of the second and third set of headings over three lessons.

Appendix 4e: Teacher E, School E

4e.1: Analysis of headings in Set I for Teacher E's four groups.

Set I - Headings	Meta-Categories	Sources of evidence	Summary of evidence
Main Source of materials for Classroom Work	**7X: 4**	Interview	'it depends on the group... for the 'top' group we use a textbook, and for the other groups we use a series of worksheets, mainly from printed sources...'
	10L: 4	School Scheme	The school scheme of work does not determine which material should be used, except for year seven, to which a list of investigations to be developed during the year has been assigned.
	9M: 4 **11T: 7**	Reports of the Lessons	Group **11T** worked through the textbook during all observed lessons. Group **10L** worked through a resource book during the first lesson and through worksheets during remaining lessons. Group **9M** worked from worksheets during the observed lessons, and Group **7X** worked through a worksheet that is preparatory to an investigation during the observed lessons.
Progression of the Contents	**7X: 4**	Interview	'the school scheme determines the progression... except for those students using the textbook... in this case the textbook takes care of it'
	10L: 4	Scheme	The school scheme suggests the topics and order of the contents, but does not determine the material to be used nor suggest how the links should be established.
	9M: 4 **11T: 7**	Report of the Lessons	Teacher based the progression of contents for **11T** on the textbook, and on the school scheme for all other groups, deciding himself which material should be used.
Materials given to the students for the purpose of reference	**7X: 4**	Interview	Students in the 'top' groups keep the books all the time... the 'medium groups keep the notebooks and some of the worksheets... and the students in the bottom group do not take any material home.
	10L: @	Reports of the Lessons	Teacher did not allow students in group **10M** to take any material home. Students in groups **7X** and **9M** are allowed to take their notebooks home, and also some of the worksheets they use during the lessons. Students in group **11T** keep the notebooks and the textbooks all the time.
	9M: 4 **11T: 7**	Notebooks	**7X:** students notebooks contain some worksheets annexed, and also some explanations copied from the board. **10L:** the students notebooks contain mainly solution to exercises. **9M:** students notebooks contain certain worksheets annexed, and also explanations copied from the board. **11T:** students notebooks contain solutions to exercises from the textbook and also explanations copied from the board.

Table 4e.1: School E, Teacher E - analysis of the first set of headings, concerning general decisions on materials taken by the teacher for the school year.

4e.2: Analysis of headings in Sets II and III for EE7X group.

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities	4 4 4	4 4 7	General Description: During the three observed lessons the teacher had a teacher in training working with him. On the last lesson before the observations, the teacher had introduced numerical expressions and the rules to decide in which order the operations should be done, in preparation for the investigation to be developed. **L1:** The teacher introduces the lesson to the whole group by asking the rules for the order in which the operations are made in a expression, and then suggests that they should try to find different results using only the operations and 1, 2, 3 and 4. He gives examples and asks students to start the investigation in their notebooks. During the lesson the teacher helped the students who had difficulties or could not understand the results they were obtaining on the calculators. **L2:** Teacher C introduced the lesson by reminding the students of what they have been doing in the previous lesson and by giving more examples. He also says that he is only interested in whole numbers, so they should leave out things such as 1+2+3+4. **L3:** Teacher C introduces the lesson by giving the students the worksheet on which he has been basing the lessons, and by defining the objectives of the investigation. He also asked the students for examples. Teacher finishes the lesson by proposing a guessing game. The student teacher helped the teacher during the lessons, by supporting students with difficulties and by giving new tasks to the students.
Introduce a new topic.	4 - -	4 - -	
Introduce a lesson.	4 4 4	4 4 7	
Reference in Classroom.	1 1 1	1 1 1	
Exemplification.	4 4 4	4 4 5	
Conclude a topic.	- - @	- - @	the teacher did not aim to conclude the topic, the investigation was supposed to be kept open ended.
Promote links between related topics.	1 1 1	1 1 1	teacher established links between the investigation and the solution to expressions involving the four operations
Promote differentiation within class.	@ @ @	@ @ @	No differentiation was noticed. All students were supposed to work through the same investigation, and the same aims were given to all students during the third lesson.
Remedial Material.	- - -	- - -	No 'crisis' situation was observed. All students were able to work through the investigation, with eventual help from the teacher.
Source of Homework.	@ 4 @	@ 4 @	During the second lesson the teacher intructed students to work through the investigation for 20 minutes as homework. No homework assigned during the other lessons.
Provide Feedback on Exercises.	1 1 1	1 1 1	The teacher provided feedback during the lesson by checking student's work and offering suggestions. No material was used to provide feedback.

Table 4e.2: EE7X Group - analysis of the second and third set of headings over three lessons.

4e.3: Analysis of headings in Sets II and III for EE10L group.

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities	4 4 4b	7ab 7ab 7ab	General Description: There are seventeen students in this group. Teacher E proposes the same activity for all of them, and says he has to change material frequently, otherwise the students lose interest. During the observed lessons, teacher E was working on 'intervals of time' with these students, and used three different materials, taken from different series of books, all with the same objective. **L1:** Teacher introduces the work by using the book Everyday Maths Practice, and working through timetables. The teacher did not introduce the lesson and spent most of the time offering help to students. **L2:** Teacher introduces the lesson by introducing a new material on time interval. He tells students that they are expected to apply what they learned in the previous lesson in order to plan time intervals in several cases. **L3:** Teacher E introduces the lesson by telling students that he brought another worksheet on timetables and time intervals. He says that the students who finish the previous one will be using this new one and the others can choose if they prefer to continue on the previous worksheet or change fto this new one. The students were not allowed to take any material home. Teacher E did not conclude the topic. He offered explanations to small groups, usually those sitting at the same table.
Introduce a new topic.	@ - -	@ - -	
Introduce a lesson.	4 4 4b	7ab 7ab 7ab	
Reference in Classroom.	4 4 4	7ab 7ab 7ab	
Exemplification.	4 4 4	7ab 7ab 7ab	
Conclude a topic.	@ @ @	@ @ @	Neither the teacher nor any of the materials used offered conclusions for the topic.
Promote links between related topics.	- - -	- - -	Neither the teacher nor the materials aim to establish links.
Promote differentiation within class.	@ @ 4b	@ @ 7ab	Differentiation was promoted by the teacher by providing extra material for those students who finished the basic activities ahead of the others and by allowing students to choose between two different materials.
Remedial Material.	1 1 4b	1 1 7ab	'Crisis situations' were observed in all lessons. Teacher E used two strategies to solve them: he offered extra support to students facing difficulties during lessons and planned different material for the next lesson.
Source of Homework.	@ @ @	@ @ @	No homework was assigned to students. In fact, students are not allowed to take any material home.
Provide Feedback on Exercises.	1 1 1	1 1 1	Teacher provided feedback during the lesson by checking on students' exercises. He did not use any written material as an aid to feedback.

Table 4e.3: EE10L Group - analysis of the second and third set of headings over three lessons.

4e.4: Analysis of headings in Sets II and III for EE9M group.

Set II -Headings: Choice of material: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities	4b 3 4	2a 3 4	**L1**: Teacher E introduced the lesson by telling pupils that they were going to apply what they had learned about coordinates to develop a new activity. He gave the students a worksheet. Teacher E developed an example using the first picture on how to find the coordinates of the reflections of the picture using the y-axis and the x-axis as 'mirror lines'. Teacher E suggested the students do the same with the other pictures in the worksheet, and gave them mirrors. He suggested that they register the initial and reflected coordinates side-by-side in a table. Teacher E was using the shapes in a worksheet on coordinates and basing the activities on another worksheet, adapted by the school from printed materials. The teacher had to solve several 'crises' because the students were following the worksheet's instructions instead of his. He gave extension work to those who finished and proposed the same extension work as homework for all students. **L2:** Teacher E started the lesson by giving the students the worksheet he was basing the teaching on. He read questions 4, 5 and 6 with the students and asked them to do these activities on reflection of shapes and its coordinates. Several 'crises' were observed again, as several students started the worksheet from the first exercise. Teacher E also used the lesson time to check on homework and to explain diagonal reflection, which several students found difficult. Teacher E reinforced the relationship between the initial and reflected sets of coordinates, and told the students they should use these results instead of starting with the mirror all over again. **L3:** Teacher E marked notebooks between lessons and introduced the lesson by saying that several students were not observing the connections between a shape and its coordinates and the shape and new coordinates obtained by reflection. He instruted them to establish these relationships as homework and spent the lesson on an activity called 'Rangoli Patterns', adapted from a textbook series. The teacher introduced the activity by giving an example of a shape obtained by several reflections of lines and asked students to replicate some of these shapes themselves.
Introduce a new topic.	3 - 4	3 - 4	
Introduce a lesson.	3/4b 3 4	4/3 3 4	
Reference in Classroom.	1 3 1	1 3 1	
Exemplification.	4 @ 4	3 @ 4	
Conclude a topic.	- 3 -	- 6 -	Although the teacher did not summarise the final results of the topics for the students, he reinforced the students' findings and encouraged them to use the results obtained in new situations. He also encouraged students who did not conclude the exercise to try it as homework.
Promote links between related topics.	1 3 -	1 5 -	The teacher did establish the connection between the new activity and a previous one in coordinates. He also encouraged students to establish links between what happened with the coordinates when the shape was reflected using different axis as mirror lines.
Promote differentiation within class.	@ @ @	@ @ @	No differentiation was noticed. All students were supposed to do the same task and those who fell behind were assigned extra homework.
Remedial Material.	1 1 -	1 1 -	'Crisis' situations were observed in two of the lessons, due to conflicting instructions given by the material and by the teacher. The teacher solved these crises by himself, without the aid of any other material.
Source of Homework.	3 @ 3	4 @ 4	The teacher proposed homework based on the same material used for class work. No homework was assigned during the second lesson, when the teacher kept the notebooks for marking.
Provide Feedback on Exercises.	1 1 1	1 1 1	The teacher did not use any support material to provide feedback. He did it in two different ways: by checking students work during the observed lessons and by marking their notebooks between the second and third lesson.

Table 4e.4: EE9M Group - analysis of the second and third set of headings over three lessons.

4e.5: Analysis of headings in Sets II and III for EE11T group.

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities	7 7ab 7ab	7 7ab 7ab	General Description: This is the only group taught by teacher E that uses a textbook. Each chapter of the textbook is organised in three sections, and the teacher expects all students in the group to complete the first and at least half of the second one. The remaining part of the chapter is considered by the teacher as extension work, and only the brighter students in the group are expected to reach the end of the chapter. Teacher E keeps the group working as a whole, all students on the same chapter. **L1:** The teacher proposes to the students to start a new chapter in the Task Maths 5 book: 'Making the most of life'. the teacher asked the students to open the book and carry on with their work. The teacher spent the lesson time checking on students work, and helping those who were facing difficulties. All students in the group were able to work though the book with eventual help from the teacher. **L2:** The teacher was late because he had some extra activity in the school. When the teacher arrived in the classroom all students were already working. The teacher helped the students and said that all of them should finish the first section of the chapter during the lesson. One student who is ahead of the other is asked by the teacher to help other students for a certain period of time, which he does willingly, and which is appreciated by the other students. The teacher concluded the lesson by asking those students in the group who wanted extra support to come at lunch time for a revision on shapes. Several students used the answer book during the lesson, in their own time. During these two lessons, the teacher did not set 'formal' homework but said that he expected students to work at least for one hour on their Maths at home, doing what they felt was best (carry on with their work or revising for their exams). **L3:** Teacher E says this is the last lesson in this chapter. Most of the students are working through section B but some students have already finished it and some of them had finished the whole chapter by the end of the lesson. Teacher E asked for notebooks for marking, but students kept the textbook, and the teacher said they should carry on with their revision at home.
Introduce a new topic.	7ab - -	7ab - -	
Introduce a lesson.	7ab 7ab 7ab	7ab 7ab 7ab	
Reference in Classroom.	7ab 7ab 7ab	7ab 7ab 7ab	
Exemplification.	7ab 7ab 7ab	7ab 7ab 7ab	
Conclude a topic.	7ab 7ab 7ab	7ab 7ab 7ab	The teacher did not highlight the conclusions to the topic made by the book. The students were supposed to read them by themselves, in their own time, without being asked to do so.
Promote links between related topics.	7ab 7ab 7ab	7ab 7ab 7ab	The teacher did not highlight the links established by the book. Students were supposed to read them by themselves, in their own time, without being asked to do so.
Promote differentiation within class.	7ab 7ab 7ab	7ab 7ab 7ab	Differentiation was promoted by the book. The teacher expected that different students would reach different final points in the chapter.
Remedial Material.	- - -	- - -	No 'crisis' situations were observed. All students were able to work through the book with eventual help offered by the teacher or by their colleagues.
Source of Homework.	7 7 7	7 7 7	The teacher did not assign homework, but students are expected to work on their Maths at least one hour in between every two lessons.
Provide Feedback on Exercises.	1/7b 1/7b 1/7b	1/7ab 1/7ab 1/7ab	The teacher provided feedback in three different ways: by checking students' work during the lesson, by allowing them to use the answer book in their own time and by marking their notebooks out of class.

Table 4e.5: EE11T Group - analysis of the second and third set of headings over three lessons.

Appendix 4f: Teacher F, School F

4f.1: Analysis of headings in Set I for Teacher F's four groups.

Set I - Headings	Meta-Categories	Sources of evidence	Summary of evidence
Main Source of Materials for Classroom Work	**7X: 5** **7L: 1** **8M: 6** **8T: 7**	Interview	She says that she uses (1) the 'red track' of the NMP, with FF8T; (2) the 'blue track' with FF8M, with several complementation and 'lots of class teaching'; (3) the NMP for year seven with FF7X for about half of the lessons, with the other half being covered by other sources; and (4) no textbook or any other written materials with FF7L.
		School Scheme	School Scheme that does not determine which material should be used. The Scheme is organised by groups of students 'within the same attainment level' and not by year group.
		Reports of the Lessons	Same information during interview in cases FF7X, FF7L, FF8T. Group FF8M was developing an investigation during the observed week, returning to the textbook afterwards.
		Notebooks	Same information given during interview for all groups. The material used in general can be described as above.
Progression of the Contents	**7X: 5** **7L: 2** **8M: 5** **8T: 7**	Interview	'the school scheme tries to reflect what each group of students can do ... I use it ... but I also take my own decisions'... and there is the book as well... at least for the brighter students.'
		Scheme	see above
		Report of the Lessons	Observed progression based on the book for **8T** and **8M**, adapted from the book and the school scheme for **7X**, and decided by the teacher, based on the school scheme for **7L.**
Materials given to the students for the purpose of reference	**7X: 1** **7L: 1** **8M: 4** **8T: 4**	Interview	the school does not allow the students to take textbooks home, 'because they come back destroyed... not even with the 'top' group'. All students are allowed to take their notebooks home.
		Reports of the Lessons	Students are not allowed to take the textbook home. All of them are allowed to take notebooks home. Evidence of little complementation of material (worksheets) for groups **8M** and **8T.** Notebooks regularly marked by the teacher, with evidence of some reference in the cases of **8M** and **8T,** and .some examples in the cases of **7X** and **7L.**

Table 4f.1: School F , Teacher F - analysis of the first set of headings, concerning general decisions on materials taken by the teacher for the school year.

4f.2: Analysis of headings in Sets II and III for FF7X group.

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities	1 1 1	1 2 2	**L1:** Teacher proposes the activities on volume without using any written material. **L2** and **L3:** Teacher proposes activities in capacity, establishing the relationship between volume and capacity, and giving the units equivalence. Activities proposed modifying the ones in the textbook. No copy of the book given to the students. **L1** and **L3:** no new topic introduced. **L2:** Teacher introduces capacity, without giving a copy of the book to the students, whilst modifying the proposed introduction. **L1:** Activity (a table with dimensions and volume of several cuboid solids) introduced by the teacher, without using written materials. **L2:** Capacity introduced by the teacher, adapting from the material and proposing own created activities (make three boxes that holds one litre) without using written materials. **L3:** Teacher gives continuity to the previous lesson's activity, proposing extensions for those who finished (make a box that holds one pint). Several modifications of the book's content.
Introduce a New Topic.	- 5 -	- 3 -	
Introduce a Lesson.	1 5 1	1 3 2	
Reference in Classroom.	@ 1 1	@ 1 1	**L1:** Activity using concrete materials, no reference given in notebooks. **L2** and **L3:** Activity in capacity. Teacher wrote the relationship between volume and capacity, and the equivalence between cubic centimetres and millilitres on the board.
Exemplification.	1 1 1	1 2 2	**L1:** Teacher illustrated the activity without using any material. Activity not proposed by the book. **L2** and **L3:** Teacher provides different examples (calculating the capacity of given boxes) to those in the book.
Conclude a Topic.	- - @	- - @	**L1** and **L2:** No topic concluded. **L3:** Teacher says that it is the last lesson on volume and capacity, but does not present any conclusion to the topic.
Promote links between related topics.	- 1 -	- 2 -	**L1** and **L3:** Teacher did not aim to establish links. **L2:** The links between the concepts of volume and capacity established by the teacher, the equivalence between volume units and capacity units were also establish by the teacher, modifying the textbook's content.
Promote differentiation within class.	@ @ 1	@ @ 1	**L1** and **L2:** No differentiation took place. All students worked through the same task. **L3:** During the third lesson, an extension was proposed, and six students started it, with no final result.
Remedial Material.	- - -	- - -	**All:** No crisis observed during the lessons. All the students worked through the task and asked teacher's help when needed. Planned extension work proposed for those who finished the tasks.
Source of Homework.	4 4 1	7 7 1	**All:** Teacher has previously introduced the topic 'volume' using a project sheet. After working one lesson on the project, it was set as homework for one whole week. The next homework involves measuring capacity from cuboid objects at home or at the shops. All homework linked with class work.
Provide Feedback on Exercises.	1 1 1	1 1 1	**All:** Teacher provided all the feedback by checking students work and offering help and suggestions for continuity. No material used to provide feedback.

Table 4f.2: FF7X Group - analysis of the second and third set of headings over three lessons.

4f.3: Analysis of headings in Sets II and III for FF7L group .

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities	1 1 1	1 1 1	**All:** All activities proposed by the teacher, without using any written materials. Students take notes and carry out the activities using concrete materials and their notebooks. **L1:** area of rectangles (cut in colour papers) using grid and multiplication. **L2:** area of right angled triangle, changed afterwards to halving. **L3:** halving, followed by area of rectangles.
Introduce a new topic.	- 1 -	- 1 -	**L1** and **L3:** No new topic introduced. **L2:** Teacher introduces area of right angled triangles based on areas of rectangles developed in the previous lesson. The students have difficulties with it, and she has to change the lesson afterwards. No written material used.
Introduce a lesson.	1 1 1	1 1 1	**All:** All lessons introduced by the teacher, without using any written material. Teacher recalled the work done in the previous lesson, and proposed new activity.
Reference in Classroom.	@ @ @	@ @ @	**All:** During all the observed lessons, the students were asked to write the title and the date in their notebooks, followed by exercises and activities. No notes in the notebooks.
Exemplification.	@ 1 @	@ 1 @	**L1** and **L3:** Teacher offered no illustration to the whole group. Illustrations were given by helping students solve the proposed exercises. **L2:** Teacher provided illustration on halving for the whole group, using different concrete materials and figures on the board.
Conclude a topic.	1 - @	1 - @	**L1:** Teacher concluded the general formula for area of rectangles. Given orally to the students, without asking them to take notes. **L2:** No topic was concluded in the second lesson. **L3:** Teacher offered no conclusion to the topic 'halving'.
Promote links between related topics.	- 1 -	- 1 -	**L1** and **L3:** teacher did not aim to establish links. **L2:** Teacher tried to establish links between area of rectangle and area of a right angled triangle, but students had difficulties in understating it. Teacher did establish the link between halving and division by two.
Promote differentiation within class.	@ @ @	@ @ @	**All:** During all the lessons, the teacher gave the same tasks to all students. No differentiation noticed.
Remedial Material.	- 1 1	- 1 1	**L1:** No crisis observed. All students working through the proposed tasks, and asking for help when needed (the teacher had an auxiliary teacher for the first two lessons observed with this group). **L2** and **L3:** Crisis observed during lesson two, when students could not associate halving with division by two. Teacher introduced halving as a new topic, work through it during the remaining of the second lesson and part of the third using calculators, and reassumed work in rectangles afterwards.
Source of Homework.	@ @ @	@ @ @	**All:** No homework given to the students.
Provide Feedback on Exercises.	1 1 1	1 1 1	**All:** No written material used for feedback. Teacher offered feedback by checking students' work and offering suggestions and help when needed.

Table 4f.3: FF7L Group - analysis of the second and third set of headings over three lessons.

4f.4: Analysis of headings in Sets II and III for FF8M group.

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities	4 4 6	8 8 5	**L1** and **L2:** Investigation developed (octagon loops), following printed material and its suggestions of use. **L3:** Teacher goes back to work from the textbook, starting a new chapter (Average, median, mode).
Introduce a new topic.	- - 5	- - 3	**L1** and **L2:** no new topic introduced. The students are continuing the investigation. **L3:** teacher introduces the concepts of average, etc.. complementing and modifying the introduction presented in the textbook.
Introduce a lesson.	4 4 5	8 8 3	**L1** and **L2:** Teacher introduces the lessons discussing stages of the investigation, as suggested in the guide. **L3:** Teacher introduces the lesson complementing and modifying the textbook.
Reference in Classroom.	4 4 7	2 2 2	**L1** and **L2:** Teacher complements the worksheets given by suggesting on the board what should be done. The material given to the students does not include written reference. **L3:** Teacher complements the textbook by offering other illustrations to clarify the difference between average, median and mode.
Exemplification.	@ @ 6	@ @ 2	**L1** and **L2:** No examples given by the teacher (although examples were given in the first lesson) **L3:** Teacher added different examples from the ones in the text, with the objective of clarifying the topic developed by the book.
Conclude a topic.	- @ -	- @ -	**L1** and **L3:** No topic was concluded during these lessons. **L2:** Neither the material nor the teacher provided any conclusion to the investigation
Promote links between related topics.	- - -	- - -	**All:** Teacher did not aim to establish links.
Promote differentiation within class.	1 1 @	8 8 @	**L1** and **L2:** Teacher promoted differentiation as suggested by the proposed investigation material. Students were at different points throughout the investigation. **L3:** No differentiation took place during the lesson. All students working on the first section of exercises in the chapter of the book.
Remedial Material.	- - -	- - -	**All:** No crisis situations observed. All students worked through the proposed activities and asked for help when needed, there was no opportunity to observe the need for extension work.
Source of Homework.	4 4 4	7 7 7	**All:** Teacher set 'completion of investigatory work" as homework. As the material was not intended as homework, the teacher set her own targets, differentiated by students. Homework matched with class work.
Provide Feedback on Exercises.	4 4 1	8 8 1	**L1** and **L2:** Teacher provided feedback based on suggestions in the investigation guide, although the material was not brought to class. **L3:** Although the answer book was in class, feedback during lessons was provided by the teacher, on an individual basis.

Table 4f.4: FF8M Group - analysis of the second and third set of headings over three lessons.

4f.5: Analysis of headings in Sets II and III for FF8T group.

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities	7 7 7	7 5 7	**L1** and **L3:** Teacher proposes activities on bearings (L1) and on use of symbolic language (L3) taken from the material, with evidence of not following the suggestions in the guides. She proposed pages and exercises. **L2:** Teacher complements the textbook by giving a own produced mental test on the topic, based on the text, and matched to previous activities.
Introduce a new topic.	- - 7	- - 2/7	**L1** and **L2:** No new topic was introduced. **L3:** Teacher introduced the new topic (use of symbolic language) complementing the material, and adding new aspects. After this introduction, the students were asked to read the book introduction.
Introduce a lesson.	7 5 7	7 5 2/7	**L1:** Teacher introduced the lesson by recalling what they have done in the previous lesson, reminding them of results. She asked them to read an information table from the book, taken notes if necessary. After that, she gave them a task taken from the text. **L2:** Teacher introduced the lesson by giving a mental test based on the textbook, proposing activities taken from the book after that. **L3:** Introduction of a new topic. See above.
Reference in Classroom.	7 7 7	7 7 7	**All:** Teacher asked the students to read explanations and examples from the book, giving them time to do so. She also asked them to take notes, if they found a new idea, of which they had previously been unwaware, gaving them time to do so.
Exemplification.	6 6 6	5 5 2	**L1** and **L2:** Teacher gives examples close to the ones offered in the text. **L3:** Teacher gives examples to complement some aspects not considered in the textbook, she also uses some of the exercises the students have already solved to lead them to certain conclusions.
Conclude a topic.	- 3 5	- 8 2	**L1** and **L3:** These lessons did not conclude a topic. Nevertheless, during L3 the teacher stopped the students to check if they had drawn any conclusions from exercises with identities (such as p+p=2p). **L2:** This was the final lesson on the topic, and the teacher used an own produced mental test to reinforce the main aspects of its content.
Promote links between related topics.	- - -	- - -	**All:** The teacher did not aim to establish links.
Promote differentiation within class.	7 7 7	6 6 6	**All:** The teacher promoted differentiation by proposing different activities selected from the book to students who had finished the 'core' section of the book.
Remedial Material.	- - -	- - -	**All:** No crisis was observed during the lessons. All students worked through the proposed exercises, asked help from the teacher when needed, whilst those who finished early carried out planned extension work taken from the book.
Source of Homework.	@ @ 1	@ @ 1	**L1** and **L2:** No homework given . **L3:** Teacher asked students to revise their work on bearings (the previous chapter) for a test during the following lesson. The students are not allowed to take the book home, so the review was made from notes and exercises in their notebooks.
Provide Feedback on Exercises.	6/1 1 1	5/1 1 1	**L1:** Teacher started the lesson by discussing answers to exercises given during the previous lesson with the whole group. **L2** and **L3:** Although the answer book was in the class, feedback was provided by the teacher, most of the time individually, but sometimes to the whole group, discussing their answers to lead them to drawn their own conclusions.

Table 4f.5: FF8T Group - analysis of the second and third set of headings over three lessons.

Appendix 4g: Teacher G, School G

4g.1: Analysis of headings in Set I for Teacher G's two groups.

Set I - Headings	Meta-Categories	Sources of evidence	Summary of evidence
Main Source of materials for Classroom Work	**7M: 6** **8T: 6**	Interview	'we use the SMP booklets...but usually one lesson per week is dedicated to mental arithmetic and table skills'
		School Scheme	Based on the SMP booklets, and complemented by extra materials for mental arithmetic, tables skills and investigations.
		Reports of the Lessons	**7M:** Series of lessons on angles, with the topic introduced by the teacher during the first lesson, and based on SMP booklet 'angles 2' and on LOGO activities at the computer during the week's other lessons. **8T:** Series of lessons on decimals, introduced by the teacher during the first lesson and based on SMP booklet 'fractions and decimals 1' and its extensions for the week's other lessons.
		Notebooks	**7M:** Notebooks basically have the titles of the contents and exercises solved, marked regularly by the teacher. About 80% of notes in notebooks are answers to SMP booklets' questions. **8T:** Notebooks not only have the solution to exercises but also some copy of references from the board, containing definitions and examples. In several notebooks it is possible to observe extensions of the SMP booklets being used. Notebooks marked regularly, but less frequently than in group **7M**.
Progression of the Contents	**7M: 6** **8T: 6**	Interview	'the booklets offer a good progression,...and we follow it most of the time...sometimes we change it, adding new things...but not much'.
		Scheme	School scheme that bases it progression on the SMP progression, with investigations and some other activities added, based on other materials.
		Report of the Lessons	Both groups were using SMP booklets as planned in the school scheme.
Materials given to the students for the purpose of reference	**7M: 1** **8T: 1**	Interview	'we do not allow the booklets to be taken home... and the notebooks are only taken home when they have homework to do...because they keep loosing them... we decided it was better to keep them here... the homework is usually a worksheet... they do not write in the worksheet... they do the task in their notebooks'
		Reports of the Lessons	Notebooks collected by the teacher at the end of the lesson, unless homework is assigned to the students.
		Notebooks	**7M:** Notebooks mostly contain solutions to exercises. Few notes added by the teacher or copied from the board by the students. **8T:** Notebooks contain not only solved exercises but also regular notes copied from the board.

Table 4.g.1: School G , Teacher G - analysis of the first set of headings, concerning general decisions on materials taken by the teacher for the school year.

4g.2: Analysis of headings in Sets II and III for GG7M group.

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities	1 8/4 7ab/4 1	1 8/7 7ab/7 1	**L1:** Teacher introduces the concept of angle as an 'amount of turn', without using any material (she had planned to use LOGO, but some students had not see it before and she changed her mind) Activities developed from shapes draw on the board and no material used. **L2:** Teacher discusses homework and after that introduces the first activity of the booklet, reading it with the whole group and presenting the concrete material to be used. After that the group was split in two: some working in pairs from the booklets and others working with LOGO at the computers available in the classroom, using a worksheet. **L3:** Teacher gave continuity to both activities developed during the previous lessons: she assigned activities for the ones using LOGO and the students using the booklet were asked to 'keep going'. **L4:** Teacher started the lesson by discussing different triangles. After that, she asked the students to draw their own triangle on a piece of coloured paper, cut it in three pieces (so that each piece had a corner in it), and put the corners side by side against a ruler. When the activity was finished, she gave the students their notebooks, asking them to copy (and complete) a series of exercises on definitions of different types of triangles. After that, she asked them to glue their ripped triangle in their notebooks and conclude the rule on the sum of internal angles of a triangle.
Introduce a new topic.	1 8/4 - 1	1 8/5 - 1	
Introduce a lesson.	1 8 7ab/4 1	1 8 7ab/7 1	
Reference in Classroom.	@ 8/4 7ab/4 1	@ 8/7 7ab/7 1	
Exemplification.	1 8 @/4 1	1 6 @/2 1	
Conclude a topic.	1 5 4 5	1 2 3 2	**L1:** Teacher concluded the lesson by discussing the results obtained and by assigning a related homework. **L2:** Teacher finished the lesson by comparing the two experiences on supplementary angles (provided by the booklet and by LOGO). She concluded the rule. **L3:** Teacher finished the lesson by discussing the process to draw an equilateral triangle using LOGO with the whole group. She concluded the internal and external angles of these types of triangles. **L4:** teacher concluded the topic by giving students a summary of types of triangles and the 'sum of internal angles' rule.
Promote links between related topics.	- 5/4 5/4 -	- 2 2 -	**L1** and **L4:** The teacher did not aim to establish links. **L2** and **L3:** Teacher established links between the different activities developed by the students by highlighting and comparing the different activities.
Promote differentiation within class.	@ 7ab/4 7ab/4 @	@ 7ab/7 7ab/7 @	**L1** and **L4:** No differentiation noticed. All students were supposed to develop the same activities with the same aims. **L2** and **L3:** Students who had never used LOGO before worked at the computer instead of using the booklet. All students were allowed to work at their own pace
Remedial Material.	1 4 4 -	1 7 7 -	**L1:** Teacher had deviate from her plans, when finding out that some students in the group had never used LOGO (which was an IT activity planned for the previous half-term). She adapted the lesson, without the introduction of any material and moved those students to work with LOGO in lessons **L2** and **L3.**
Source of Homework.	1 @ @ @	1 @ @ @	**L1:** Homework assigned for the following day, before the next lesson. Homework based on class activities. **L2, L3** and **L4:** No homework assigned.
Provide Feedback on Exercises.	1 1 1 1	1 1 1 1	**All**: Teacher did not use the SMP answer book to provide feedback. All feedback given by the teacher, usually on an individual basis. The feedback on homework assigned in the first lesson was the introduction to the second lesson for the whole group.

Table 4g.2: GG7M Group - analysis of the second and third set of headings over four lessons.

4g.3: Analysis of headings in Sets II and III for GG8T group.

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities	1 8 7ab	1 8 7ab	**L1:** Teacher introduces decimals by playing a game of discovering the number. Students started by imagining it was a whole number but changed quickly to decimals when they discovered it was not an integer. A sheet with labelled and not labelled number lines was used to help the students. Teacher also discussed the relationship between fractions and decimal numbers, and introduced place value representation using decimal places. Finally comparison between units was introduced. **L2:** Teacher introduces the booklet 'fractions and decimals 1' to the whole group. She started by developing the first activity with them, distributed the booklets and read with them all exercises in section A. After that, students worked individually or in pairs at their own pace. A complementary activity: decimal dominoes was carried out with groups of students. The teacher supervised this activity with one group of students at a time. **L3:** The students were asked to 'carry on' with their work from the SMP booklet, and the teacher supervised the decimal dominoes activity with those who had not done it in the previous lesson. During this lesson, three pairs of students finished the booklet, and two of them had time enough to start an extension booklet during the lesson.
Introduce a new topic.	1 - -	1 - -	
Introduce a lesson.	1 8 7ab	1 8 7ab	
Reference in Classroom.	@ 8 7ab	@ 8 7ab	
Exemplification.	1 8 @	1 8 @	
Conclude a topic.	- - 7ab	- - 7ab	**L1** and **L2**: No topic was concluded during this lesson. **L3:** Teacher concluded the lesson by assigning a review sheet of the booklet as homework. Teacher did not offer any other conclusion.
Promote links between related topics.	1 - -	1 - -	**L1:** Teacher linked the work done in decimals during the lesson with previous work in decimal fractions and place value representations of numbers. **L2** and **L3:** neither the teacher nor the booklet aim to establish links.
Promote differentiation within class.	@ 7ab 7ab/8	@ 7ab 7ab/8	**L1:** All students worked on the same activity, at the same time, with the same aims. No differentiation noticed. **L2** and **L3**: Students worked at their own pace through the booklets. Extension material provided for those who finished ahead of the others. Students that were late in their work were assigned extra homework to ensure that all completed the 'basics' of the topic.
Remedial Material.	- - -	- - -	**All:** No crisis situation observed. The teacher was able to apply her plans successfully. During the third lesson, the teacher offered the extension material from SMP to those who finished the proposed booklet.
Source of Homework.	@ @ 8	@ @ 8	**L1** and **L2:** No homework assigned. **L3:** The review sheet associated with the booklet was assigned as homework. Those students that had not started the final section of the booklet had to finish the booklets as complementary homework.
Provide Feedback on Exercises.	1 7a 7a/7ab	1 7 7/7ab	**L1:** Teacher gave feedback on the activities proposed without using any written materials. **L2** and **L3:** Feedback given using the answer book at the end of the lesson. The teacher insisted that students should correct their work, before going further. During **L3**, the students who finished the booklet before the end of the lesson were allowed to use the answer book by themselves. They were asked by the teacher to do the corrections before starting the extension material.

Table 4g.3: GG8T Group - analysis of the second and third set of headings over three lessons.

Appendix 4h: Teacher H, School H

4h.1: Analysis of headings in Set I for Teacher H's four groups.

Set I - Headings	Meta-Categories	Sources of evidence	Summary of evidence
Main Source of materials for Classroom Work	7X: 5 10L: 7 9M: 7 10T: 7	Interview	'we use the textbook series... but for year seven it is complemented with the computer system and we also change materials for several subjects... for year eight we also use several booklets of SMP to complement the book... and for the elder students the book become almost the only source...'
		School Scheme	The school scheme for years seven and eight is topic-based, and suggests the material that should be used for the core, support and extensions. The book is not the only resource and several changes are made in the order that the topics are presented in the book. The school scheme for the 'low attainers' from year nine upwards is not organised by year, but establishes that the SMP green track should be used on an individualised bases, one book after the other, up to year eleven. The school scheme for other students in year nine is based on the textbook, folowing the order suggested by the book and proposing the book as the main resource. Finally, the are three school schemes for years ten and eleven, each one based on one track of the textbook series, and strongly based on the book, including the order in which the contents are presented.
		Reports of the Lessons	The activities observed during the lessons were organised in the way proposed by the school scheme, except for year nine, medium group, where the teacher was developing a topic based on a series of school produced worksheets, with the objective of integrating the science and mathematics contents.
Progression of the Contents	7X: 5 10L: 7 9M: 7 10T: 7	Interview	'...we decide the progression for years seven and eight... it is not the one suggested in the book... years nine, ten and eleven... the progression is the one suggested in the book... except for some occasional complementation in year nine... but it is rare.'
		Scheme	the school scheme changes the order of the contents in years seven and eight, and it also suggests changes of materials. For years nine upwards, the school scheme is a copy of the contents in the book, respecting the order in which they are presented in the book.
		Report of the Lessons	The teacher used SMP, computer system and the adopted textbook with year seven. He used the SMP green series for all activities observed with the year ten 'bottom' group. He used a series of worksheets developed by the school and the textbook with year nine 'medium' group and he used the textbook page by page with year ten 'top' group.
Materials given to the students for the purpose of reference	7X: 8 10L: 8 9M: 8 10T: 8	Interview	'it is the school policy that each student should have a copy of the textbook for the whole year... it is for them to keep... and we usually give them a copy of the worksheets as well... we cannot give them copies of other materials used, as the SMP booklets, for example...'
		Reports of the Lessons	The students have a copy of the book, which they bring to lessons. Other materials used by the teacher were given during the lessons, and the students returned the books used, but kept copies of the worksheets.
		Notebooks	Students notebooks show that materials other than the book are used regulary with year seven, but are not used with the other groups. The students notebooks contain mainly the solutions to exercises proposed by the book, with regular marking done by the teacher.

Table 4h.1: School H, Teacher H - analysis of the first set of headings, concerning general decisions on materials taken by the teacher for the school year.

4h.2: Analysis of headings in Sets II and III for HH7X group.

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities	4ab 1 1/6	7ab 1 1/2	During all lessons observed, the 'bottom-half' of the students started by working with the teacher while the 'top-half' was working at the computers. After half lesson, the students changed places, and the ones working at the computers started to work with the teacher. **L1:** The students were working through the SMP booklets on decimals.
Introduce a new topic.	- 5 5	- 2 2	At the beginning of the lesson the teacher asked them which booklet they were using and gave the correct copy to each student. They were asked to 'keep going' with their work. The teacher had an auxiliary teacher to help with one student who had 'medically proved learning difficulties' and who worked individually. **L2:** The teacher was not
Introduce a lesson.	4ab 5 5	7ab 2 2	expecting to use computers during this lesson, but had to change class due to an A-level exam, and decided to allow the students to play some games on the computer during half lesson. The games were not part of the usual system they work on and were all mathematics related. During this lesson the teacher introduced a
Reference in Classroom.	4 7 7	7 2 2	handling data activity and asked the students to do a tally and create a frequency table on size of words in a fifty words text. He instructed them to finish the table for homework. **L3:** The teacher decided not to use the table the students developed in the previous lesson, but to introduce the concept of 'mean' instead, by giving the definition and
Exemplification.	4ab 1/5 1/5	7ab 1/2 1/2	asking the students to do a series of own proposed exercises. Those who finished these exercises were asked to work on exercises from the book and homework was set to calculate the mean of size of the
Conclude a topic.	4ab - 5	7ab - 2	words on the table developed in the previous lesson. **L1:** Final lesson using the SMP booklets on decimals. The teacher did not provide conclusions and did not recommend students to read it in the booklet used. **L2:** No topic concluded during this lesson. **L3:** The teacher provided some conclusions complementing the ones in the material.
Promote links between related topics.	4ab 5 5	7ab 2 2	**L1:** All links established by the material used. The teacher did not reinforce them. **L2** and **L3:** The teacher complemented the material by establishing links with previous work done in AT5.
Promote differentiation within class.	4ab 1/4ab 1/4ab	7ab 1/7ab 1/7ab	Differentiation was observed in several ways: (a) the teacher divided the group in halves by ability levels; (b) the teacher allowed the students to work at their own pace through the material selected for the first lesson observed; (c) the teacher complemented the exercises proposed by the book with own created exercises, differentiated by the sub-groups.
Remedial Material.	- - -	- - -	No 'crisis' situations were observed. All students were able to work through the proposed activities, with eventual help given by the teacher.
Source of Homework.	@ 1 1	@ 1 1	No homework was set during the first lesson observed, and homework set during other lessons was based on class activities created by the teacher, without using the book (complementary class activities)
Provide Feedback on Exercises.	4b 1/4b 1/4b	7ab 1/7ab 1/7ab	The computer activity provides immediate feedback. The teacher provided feedback by checking students' work during the lesson. During the first lesson students were also allowed to use the answer book when finishing the booklet.

Table 4h.2: HH7X Group - analysis of the second and third set of headings over three lessons.

4h.3: Analysis of headings in Sets II and III for HH10L group.

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities	7ab 7ab 7ab	7ab 7ab 7ab	During all observed lessons, the nine students worked individually using the SMP green track. As the students have been working this way since the beginning of year nine, the differences in pace are notable because three different volumes of the book are being used at the moment (books 3, 4 and 5). There were no differences in the way materials were chosen and used observed in different lessons. It seems that a routine has been established and is followed: the teacher asks the students to 'keep going'. If some student finish one section of the book he/she is given an assessment activity. The other students carry on with their work, with eventual help offered by the teacher. Homework was set for those students who finished a section during the lesson, and was based in the book (usually the revision section). Each student had at least one turn with the teacher in each lesson. The teacher marked the previous work and offered explanations on the exercises with which the students had difficulties.
Introduce a new topic.	7ab 7ab 7ab	7ab 7ab 7ab	
Introduce a lesson.	7ab 7ab 7ab	7ab 7ab 7ab	
Reference in Classroom.	7ab 7ab 7ab	7ab 7ab 7ab	
Exemplification.	7ab 7ab 7ab	7ab 7ab 7ab	
Conclude a topic.	7ab 7ab 7ab	7ab 7ab 7ab	The teacher offered no conclusion of topics, but recommended those students who finished a topic to do the revision exercises as homework.
Promote links between related topics.	7ab 7ab 7ab	7ab 7ab 7ab	No links established by the teacher. Students are supposed to read the links established by the book without being asked by the teacher to do so.
Promote differentiation within class.	7ab 7ab 7ab	7ab 7ab 7ab	Differentiation is promoted by allowing students to work at their own pace through the series of textbook chosen.
Remedial Material.	1 1 1	1 1 1	No remedial material introduced. The teacher noticed some students facing difficulties but provided extra explanations by himself, without changing material.
Source of Homework.	@/7 @/7 @/7	@/7 @/7 @/7	Homework was set for different students each lesson. The teacher uses the homework for revision work, and each student who finishes one section is asked to work through the revision exercises proposed by the book.
Provide Feedback on Exercises.	1/7b 1/7b 1/7b	1/7ab 1/7ab 1/7ab	The teacher did not use the answer book to provide feedback. Nevertheless, answer books are available in class and students are allowed to use them when finishing a chapter. The teacher reinforces the need to work though correction before starting a new chapter.

Table 4h.3: HH10L Group - analysis of the second and third set of headings over three lessons.

4h.4: Analysis of headings in Sets II and III for HH9M group.

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories Set II	Set III	**Summary of evidence from lesson reports.**
Source of Class Activities	2 2 2	6 6 6	During the observed lessons the teacher was using a series of worksheets developed by the department, which were not based on any printed materials. The material was developed as a response to the needs of the science department: the students will need to have some basic skills in plotting graphs of linear functions from a set of experimental points given. As the previous work done with these students using the textbook did not reinforce graphs skills, the mathematics department has developed these series of worksheets to complement the textbook. Teacher H did not use the worksheets in the way they were developed. During the observed lessons several complementary explanations were offered by the teacher, and students were asked to take notes in their notebooks. It was also observed that the teacher selected exercises from the ones proposed by the worksheet. The teacher did not use the textbook during these lessons, and homework was also set from the material used, which was kept by the students.
Introduce a new topic.	2 - 2	5 - 5	
Introduce a lesson.	2 2 2	5 5 5	
Reference in Classroom.	1/2 1/2 1/2	2 2 2	
Exemplification.	2 2 2	6 6 6	
Conclude a topic.	- - 2	- - 5	No conclusions were offered during the first two lessons observed. The teacher complemented the conclusions offered by the material by asking students to take notes of the gradient formula on their notebooks.
Promote links between related topics.	2 2 2	5 5 5	The worksheet used some example based on science linear models and the teacher also stressed that skills developed during these lessons were to be applied in science classes.
Promote differentiation within class.	@ 1 @	@ 1 @	Differentiation was observed by allowing students to work at their own pace in a series of selected exercises during the second lesson. Nevertheless, all the students were supposed to do all other activities proposed during the lessons, with no differentiation noticed.
Remedial Material.	- - -	- - -	No remedial material was introduced and no 'crisis' situations were observed. Students were able to work through the proposed material with eventual help from the teacher.
Source of Homework.	@ 2 @	@ 6 @	No homework was set in the first and the third lessons observed. Teacher H set homework from one of the worksheets during the second lesson, with evidence of simple selection.
Provide Feedback on Exercises.	1 1 1	1 1 1	The teacher used no answer book to provide feedback on exercises. He provided feedback by checking students work during the three lessons observed, by collecting their graph work at the end of the second lesson for marking and by providing answers to exercises on the blackboard during the third lesson.

Table 4h.4: HH9M Group - analysis of the second and third set of headings over three lessons.

4h.5: Analysis of headings in Sets II and III for HH10T group.

Set II -Headings: Choice of material to: **Set III-Headings:** Use of material to:	Meta-Categories **Set II**	**Set III**	**Summary of evidence from lesson reports.**
Source of Class Activities	7 7 7	7 7 7	**L1:** The teacher introduces the sine rule, by proving the result and applying it in some direct examples. Both the proof and the examples were based on the textbook. The students were asked to do some practical exercises and the first two problems proposed by the book. At the end of the lesson, the teacher asked the students who had not finished the task to complete it at home. **L2:** The teacher introduces the lesson by developing some examples on the use of the sine rule as a way to show that some triangles are impossible to construct.
Introduce a new topic.	5 5 5	5 5 5	
Introduce a lesson.	5 5 5	5 5 5	Teacher H asks the students to solve some problems proposed by the book on the subject, and offered a tutorial to students who missed the previous lesson on the sine rule. No homework was set, except for those students who missed the previous lesson, who were told to finish the chapter at home. **L3:** Teacher H introduces the cosine rule, without proving it (he suggests the students should read the proof from the book at home and ask him if they have problems in understanding it during the next lesson) He applies it in examples taken from the book and suggests the students practice by using the first two proposed exercises and doing the problems. The teacher also says that students are supposed to finish the chapter at home.
Reference in Classroom.	7 7 7	5 5 5	
Exemplification.	7 7 7	6 6 6	
Conclude a topic.	6 6 6	5 5 5	The teacher provided conclusions and summaries for the topics developed, based on the textbook. The students have complementary notes in their notebooks, copied from the board.
Promote links between related topics.	6 6 6	5 5 5	The teacher reinforced the links established by the book, by starting the topics using previous knowledge. He also established links between Pythagoras' theorem and the cosine rule.
Promote differentiation within class.	@ @ @	@ @ @	No differentiation was noticed. All students are supposed to develop the same activities, and homework is used to keep students together. The teacher allowed students to decide how many practical exercises they wanted to do, but they were all supposed to do all the application problems.
Remedial Material.	- - -	- - -	No crisis situations observed. All students were able to develop the proposed activities with eventual help from the teacher.
Source of Homework.	7 7 1/7	7 7 1/7	The teacher proposed that they finish the class activity as homework during all observed lessons. He also proposed preparatory homework for the next lesson during the last lesson observed.
Provide Feedback on Exercises.	7b 7b 1/7b	7ab 7ab 1/7ab	The textbook provide answers at the end of the book. The teacher asked the students to check their answers before moving to the next exercises. The teacher complements feedback provided by the book when asked by the students, usually because they disagree with the answer provided by the book.

Table 4h.5: HH10T Group - analysis of the second and third set of headings over three lessons.

APPENDIX 5
SUMMARY OF THE RESULTS OF THE TESTS

Appendix 5a: Teacher A, School A

5a.1: Results of the test of group AA8X:

Stud. \Quest.	1a *	1b*	2a	2 b	3a**	3b* *	4**	5	6	7	8* *	9*	10**	% part 1	% part 2	% grade
1	1	1	0	0	0	0	1	0	0	0	0	0	0	43	0	23
2	1	1	1	1	1	1	1	1	1	1	1	1	0	100	83	92
3	1	1	1	1	1	1	1	1	1	1	1	1	1	100	100	100
4	1	0	1	0	0			0	1	1	0			29	33	31
5	1	1	1	1	1	0		0	0	1	0	1	0	71	33	54
6	1	1	1	1	1	1	1	1	1	1	1	1	1	100	100	100
7	1	0	1	0	0	0	0	1	1	1	1	1	0	29	83	54
8	1	1	1	1	0	0	0	1	1	1	1	1	0	57	83	69
9	1	1	1	0	1	1	1	1	1	1	1	1	1	86	100	92
10	1	1	0	0	0	0	0	0	1	1	1	1	0	29	67	46
11	1	1	1	0	1	0		1	1	1	0	1	0	57	67	62
12	1	0	0	0						1				14	17	15
13	1	1	1	0	1		1	0	1	1	1	1		71	67	69
14	1	1	1	0	0	0	1	1	1	1	0	1	0	57	67	62
15	1	1	1	0	1		1	0	1	1		0	1	71	50	62
16	1	0	0	0	0	0	1	0	0	1	0	0	0	29	17	23
17	1	1	1	0	1	1	1	1	1	1	1	1	1	86	100	92
18	1	1	1	1	1	1	1		1	0	1	1	0	100	50	77
19	1	1	1	1	1	0	1	0	1	1	1	1	0	86	67	77
20	1	1	1	0	1	1	0	0	0	1	1	1	0	71	50	62
21	1	0	1	0	0	0	0	0	0	0	0	0		29	0	15
22	1	1	1	1	1	0	1	1	1	1	0	1	0	86	67	77
23	1	1	0	0	0	0		1	0	1	0	1		29	50	38
24	1	0	1	0	0	0	0	1	1	0	1	0		29	50	38
25	1	1	1		1	0		0	1	1	1	1	1	57	83	69
26	1	1	1	0	1	1	0	0	1	0	1	1	1	71	67	69
N correct	26	20	21	8	15	8	13	12	19	21	15	19	7			
N try	26	26	26	25	25	22	20	24	25	26	24	24	20			
% correct	100	77	81	31	58	31	50	46	73	81	58	73	27			
% try	100	100	100	96	96	85	77	92	96	100	92	92	77			
N. C. Level	4	4	4	4	6	6	6	4	5	4	5	4	5			
N. C. ATs.	4	4	4	4	4	4	4	3	3	3	3	3	3			

in the table above, questions 1 to 4 are related with the first choice of materials (draw transformed shapen on isometric paper, based on textbook series),while questions 5 to 10 are related with the second choice (primes, squares, roots, etc. using no written material)

table 5a.1: results of the test for group AA8X.

5a.2: Results of the test of group AA9L:

Stud/Qst .	1a*	1b*	2a*	2b*	3a	3b	4a**	4b**	5a	5b	6a*	6b*	7a	7b	8a**	8b**	9a	9b	%p1	%p2	% gr.
1	1	1	1	1	1	0			1	0	1								60	17	50
2	1	0			1	0			1	0									30	0	21
3	1	1	1	1	1	0			1	0	1	1							60	33	57
4	0	0	1	1							1								20	17	21
5	1	0	1	0	1	0			1	0	1	1	1	0			1	0	40	67	57
6	1	1			1	0	1	0			1								40	17	36
7	1	0	1	1	1	0			1	0									50	0	36
8	1	1	1	1							1	1							40	33	43
9	1	1			1	0													30	0	21
10	1	1	1	1	1	1			1	0	1	1	1	0			1	0	70	67	79
11	1	1	1	1	1	1	1	1	1	1	1	1	1	0					100	50	93
12	1	1	1	1	1	0					1	1							50	33	50
13	1	1	1	0							1	1							30	33	36
14	1	1	1	1	1	0			1	0	1	1							60	33	57
15	1	1	1	1	1	0			1	0	1	1							60	33	57
16	1	1	1	1	1	1	1	1	1	1	1	1							100	33	86
17			1	0								1							10	17	14
18	1	1	1	1	1	0	1	1			1		1	1	1	1			70	83	86
N correct	16	13	15	12	14	3	4	3	10	2	14	11	4	1	1	1	2	0			
N try	17	17	15	15	14	14	4	4	10	10	14	11	4	4	1	1	2	2			
% correct	89	72	83	67	78	17	22	17	56	11	78	61	22	6	6	6	11	0			
% try	94	94	83	83	78	78	22	22	56	56	78	61	22	22	6	6	11	11			
N. C. Level	5	5	3	3	5	5	6	6/3	6	6	6	6	4	4	6	6/3	5	6			
N. C. ATs.	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1			

in the table above, questions 1 to 5 are related with the initial investigation and question 6 to 9 are related with the extension of the investigation. Each 'question' represents one step in the investigation (Algebra related)

table 5a.2: results of the test for group AA9L

Observation: In this case, the teacher did not expect the students to do all the items, but to choose some of them. So, the percent grade was calculate considering 14 as the total number of items, instead of 18. Notice that no student tried more than 14 items. Percentage grade in P2 was calculated for a total of six items.

5a.3: Results of the test of group AA10M:

Stud./Quest.	1*	2*	3	4	5	6**	7**	8**	9**	% grade
1	1	1	0			0				22
2	1	1	1	1	1	1				67
3	1	1	1	1	1	1				67
4	1	1	1	1	1	1				67
5	1	1	0		1	1				44
6	1	1	1	1	1	0				56
7	1	1	1	0		0				33
8	1	1	1	1		0				44
9	1	1	0			0				22
10		1	1		1	1	1			56
11	1	1	1	0		0				33
12		1	1	1	1	0				44
13	1	1	1		1	0				44
14	1	1	0			0				22
15	1	1	1			0				33
16	1	1	1	1	1	0				56
17	1	1	0			0				22
18	1	1	0			0				22
N correct	16	18	12	7	9	5	1	0	0	
N try	16	18	18	9	9	18	1	0	0	
% correct	89	100	67	39	50	28	6	0	0	
% try	89	100	100	50	50	100	6	0	0	
N. C. Level	5	3	5	6	5	3	5	6	4	
N. C. ATs.	1	1	1	1	1	1	1	1	1	

in the table above, each 'question' represents one step in the investigation

table 5a.3: results of the test for group AA10M

5a.4: Results of the test of group AA10T:

Stud./Quest.	1a*	1b	2a	2b**	2c**	3a*	3b	3c**	3d**	% grade
1	0	0	1	0	1	1	0	0	0	33
2	1		0	0						11
3	1	1	0			1	1			44
4	1	0	1			1	1	1		56
5	1					1	1	1		44
6	0		0	0						0
7	1					1	1			33
8	1	0	0	0	0	1				22
9	1	0				1	1			33
10	1	1	1	1	0	1	1	1	1	89
11	1	0	1	0	1	1				44
12	1	0				1	1			33
13	1	0	0			1				22
14	1	1	0			1	1			44
15	1	1	0			1	1	1	1	67
16	0	0				1	1			22
17	0					1	1			22
18	1	0	0			1	1			33
19	1	1	0			1				33
20	1	0	0			0				11
21	1	0	1	0		1	1	1	1	67
22	1	1	0	1	0	1	1			56
23	1	0	0	1		1	1			44
24	1	0	0			1	1			33
25	1	1	0	1		1				44
26	1	1	0			1	1			44
27	1	1	0			1	1			44
28	0		0			1	1			22
29	1	0				1				22
N correct	24	9	5	4	2	26	19	5	3	
N try	29	23	22	10	5	27	20	6	4	
% correct	83	31	17	14	7	90	66	17	10	
% try	100	79	76	34	17	93	69	21	14	
N. C. Level	5	5	5	6	6	6	6	6	6	
N. C. ATs.	4	4	4	4	4	4	4	4	4	

in the table above, questions 1 and 2 are related with internal and external angles of regular polygons and question 3 is related with procedures in LOGO to draw regular polygons.

table 5a.4: results of the test for group AA10T

Appendix 5b: Teacher B, School B

5b.1: Results of the test of group BB10X:

Observation: During the observed lessons, this group was split into two sub-groups, using different tracks of the textbook (SMP): 'yellow' and 'red'. Figure 5b.1 compare the results of the sub-groups.

Red Sub-group:

Stud/Quest.	1a*	1b	1c	1d	1e**	2a*	2b	2c	3**	%grade
red1	1	1	1	0		1	1	0	0	56
r2	1	1	0	1	0	1	1	0		56
r3	0	1	1	1	1	1	0	0	0	56
r4	1	1	1			1	1			56
r5	1	1	1	1	1	1	1	1	1	100
r6	1	0	0	0		0		0		11
r7	1	1	1	1	1	1	1	1	0	89
r8	1	1	0	0	0	1	1	1		56
r9	1	1				1	1	1	1	67
r10	1	1	0	1	1	0	1	1	1	78
r11	1	1	1	1	0	1	1	0	0	67
r12	1		1	1	0	1	1			56
r13	1	1	1	1	1	1	1	1	1	100
r14	1					1	1	0	0	33
r15	1	1	1	0	0	0	1	1	1	67
r16	1	1	1	1	1	1	1	0	0	78
r17	1	1	0	1	1	1	0	1	1	78
N correct	16	14	10	10	7	14	14	8	6	
N try	17	15	15	14	12	17	16	15	12	
% correct	94	82	59	59	41	82	82	47	35	
% try	100	88	88	82	71	100	94	88	71	
N.C. levels	6	6	6	6	6	6	6	6	8	
N.C. ATs	4	4	4	4	4	2	2	2	3	

in the table above, question 1 is on angles and questions 2 and 3 are on proportionality.

table 5b.1: results of the test for Group BB10X - Red Sub-group

Yellow Sub-group:

Stud/Quest	1a*	1b	2a	2b**	3a*	3b	3c	3d**	% grade
yellow 1	1	1	1	0	1	1	1	1	88
y2	1	1	0		1	1	1	0	63
y3	1	0	0		1	1	1		50
y4	1	1	1	1	1	1	1		88
y5	1	1	1	1	1	1	1	1	100
y6	1	1	0	0	1	0	0	1	50
y7	1	1	1	1	0	0	1	0	63
y8	1	1	1	1	1	1	1	1	100
y9	1				1	1			38
y10	1	1	0	0	1	1	1	1	75
y11	1	1	1	0	1	1			63
y12	1	1	1	1	1	1	0	1	88
N correct	12	10	7	5	11	10	8	6	
N try	12	11	11	9	12	12	10	8	
% correct	100	83	58	42	92	83	67	50	
% try	100	92	92	75	100	100	83	67	
N.C. Levels	8	8	8	8	7	7	7	7	
N.C. ATs	3	3	3	3	3	3	3	3	

in the table above questions 1 and 2 are on proportionality and question 3 is on gradient

table 5b.2: results of the test for Group BB10X - Yellow Sub-group

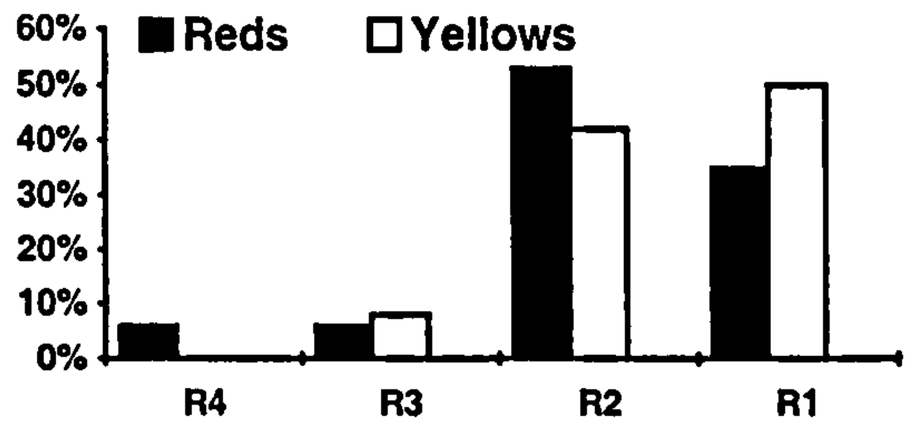


figure 5b.1: Bar chart of the percentage distribution of the students into the ranges

5b.2: Results of the test of group BB9L:

Stud/Quest.	1a*	1b*	2a	2b	3a**	3b**	4*	5	6**	% part 1	% part 2	% grade
1	0	0	1	0	1	1	1	0		50	33	44
2	1	1	1	1	1	1	1	1	0	100	67	89
3	1	1	1	1	1	1	1	0		100	33	78
4	1	1	1	1	0	1	0	1	0	83	33	67
5	1	1	1	1	0	0	1	1	1	67	100	78
6	1	1	1	0	0	0	0			50	0	33
7	1	1	0	0	1	1	1	1	0	67	67	67
8	1	1	1	1	1	1	1	1	1	100	100	100
9	1	1	1	0	1	1	1	0		83	33	67
10	1	1	1	1	1	1	1	1		100	67	89
11	1	1	1	0	1	0	1	0	1	67	67	67
N correct	10	10	10	6	8	8	9	6	3			
N try	11	11	11	11	11	11	11	10	6			
% correct	91	91	91	55	73	73	82	55	27			
% try	100	100	100	100	100	100	100	91	55			
N.C. levels	3	3	3	3	3	3	3	4	4			
N.C. ATs	2	2	2	2	2	2	2	2	2			

in the table above, questions 1,2 and 3 were common for all students and related with paying market bills and giving change. questions 4, 5, and 6 were individualised questions taken from SMP G3

table 5b.3 results of the test for group BB9L

5b.3: Results of the test of group BB11M:

Observation: During the observed lessons, this group was split into two sub-groups, using different tracks of the textbook (SMP): 'red' and 'blue'. Figure 5b.2 compare the results of the sub-groups.

Red Sub-group:

Stud/Quest.	1*	2a	2b	3a	3b	4a**	4b**	% grade
r1	0	1		1	1	1	.	57
r2	1	1	1	1	1	1	1	100
r3	1	1	1	1	1	1	1	100
r4	1	1	1	1	1	1	1	100
N correct	3	4	3	4	4	4	3	
N try	4	4	3	4	4	4	3	
% correct	75	100	75	100	100	100	75	
% try	100	100	75	100	100	100	75	
N.C. levels	6	6	6	6	6	6	6	
N.C. ATs	2	2	2	2	2	2	2	

in the table above, all questions were individualised questions taken from SMP red track.

tables 5b.4: Results of the test from group BB11M - Red sub-group.

Blue Sub-group:

Stud./Quest.	1	2a	2b**	2c**	3	4a*	4b*	4c	4d	4e	5**	% grade
b1						1	1	1	1	1	0	45
b2	0					1	1	1	1	1	1	55
b3	1	1	1	1	1	1						55
b4	0	0	0		0	0	0	0	0	0	1	9
b5						1	1	1	1	1		45
b6	1	1	1	1	1	1	0	0	0	0	1	64
b7		0	0	0	0	1	1	1	1	1	1	55
b8	0				0	1	1	1	1	1		45
b9	0		0	0	0						0	0
N correct	2	2	2	2	2	7	5	5	5	5	4	
N try	6	4	5	4	6	8	7	7	7	7	6	
% correct	22	22	22	22	22	78	56	56	56	56	44	
% try	67	44	56	44	67	89	78	78	78	78	67	
N.C. levels	6	6	6	6	6	6	6	6	6	6	6	
N.C. ATs	2	2	2	2	2	2	2	2	2	2	2	

in the table above, questions 1 and 2 were on 'value for money', and question 4 was on 'mapping' and the others were individualised ones, taken from the SMP blue track.

table 5b.5: Results of the test from group BB11M - Blue sub-group.

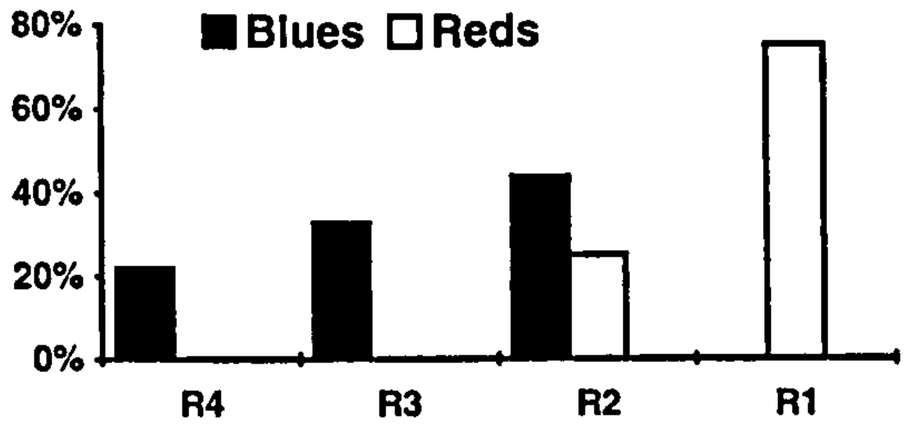


figure 5b.2: Bar chart of the percentage distribution of the students into the ranges

5b.4: Results of the test of group BB9T:

Stud./Quest	1a*	1b*	2a	2b	2c	2d**	3a*	3b	3c**	3d**	% grade
1	1	1	1	1	0	1	1	1			70
2	1	1	1	1	1	1	1	1	1	1	100
3	1	1	0	0	1	0	1	1	1	1	70
4	1	0	1	1	0	1	1	1	1	0	70
5	1	1	1	1	1	0	1	0	0	0	60
6	1	1	1	1	1	1	1	1	1	1	100
7	1	1	1	1	1	0	1	1	1	1	90
8	1	1	1	1	1	0	1	1	1	1	90
9	1	1	1	0	0	0					30
10	1	1	1	0	0	1	1	1	1	1	80
11	1	1	1	0	0	1	1	1	1	1	80
12	1	0	1	1	1	1	1	1	1	1	90
13	1	1	1	1	1	0	1	1	1	1	90
14	1	1	1	0	0	0	1	1	1	1	70
15	1	1	1	1	1	1	1	1	1	1	100
16	1	1	1	1	1	0	1	1	1	1	90
17	1	1	1	1	1	1	1	1	1	1	100
18	1	1	1	0	0	0	1	1	1	1	70
19	1	1	1	1	1	1	1	1	1	1	100
20	1	1	0	0	0	0	1	0	0	1	40
21	1	1	1	1	1	1	1	1	1	1	100
22	1	1	0	1	1	0	1	1	1	1	80
23	1	1	0	0	0	1	1	1	1	1	70
24	1	1	1	1	1	1	1	1	1	1	100
25	0	1	1	1	1	1	1	1	1	1	90
26	1	1	1	1	1	1	1	1	1	1	100
27	1	0	0	0	0	0	1	1	0	0	30
28	1	1	1	1	1	1	1	1	1	1	100
N correct	27	25	23	19	18	16	27	25	23	23	
N try	28	28	28	28	28	28	27	27	26	26	
% correct	96	89	82	68	64	57	96	89	82	82	
% try	100	100	100	100	100	100	96	96	93	93	
N.C. levels	8	8	8	8	8	8	7	7	7	7	
N.C. ATs	2	2	2	2	2	2	3	3	3	3	

in the table above, questions 1 and 2 are on Standard Index Form and question 3 was on decimal searching.

table 5b.6: Results of the test for BB9T.

Appendix 5c: Teacher C, School C

5c.1: Results of the test of group CC8X:

Stud/Quest.	1*	2*	3	4	5	6	7**	8**	% grade
1	1	1	1	0	0	1	1	0	63
2	1	1	1	1		1			63
3	1	1	1	1	1	1		1	88
4	1	0	1	1	0	1	0	0	50
5	1	1	1	1	1	0	1	1	88
6	1	1	0	0	1	0			38
7	1	1	1	1	1	1	1	1	100
8	0	1	1			1			38
9	1	1	1	1	1	1	0	0	75
10	1	1	0	1	0	1	1	0	63
11	1	1	1	1	1	1	1	1	100
12	1	0	1	0	0				25
N correct	11	10	10	8	6	9	5	4	
N try	12	12	12	11	10	11	7	8	
% correct	92	83	83	67	50	75	42	33	
% try	100	100	100	92	83	92	58	67	
N.C. levels	4	4	4/5	4/5	4/5	4/5	4/5	4/5	
N.C. ATs	#	#	#	#	#	#	#	#	

in the table above, all questions were individualised, taken from SMP booklets.

table 5c.1: results of the test for group CC8X.

5c.2: Results of the test of group CC8L:

Quest. Stud.	1a *	1b *	2a	2b **	3a	3b **	4 *	5 *	6	7	8 *	9	10	11 **	12 *	13	14 **	15 **	% P1	% P2	% grade
1	1	1	1	0	1	0	1	1	0	0	1	0			0	0	0	0	67	25	39
2	1	1	1	0	1	0	1	1	1	1	1	1	1	0	0	0			67	58	61
3	1	1	1	0	1	1	1	1	0	1	1	0	1	0	1	1	1	0	83	67	72
4	1	1	1		1	1	1	1	1	0	0	0	1	0	0	1		0	83	42	56
5	1	1	1		1	1	1	1	1	1	1	1	1	1	1	1	0	0	83	83	83
6	1	1	1	0	1	0	1	1	1	1	1	1	1	1	1	0	1		67	83	78
7	1	1	1	1	1	0	1	1	1	1	1	1	1	0	1	1	0	0	83	75	78
8	1	1	1		1	0	1	1		1	0	0	1	0	0	0	0	0	67	33	44
9	1	1	1		1	1	1	1	1	1	0	1	1	1	0	0			83	58	67
10	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	0	1	100	83	89
11	1	1	1	0	1	0	1	1	1	1	1	1	1	0	1	1		1	67	83	78
12	1	1	1	0		1	1	1	1	1	1	1	1		1	1	1	1	67	92	83
13	1	1	1		1	0	1	1	1	1	1	1		0	1	1			67	67	67
14	1	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	33	8	17
15	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	83	92	89
N correct	15	13	15	3	13	7	14	14	12	12	11	10	12	4	9	9	3	4			
N try	15	15	15	10	14	15	15	15	14	15	15	15	13	13	15	15	10	11			
% correct	100	87	100	20	87	47	93	93	80	80	73	67	80	27	60	60	20	27			
% try	100	100	100	67	93	100	100	100	93	100	100	100	87	87	100	100	67	73			
N.C. levels	2	2	2	2	2	2	3	3	3	3	3	3	3	3	4	4	4	4			
N.C. ATs	4	4	4	4	4	4	2	2	2	2	2	2	2	2	2	2	2	2			

in the table above, questions 1,2 and 3 are about recognising polygons, questions 4 to 7 are on addition, questions 8 to 11 are on subtraction and questions 12 to 15 are on division.

table 5c.2 results of the test for group CC8L

5c.3: Results of the test of group CC10M:

Stud/Quest	1a*	1b*	2a	2b	3a	3b	4**	5a	5b**	5c**	6*	7	8	9**	% P1	%P2	% grade
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	100	100	100
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	100	100	100
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	100	100	100
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	100	100	100
5	1	1	1	1	1	1	1	1	1	1	1	1	1	0	100	75	93
6	1	1	0	1	1	1	1	1	1	1	1	1	1	1	90	100	93
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	100	100	100
8	1	1	1	1	1	1	1	1	1	1	1	1	1	0	100	75	93
9	1	1	1	1	1	1	1	1	1	0	1	1	1	1	90	100	93
10	1	1	1	1	1	0	0	1	1	1	1	1	1	0	80	75	79
11	1	1	1	1	1	1	1	1	1	1	1	1	0	1	100	75	93
12	1	1	0	1	1	1	1	1	1	0	1	1	1	0	80	75	79
13	1	1	1	1	1	1	1	1	1	1	1	1	1		100	75	93
14	1	1	1	1	1	0	1	1	1	0	1	0	1	1	80	75	79
15	1	1	1	1	1	1	0	1	0	0	1	1	1	0	70	75	71
16	1	1	1	1	1	0	1	1	1	1	1	1	1	1	90	100	93
17	1	1	1	1	1	1	1	1	1	0	1	1	1	1	90	100	93
18	1	1	1	1	1	0	0	1	1	0	1	1	0	1	70	75	71
19	1	1	1	1	1	1	1	1	1	1	1	1	1		100	75	93
20	1	1	1	1	1	1	1	1	1	0	1	0	1	1	90	75	86
21	1	1	1	1	1	1		1	1	1	1	1	1	0	90	75	86
22	1	1	0	1	1	0	0	1	0	0	1	1	1	0	50	75	57
23	1	1	0	1	1	1	0	1	1		1	0	1	1	70	75	71
24	1	1	1	1	1	1	1	1	1	0	1	1	1		90	75	86
25	1				1	1	0	1	1	1	1	1	1	0	60	75	64
26	1	0	0	1	1	1	1	1	1	0	1	0	1	1	70	75	71
27	1	1	1	1	1	1		1	1	0	1	0	1	0	80	50	71
28	1	0	1	1	1	1	0	1	1		1	1	0		70	50	64
N correct	28	25	22	27	28	23	19	28	26	15	28	23	25	15			
N try	28	27	27	27	28	28	26	28	28	26	28	28	28	24			
% correct	100	89	79	96	100	82	68	100	93	54	100	82	89	54			
% try	100	96	96	96	100	100	93	100	100	93	100	100	100	86			
N.C. Levels	6	6	6	6	6	6	6	6	6	6	6	6/7	6/7	6/7			
N.C. ATs.	4	4	4	4	2	2	2	3	3	3	#	#	#	#			

in the table above, questions 1,2 are on angles, questions 3 and 4 are on proporcionality, question 5 is on equations and questions 6 to 9 are individualised, taken from the textbook (SMP).

table 5c.3 results of the test for group CC10M

5c.4: Results of the test of group CC11T:

Stud/Quest	1*	2*	3	4	5	6	7**	8**	% grade
1	1	1	1	1	1	1	1	0	88
2	1	1	1	1	0	1	1	1	88
3	1	1	1	1	1	1			75
4	1	1	1	1	1	1	1	1	100
5	1	0	1	1	1	1	1		75
6	1	1	1	1	1	1	1	1	100
7	1	1	1	1	1	0	0	1	75
8	1	1	1	1	1	1	1	1	100
9	1	1	1	1	1	1	1	1	100
10	1	1	0	0	1	1	0	0	50
11	1	1	1	1	0	1	1		75
12	1	1	1	1	1	1	1	1	100
N correct	12	11	11	11	10	11	9	7	
N try	12	12	12	12	12	12	11	9	
% correct	100	92	92	92	83	92	75	58	
% try	100	100	100	100	100	100	92	75	
N.C. levels	7/8	7/8	8	8	8	8/9	8/9/10	8/9/10	
N.C. ATs	#	#	#	#	#	#	#	#	

in the table above, all questions are individualised, taken from the textbook (SMP) or from review material based on the book..

table 5c.4 results of the test for group CC11T

Appendix 5d: Teacher D, School D.

5d.1: Results of the test of group DD7X:

Stud/Quest	1a*	1b*	2	3a	3b	4a**	4b**	5*	6	7	8**	% P1	% P2	% grade
1	1	1	0	1	1	0	0	1	1	0	0	57	50	55
2	1	1	0	1	1	0	0	1	0	1	0	57	50	55
3		1	1	1	1	1	1	1	1	1	1	86	100	91
4	1	1	1	1	1	1	0	1	1	1	0	86	75	82
5	1	1	1	1	0	1	1	1	0	1	1	86	75	82
6	1	1	1	1	1	0	0	1	1	0	0	71	50	64
7	1	1	1	1	0	1	1	1	1	1	0	86	75	82
8	1	1	0	1	0	1	1	1	0	0	1	71	50	64
9	1		0	1	0	0	0	1				29	25	27
10	1	1	1	1	1	1	1	1	1	1	1	100	100	100
11	1	1	1	1	1	1	1	1	1	0	1	100	75	91
12	1	1	1	1	1			1	1	1		71	75	73
13	1	1	1	0	0			1		0		43	25	36
14	1	1		1	0			1	0			43	25	36
15	1	1	0	1	1	1	0	1	1	0	0	71	50	64
16	1	1	0	1	1	1	0	1	1	1	0	71	75	73
17	1	1	0	1	1	1	1	1	1	1	1	86	100	91
18	1	1	0	1	1	1	1	1	0	1	0	86	50	73
19	1	1	1	1	1	1	0	1	1	1	0	86	75	82
20	1	1	1	1	1	1	1	1	1	1	1	100	100	100
21	1	1	0	1	0			1	0			43	25	36
22	1	1	1	1	0			1	1	0		57	50	55
23	1	1	1	1	0	0	0	0	1	0	0	57	25	45
N correct	22	22	13	22	14	13	9	22	15	12	7			
N try	22	22	22	23	23	18	18	23	21	20	17			
% correct	96	96	57	96	61	57	39	96	65	52	30			
% try	96	96	96	100	100	78	78	100	91	87	74			
N.C. level	5	5	5	5	5	5	5	4	4/5	4/5	4/5			
N.C. ATs	2	2	2	2	2	2	2	#	#	#	#			

in the table above, questions 1 to 4 are on ratio and the others are individualised, taken from the SMP booklets

table 5d.1: results of the test for group DD7X

5d.2: Results of the test of group DD10L:

Stud/Quest	1*	2*	3	4	5	6	7**	8**	% grade
1	1	1	1	1	1	0	0	0	63
2	1	1	1		1		1		63
3	0	1	1	0	0	1		0	38
4	1	1	1	1	1	1	0	0	75
5	1			1	1	0	0	0	38
6	1	0	1	1	1	0	1		63
7	1	1			1	1		1	63
8	1	1	1	1	1	1	1	1	100
9	1	1	1	1	1	1	1	1	100
10	1	1	0	0	1	0	1	0	50
11	1	1	1	0	0	1	0		50
12	1	1	1	1	0	1			63
N correct	11	10	9	7	9	7	5	3	
N try	12	11	10	10	12	11	9	8	
% correct	92	83	75	58	75	58	42	25	
% try	100	92	83	83	100	92	75	67	
N.C. levels	3/4	3/4	3/4	4	4	4	4	4	
N.C. ATs	2/4	2/4	2/4	2/4	2/4	2/4	2/4	2/4	

in the table above, all questions are individualised, taken from the textbook

table 5d.2: results of the test for group DD10L.

5d.3: Results of the test of group DD9M:

Stud/Question	1*	2*	3	4	5	6	7**	8**	% grade
1	1	1	1	1	0	1	1	0	75
2	1	1	0	1	1	1	0	0	63
3	1	1	0	1	1	0	0		50
4	1	1	1		0	0			38
5	1	1	1	0	0		1		50
6	1	1	1	1	1	1			75
7	1	1	0	1	1	1	1	0	75
8	1	0		1		1	0		38
9	1	1	1	1	1	1		1	88
10	1	1	0	1		1	0		50
11	1	1	1	1	1	0			63
12	1	1	1	1	1	1	0	1	88
N correct	12	11	7	10	7	8	3	2	
N try	12	12	11	11	10	11	8	5	
% correct	100	92	58	83	58	67	25	17	
% try	100	92	92	83	92	67	66	42	
N.C. levels	5	5	5	5	5	5	5/6	5/6	
N.C. ATs	2, 3 or 4								

in the table above, all questions are individualised, taken from SMP textbook

table 5d.3: results of the test for group DD9M.

5d.4: Results of the test of group DD9T:

Stud/Quest	1a*	1b*	2a	2b	3a	3b	3c**	3d**	4a*	4b	4c	4d**	4e**	% grade
1	0	0	1	1	1	1	1	1	1	1	1	1	1	85
2	1	1	1	1	1	1	1	1	1	1	1	1	1	100
3	1	0	1	1	1	1	0	1	1	1	1	1	1	85
4	1	1	0	0	1	1	0	1	1	1	1	0	1	69
5	0	0	1	0	1	0	0	1	1	1	0	1	1	54
6	1	1	1	0	1	1	1	1	1	1	1	1	1	92
7	1	0	1	1	1	1	0	1	1	1	0	1	1	77
8	1	1	1	1	1	1	0	1	1	1	1	1	1	92
9	1	1	1	1	1	1	1	1	1	1	1	0	1	92
10	0	0	1	1	1	1	1	1	1	1	1	0	1	77
11	1	0	0	0	0	1	0	0	1	1	1	0	1	46
12	1	0	1	1	1	1	0	1	1	1	1	1	1	85
13	1	1	1	1	1	1	1	1	1	1	1	0	1	92
14	1	1	1	1	0	0	0	0	1	1	1	0	1	62
15	1	1	1	1	1	1	1	1	1	1	1	1	0	92
16	1	1	1	1	1	1	1	0	1	1	1	1	1	92
17	1	1	1	1	0	0	0	1	1	1	1	1	1	77
18	1	1	1	1	1	1	1	1	1	1	1	0	1	92
19	1	1	1	1	1	1	1	1	1	1	1	1	1	100
20	1	1	1	1	1	1	0	0	1	1	1	1	1	85
21	1	1	1	1	1	1	1	1	1	1	1	0	1	92
22	1	0	1	1	0	0	1	1	1	1	1	1	1	77
23	1	1	1	1	1	1	1	1	1	1	1	1	1	100
24	1	0	1	1	1	1	0	1	1	1	0	1	1	77
25	1	0	1	1	1	1	1	1	1	1	1	1	1	92
26	1	1	0	0	1	1	1	1	1	1	1	0	1	77
27	1	1	1	1	1	1	1	1	1	1	1	0	1	92
28	1	1	1	1	1	1	0	1	1	1	1	0	1	85
29	1	1	1	1	0	1	0	1						46
30	1	1	1	1	1	1	1	1	1	1	1	1	1	100
N correct	27	20	27	25	25	26	17	26	29	29	26	18	28	
N try	30	30	30	30	30	30	30	30	29	29	29	29	29	
% correct	90	67	90	83	83	87	57	87	97	97	87	60	93	
% try	100	100	100	100	100	100	100	100	97	97	97	97	97	
N.C. levels	6	6	6	6	6	6	6	6	8	8	8	8	8	
N.C. ATs	5	5	5	5	5	5	5	5	3	3	3	3	3	

in the table above, questions 1 to 3 are on probability and question 4 is on sketch a graph

table 5d.4: results of the test for group DD9T

Appendix 5e: Teacher E, School E.

5e.1: Results of the test of group EE7X:

Stud/Quest	1a*	1b**	1c*	1d	1e	1f**	2a*	2b	3a	3b	3c**	% P1	%P2	% grade
1	1	0	1	1	1	1	1	1	1	1	1	83	100	91
2	1	1	1	1	1	1	1	1	1	1	1	100	100	100
3	1	1	1	1	1	1	1	1	1	1	1	100	100	100
4	1	0	1	0	1	0	1	1	1	1	1	50	100	73
5	1	0	1	1	1	1	1	1	1	0	1	83	80	82
6	1	0	1	0	1	0	0	0	0	0	0	50	0	27
7	1	0	1	0	1	0	1	1	1	1	1	50	100	73
8	1	0	1	0	1	0	1	1	1	0	1	50	80	64
9	1	0	1	0	1	0	1	1	1	1	1	50	100	73
10	1	0	1	0	1	0	1	1	1	1	1	50	100	73
11	1	0	1	0	1	0	1	1	1	1	1	50	100	73
12	1	0	1	0	1	0	1	0	0	0	1	50	100	45
13	1	1	1	1	1	1	1	1	1	1	1	100	100	100
14	1	0	1	0	1	0	1	1	0	0	0	50	40	45
15	1	0	1	0	1	0	1	1	0	1	1	50	80	64
16	1	0	1	1	1	0	1	1	0	1	1	67	80	73
17	1	0	1	0	1	0	1	1	1	0	1	50	80	64
18	1	1	1	1	1	1	1	1	1	1	1	100	100	100
19	1	1	1	1	1	1	1	1	0	1	1	100	80	91
20	1	1	1	1	1	0	1	1	1	1	1	83	100	91
21	1	0	1	0	1	0	1	1	1	1	1	50	100	73
22	1	1	1	1	1	0	1	1	1	1	1	83	100	91
23	1	1	1	1	1	0	1	1	1	1	1	83	100	91
24	1	0	1	0	1	0	1	1	1	1	1	50	100	73
25	1	0	1	0	0	0	1	0	1	1	1	33	80	55
26	1	0	1	1	1	1	1	1	1	1	1	83	100	91
27	1	0	1	0	0	0			0	1		33	20	27
28	1	0	1	0	1	0	1	1	1	1	1	50	100	73
29	1	0	1	1	1	0	1	1	1	1	1	67	100	82
30	1	0	1	0	0	1	1	1	1	1	1	50	100	73
31	1	0	1	0	1	0	1	1	0	1	1	50	80	64
N correct	31	8	31	13	28	9	29	27	23	25	28			
N try	31	31	31	31	31	31	30	30	31	31	30			
% correct	100	26	100	42	90	29	94	87	74	81	90			
% try	100	100	100	100	100	100	97	97	100	100	97			
N.C. levels	4	4/5	4	4/5	4	4/5	4	4	4	4	4			
N.C. ATs	2	2	2	2	2	2	1	1	1	1	1			

in the table above, questions 1 is on rules of usage of parentesis on expressions, questions 2 and 3 are on the '1, 2, 3, 4' investigation.

table 5e.1: results of the test for group EE7X.

5e.2: Results of the test of group EE10L:

Stud./Quest.	1*	2	3	4	5**	6**	% grade
1	0	1	0	1	1		50
2	1	1	1	1	1	1	100
3	1	1	1	1	0	0	67
4	0	1	0				17
5	0	1	0	1	1	0	50
6	1	1	1	1	1	1	100
7	0	1	1	1	1		67
8	0	1	1	1	0	1	67
9	0	1	0	1	0		33
10	0	1	1	1	1	0	67
11	1	1	1	1	1	1	100
12	1	0	1	1	1	1	83
13	1	1	1	1	1		83
14	1	1	1	1	1	1	100
15	1	1	1	1	1	1	100
16	1	1	1	1	1	1	100
17	0						0
18	0						0
19		0					0
N correct	9	15	12	15	12	8	
N try	18	17	16	15	15	11	
% correct	47	79	63	79	63	42	
% try	95	89	84	79	79	58	
N.C. levels	4	4	4	4	4	4	
N.C. ATs	2	2	2	2	2	2	

in the table above, all questions are on 'planning' (time intervals), taken from the printed materials used during the lessons.
table 5e.2: results of the test for group EE10L.

5e.3: Results of the test of group EE9M:

Stud./Quest.	1a*	1b*	2a	2b	3a	3b	4a	4b	5a	5b	5c	6**	% grade
1	1	1	0	0	1	1	1	1	1	1	1		75
2	1	1	0	0	1	1	1	1	1	1	1		75
3	1	1	0		1	0	1		1	0	0		42
4	1	0	0	0		1		0	1	0	0	1	33
5	1	1		0	0	0	0	0	1	1	1		42
6	0	1	1	1		1	0	0	1	1	1		58
7	1	1	0	0	0	1	0	0	1	1	0		42
8	1	1	1	1	1	1	1	1	1	1	1	1	100
9	1	1	1	1	1				1	0			50
10	1	1	0	0	1	0	0	1	1	1	1		58
11	0	1	0		1	1	1	1	1	1	1		67
12	1	1	1	1	1	1	1	1	1	1	1		92
13	1	1	0	0	0		0	0	1	1	1		42
14	1	1	0		1	1	1	1	1	1	1		75
15	1	1		0	1		0		1	1	1		50
16	1	1	1	1	1	0	0	1	1	1	1		75
17	1	1	0	0		1			1	1	1	1	58
18	1	1	1	1	1	1			1				58
19	1	1	1	1	1		1		1	1	1		75
20	1	1	0	0		1			1	1	1		50
21	1	1	0	0	1	1	1	1	1	1	1	1	83
22	1	1	0	0	1	1	1	1	1	0	1		67
23	1	1	0			1			1	0	1		42
24	1	1	1	1	1	1	1	1	1	1	1	1	100
25	1	1	0	0	1	1	1	1	1	1	1	1	83
26	1	1		1	1	1		1	1	0	1		67
27	1	1	0	0	1	1	1	1	1	1	1	1	83
28	1	1	0	0	1		0	0	1	1	1		50
N correct	26	27	8	9	20	19	13	14	28	21	23	7	
N try	28	28	25	24	23	23	21	20	28	27	26	7	
%correct	93	96	29	32	71	68	46	50	100	75	82	25	
% try	100	100	89	86	82	82	75	71	100	96	93	25	
N.C. levels	4	4	4	4	4	4	4	4	4	4	4	4	
N.C. ATs	4	4	4	4	4	4	4	4	4	4	4	4	

in the table above, all questions are on coordinates of reflected shapes. The diagonal is used as mirror line in question 5 and question 6 did not have space for drawing, asking for conclusions.
table 5e.3: results of the test for group EE9M.

5e.4: Results of the test of group EE11T:

Stud/Quest	1a*	1b*	2a	2b	2c	2d	3a	3b**	4a**	4b**	% grade
1	1	1	1	1	1	1	1				70
2	1	1	1	1	1	1					60
3	1	1	1	1	1	1	1	1	1	0	90
4	1	1	1	1	1	1					60
5	1	1	1	1	1	1	1				70
6	1	1	1	1		1					50
7	1	1	1	1	1	1	0				60
8	1	1	1	1	1		0				50
9	1	1	1	1	1	1	1	1			80
10	1	1	1	1	1	1	1	1	1	1	100
11	1	1	1	1	1	1	1	1	1	1	100
12	1	0	1	1	1	1	0				50
13	1	1	1	1	1	1	1		0		70
14	0	0	1	1	1	1	1	1	1		70
15	0	0	1	1	1	1	0	0	1	1	60
16	1	0	1	1	1	1	0	0	1	0	60
17	1	1	1	1	0	1	0				50
18	1	1	1	1	1	0	1	0	1	0	70
19	1	1	1	1	1	1	1	0	1	0	80
20	1	1	1	1	1	1	1	1	1	0	90
21	1	1	1	1	1	1	1	1	1	0	90
22	0	0	1	1	1	0	0				30
23	1	0	1	1		1	1	0			50
24	1	0	1	1	1	1	1				60
25	1	1	1	1	1	1	1	1	1		90
N correct	22	18	25	25	22	22	15	8	11	3	
N try	25	25	25	25	23	24	22	13	12	9	
% correct	88	72	100	100	88	88	60	32	44	12	
% try	100	100	100	100	92	96	88	52	48	36	
N.C. levels	6	6	7	7	7	7	8	10	8	10	
N.C. ATs	2	2	4	4	4	4	3	3	3	3	

in the table above, question 1 is on place value in positive and negative numbers, question 2 is on locus, question 3a and 4a are on algebraic manipulation of equations and questions 3b and 4b are on maximun and minimun of quadractic expressions by transforming thrm in perfect squares.

table 5e.4: results of the test for group EE11T.

Appendix 5f: Teacher F, School F.

5f.1: Results of the test of group FF7X:

Stud/Quest	1a*	1b	2a*	2b	3	4**	5a	5b**	% grade
1	1	0	1	1				0	38
2	1	0	0	1	1	1	1	0	63
3	1	1	1	1	1	0	1	0	75
4	1	1	1	1	1	1	0	0	75
5	1	0	1	1	0	1			50
6	1	1	1	1			0	0	50
7	1	0	0	1	1	1	0	0	50
8	1	1	1	1	1	1	1	0	88
9	1	1	0	1	0	0	1	0	50
10	1	1	1	1	0		1	0	63
11	1	1	1	1	0	0	1	0	63
12	1	1	1	1	1	1	0	0	75
13	1	0	0		0	0			13
14	1	0	1	1	1	1	1	0	75
15	1	1	0	0	1	1	1	0	63
16	1	1	1	1	1	1	1	1	100
17	1	1	0	1	1	1	1	0	75
18	1	1	0	0	1	0	1	0	50
19	1	1	1	1	1	1	1	0	88
20	1	1	1	1	1	1	1	1	100
21	1	1	1	0	1	1	1	0	75
22	1	1	1	1	0	0	1	0	63
23	1	0	0	1	1	1	1	0	63
24	1	0	1	1	1	1	0	0	63
25	1	1	1	1	1	0	1	0	75
26	0	0	0	0	1	0	0	0	13
27	0	0	0		1	1	0	0	25
28	1	0	1	1	1	0	0	0	50
29	1	0	1	1	1	0			50
30					0	0			0
N correct	27	17	19	23	21	16	17	2	
N try	29	29	29	27	28	27	25	26	
% correct	90	57	63	77	70	53	57	7	
% try	97	97	97	90	93	90	83	87	
N.C. levels	5	5	5	5	5	5	5	5	
N.C. ATs	4	4	4	4	4	4	4	4	

in the table above, question 1 and 2 are on volumes, questions 3 and 4 ask the students to build a box with a given capacity, and question 5 is about comparison of capacities.

table 5f.1: results of the test for group FF7X.

5f.2: Results of the test of group FF7L:

Stud/Quest.	1*	2a*	2b	3a*	3b	3c	4**	5a	5b**	% grade
1	1	1	1	1	1	1	1	1	0	89
2	1	0	1	1	1	1	1	1	0	78
3	1	1	1	1	1	1	1	1	1	100
4	1	1	1	1	1	1	1	1	1	100
5	1	1	1	1	1	1	1	1	1	100
6	1	1	1	1	1	1	1	1	1	100
7	1	1	1	1	1	1	1	1	0	89
8	1	1	1	1	1	1	1	1	1	100
9	0	1	0	1	1	1	1	1	1	78
10	1	1	1	1	1	1	1	1	1	100
N correct	9	9	9	10	10	10	10	10	7	
N try	10	10	10	10	10	10	10	10	10	
% correct	90	90	100	90	100	100	100	100	70	
% try	100	100	100	100	100	100	100	100	100	
N.C. levels	4	4	2	5	2	2	5	3	5	
N.C. ATs	4	4	4	2	2	2	2	4	4	

in the table above, question 1 and 2 are on areas of rectangles, using grid and not, questions 3 and 4 (P1)are on halving and question 5 are on area of triangles.

table 5f.2: results of the test for group FF7L.

5f.3: Results of the test of group FF8M:

Stud/Quest	1*	2*	3	4	5	6	7**	8**	% grade
1	1	1	1	1	0		1		63
2	1	1	0	1	1		1		63
3	1	1	1	1	1	1	1	1	100
4	1	1	1	1	1	1	1	1	100
5	1	1	0	0	1	1	1		63
6	1	1	1	1	1	0			63
7	1	1	1	1	1	1	0		75
8	1	1	1	1	1	1	1		88
9	1	1		1	1	1			63
10	1	1	1	1	1	1	1		88
11	1	1	0	1	1	0	1		63
12	1	1	1	1	0	1			63
13	1	1	1	0	0	1	1		63
14	1	1	0		1	1	1		63
15	1	1	1		1		1		63
N correct	15	15	10	11	12	10	11	2	
Ntry	15	15	14	13	15	12	12	2	
% correct	100	100	67	73	80	67	73	13	
% try	100	100	93	87	100	80	80	13	
N.C. levels	4	4	4	4	4	5	5	5	
N.C. ATs	1	1	1	1	1	1	1	1	

in the table above, all questions are related with the proposed steps in the investigation 'octogon loops'

table 5f.3 results of the test for group FF8M.

5f.4.Results of the test of group FF8T:

Stud/Quest	1a*	1b*	2a*	2b	3a	3b	4a	4b	5a	5b	6a	6b**	7a**	7b**	% grade
1	1	0	1	1	1	0	0	0	1	1	1	1	0	0	57
2	0	0	0	1	1	1	0	0	0		1	0	0		29
3	1	1	0	1	1	1	1	1	0	1	1	1	0	0	71
4	0	0	0	1	1	1	0	0	0	0	0	0	0	0	21
5	1	1	1	1	1	1	0	0	0	0	0	0	0	0	43
6	1	0	1	0	0	0	1	1	0	0	0	0	0	0	29
7	0	0	0	0	0	0	0	0	0				0	1	7
8	1	1	1	1	1	1	1	1	1	1	0	0	1	1	86
9	0	0	0	0	0	0	0	0	0	0	1	1	0	0	14
10	1	0	0	0	1	1	0	0	0	0	0	0	0	0	21
11	0	1	0	1	1	1	0	0	1	1	1	0			50
12	1	1	1	1	1	1	0	0	0	1	1	0	0	0	57
13	1	0	1	1	1	0	1	1	1	1	1	1	0	0	71
14	1	0	1	1	1	1	1	0	0	0	1	1	0	0	57
15	0	0	0	0	1	1	0	1	1	0	0	0	1	0	36
16	1	1	1	1	0	1	1	0	0	0	1	0	0	0	50
17	1	1	1	1	1	1	1	1	1	1	1	1	1	0	93
18	1	1	1	1	0	1	1	0	1	1	1	1	0	0	71
19	1	1	1	1	0	1	1	1	1	1	1	1	0	0	79
20	1	1	1	1	1	1	0	1	1	1	1	0	1	0	79
21	1	1	1	1	0	1	0	0	1	1	0	0	1	0	57
22	0	0	1	0	0	0	0	0	1	1	0	1	1	1	43
23	0	0	0	0	1	0	1	0	0	0	1	1	0	0	29
N correct	15	11	14	16	15	16	10	8	11	12	14	10	6	3	
N try	23	23	23	23	23	23	23	23	23	21	22	22	22	21	
% correct	65	48	61	70	65	70	43	35	48	52	61	43	26	13	
% try	100	100	100	100	100	100	100	100	100	91	96	96	96	91	
N.C. levels	6	6	6	6	6	6	6	6	6	6	6	6	6	6	
N.C. ATs	4	4	4	4	4	4	4	4	4	4	4	4	4	4	

in the table above, question all questions are on bearings, taken from the textbook.

table 5f.4: results of the test for group FF8T.

Appendix 5g: Teacher G, School G.

5g.1: Results of the test of group GG7M :

Textbook Sub-Group:

Quest. Stud.	1a *	1b *	1c	1d	1e	2a	2b	2c	2d **	2e **	3a *	3b *	3c	3d	4a	4b **	5a	5b **	5c **	% P 1	% P 2	% grade
1	0	0	0	0	1	1	0	1	1	0	1	1	1	1	1	0	1	1	1	40	89	63
2	1	1	0	1	1	0	0	1	0	0	0	1	1	1	1	1	1	1	1	50	89	68
3	1	1	0	1	1	1	0	0	0	0	0	0	0	0	1	0	0	0	1	50	22	37
4	0	1	0	1	0	1	0	1	0	0	0	0	1	0	1	0	0	0	0	40	22	32
5	0	1	0	1	0	1	0	1	0	0	1	0	0	0	1	0	0	0	1	40	33	37
6	1	1	0	0	1	1	1	1	0	0	0	1	1	0	0	0	0	0	1	60	33	47
7	1	0	1	1	1	1	0	0	0	0	0	0	0	0	1	0	0	0	1	50	22	37
8	1	1	1	1	1	1	0	0	0	0	0	0	0	0	1	0	0	0	1	60	22	42
9	1	0	1	1	1	0	0	1	0	1	1	1	1	1	0	0	1	1	1	60	78	68
10	1	0	1	1	1	0	0	1	0	1	1	1	1	1	1	0	1	1	1	60	89	74
11	0	1	0	1	0	1	1	1	0	0	1	1	1	1	0	0	0	0	1	50	56	53
12	0	1	0	1	0	1	1	1	0	0	1	1	1	1	0	0	0	0	1	50	56	53
13	1	0	0	0	1	0	1	1	0	0	1	1	1	0	0	0	0	0	0	40	33	37
N correct	8	8	4	10	9	9	4	10	1	2	7	8	9	6	8	1	4	4	11			
N try	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13	13			
% correct	62	62	31	77	69	69	31	77	8	15	54	62	69	46	42	8	31	31	85			
% try	100% all questions										100% all questions											
N.C. levels	4	4	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5	5	5			
N.C. ATs	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4			

in the table above, questions 1 and 2 are based on class teaching, and common for both sub-groups, questions 3 is based on class teaching using the textbook and questions 4 and 5 are taken from the textbook. All questions on angles.

table 5g.1: results of the test for group GG7M - textbook sub-group.

Logo Sub-Group:

Qst. Std.	1a *	1b *	1c	1d	1e	2a	2b	2c	2d **	2e **	3a *	3b *	3c	4a	4b	4c	5a	5b **	6a **	6b **	% P1	% P2	% grade
1	1	0	1	1	1	0	0	1	0	0	1	1	1	1	1	1	0	0	1	0	50	70	60
2	1	0	0	1	1	1	1	0	0	0	1	1	1	1	1	1	0	0	1	0	50	70	60
3	1	0	0	1	1	1	1	0	0	0	1	1	1	1	1	1	1	0	1	1	50	90	70
4	0	0	0	1	1	0	1	0	0	0	1	0	1	1	0	0			1	0	30	40	35
5	1	0	1	1	0	0	0	1	0	0	1	1	1	1	1	1	1	0	0	0	40	70	55
6	1	0	1	1	0	1	0	1	0	0	1	1	1	1	1	1	1	1	0	0	50	80	65
N cor	5	0	3	6	4	3	3	3	0	0	6	5	6	6	5	5	3	1	4	1			
N try	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	6	5	5	6	6			
% cor	83	0	50	100	67	50	50	50	0	0	100	83	00	100	83	83	50	17	67	17			
% try	100	100	100	100	100	100	100	100	100	100	100	100	00	100	100	100	83	83	100	100			
N.C. level	4	4	4	4	4	4	4	4	4	4	4	4	4	5	5	5	5	5	5	5			
N.C. ATs	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4			

in the table above, questions 1 and 2 are based on class teaching, and common for both sub-groups, questions 3 to 6 are on logo, related to commands Foward and Rotate.

table 5g.2: results of the test for group GG7M - Logo sub-group.

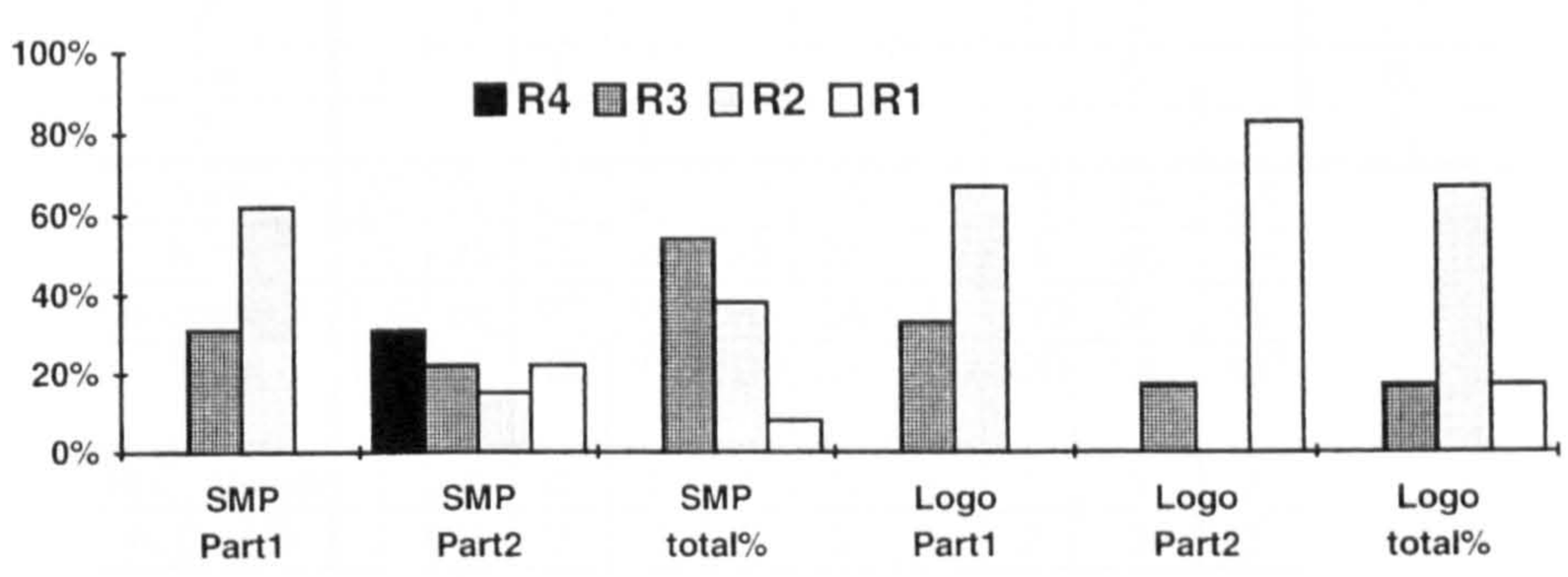


figure 5g.1: graph of the percentage of students in each range - textbook and logo sub-groups - group GG7M.

5g.2: Results of the test of group GG8T :

Stud/Quest	1a*	1b*	2a	2b	2c	3	4a	4b	4c**	5**	% grade
1	1	1	1	1	1	1	1	1	0	1	90
2	1	1	1	1	1	1	1	1	0	1	90
3	1	1	1	1	1	1	0	1	0	0	70
4	1	1	1	1	1	0	1	1	1	1	90
5	1	1	1	1	1	1	1	1	1	1	100
6	1	1	1	1	1	1	1	1	1	1	100
7	1	0	1	1	1	0	1	1	0	1	70
8	1	1	1	1	1	1	1	1	1	1	100
9	1	1	1	1	1	0	1	1	1	1	90
10	1	1	1	1	1	1	1	1	0	1	90
11	1	1	1	1	1	1	1	1	1	1	100
12	1	1	1	1	1	1	1	1	0	1	90
13	1	1	1	1	1	1	1	1	0	1	90
14	1	1	1	1	1	1	1	1	1	0	90
15	1	0	1	1	1	0	1	1	1	1	80
16	1	1	1	1	1	0	1	1	0	1	80
17	1	1	1	1	1	1	1	1	1	1	100
18	1	1	1	1	1	1	1	1	1	1	100
19	1	1	1	1	1	1	1	1	0	1	90
20	1	1	1	1	1	1	1	1	1	1	100
21	1	1	1	1	1	0	1	1	1	1	90
22	1	1	0	0	0	1	1	1	1	1	70
23	1	0	1	1	1	0	1	1	0	1	70
24	1	1	1	1	1	0	1	1	0	1	80
25	1	1	0	0	1	0	1	1	1	1	70
N correct	25	22	23	23	24	16	24	25	14	23	
N try	25	25	25	25	25	25	25	25	25	25	
% correct	100	88	92	92	96	64	96	100	56	92	
% try	100	100	100	100	100	100	100	100	100	100	
N.C. levels	6	6	6	6	6	6	6	6	6	5	
N.C. ATs	2	2	2	2	2	2	2	2	2	2	

in the table above, all questions are on decimals

table 5g.3: results of the test for group GG8T.

Appendix 5h: Teacher H, School H.

5h.1: Results of the test of group HH7X:

Stud/Quest	1*	2*	3a*	3b	3c	3d**	4a*	4b	4c	4d**	5a*	5b	5c**	5d**	Part 1	Part 2	% grade
1	1	0	1	0	1	0	1	1	0	0	1	1	0		50	50	50
2	1	1	1	1	1	0	1	1	1		1	1	1	0	80	75	79
3	1	1	1	1	1	0	1	0	1	1	1	1	0	0	80	50	71
4	1	1	1	1	1	0	1	1	1	0	1	1	0	0	80	50	71
5	1	1	1	1	1	1	1	1	1		1	0	0	0	90	25	71
6	1	1	0	1	1	0	1	1	1		1	1			70	50	64
7	1	1	1	0	1	0	1	1	1	1	1	1	0	0	80	50	71
8	1	1	1	1	0		1	0	1	0	1	1	1	1	60	100	71
9	1	1	1	1	0	1	1	1	0	0	1	1	0	1	70	75	71
10	1	1	1	1	1	0	1	1	0		0	0			70	0	50
11	1	1	1	1	1		1	1	0	1	1	0		0	80	25	64
12	0	1	1	0	1	1	1				1	0			50	25	43
13	1	1	1	0	1		1	0	0		1	1	1	1	50	100	64
14	1	1	1	1	0	1	1				1	0			60	25	50
15	1	0	1	1	0	0	1	1	0		0	0			50	0	36
16	1	1	1	1	1	1	1	0	1	1	1	1	0	1	90	75	86
17	1	1	1	1	0	0	1		0	0	1	1	0	0	50	50	50
18	0	1	0	1	1		0	1			1	1	1		40	75	50
19	1	1	1	1	1	1	1		1		1	1	0	0	80	50	71
20	1	0	1	0	1	1	1	1	0	0	1	0	0	0	60	25	50
21	1	1	1	1	0	1	1	1	1	1					80	0	64
22	1	1	1	1	0	0	0	1	0		1	0			50	25	40
23	1	1	1	1	1	0	1	1	0		1	1			70	50	64
24	1	1	1	1	1	1	1	1	1	1	1	1	1	0	100	75	93
25	1	1	1	0	1		1	0	1		1	0	1	0	60	50	57
26	1	1	0	1	1		1	1	0	0	1	0	0	0	60	25	50
27	1	1	1	1	0	0	1		1	0	1	1	0	0	60	50	57
28	1	1	1	0	1	1	1	1	0		1	1			70	50	64
29	1	1	1	1	1	0	1	1	1	0	1	1	0	1	80	75	79
N correct	27	26	26	21	22	10	27	19	14	6	26	19	6	5			
N try	29	29	29	29	29	23	29	24	26	15	28	28	19	18			
% correct	93	90	90	72	76	34	93	66	48	21	90	66	21	17			
% try	100	100	100	100	100	79	100	83	90	52	97	97	66	62			
N.C. levels	4	4	4	4	4	4	4	4	4	4	4	4	4	4			
N.C. ATs	2	2	2	2	2	2	2	2	2	2	5	5	5	5			

in the table above, questions 1 to 4 are on decimals, and question 5 is on handling data (tally, frequency and mean)

table 5h.1: results of the test for group HH7X.

5h.2: Results of the test of group HH10L:

Stud/Quest	1*	2*	3	4	5	6	7**	8**	% grade
1	1	1	1	0	1	1	0	1	75
2	1	1	1	1	1	0	1	0	75
3	1	1	0	1			0		38
4	1		1	1	0	1			50
5	1	1	1	1	1		1	0	75
6	0	1	1	1	1	1	0	1	75
7	1	1	1	1	1	1	1	1	100
8	1	1	0		0	0			25
9	1	1	0	1	0	1	0	0	50
N correct	8	8	6	7	5	5	3	3	
N try	9	8	9	8	8	7	7	6	
% correct	89	89	67	78	56	56	33	33	
% try	100	89	100	89	89	78	78	67	
N.C. levels	4	4	4	4	4	4	4	4	
N.C. ATs	2/4	2/4	2/4	2/4	2/4	2/4	2/4	2/4	

in the table above, all questions are individuallised, taken from SMP green series.

table 5h.2: results of the test for group HH10L.

5h.3: Results of the test of group HH9M:

Stud/Quest	1*	2*	3	4	5	6a	6b	6c**	6d**	7**	% grade
1	1	1	1	1	0	1	0	1	0	0	60
2	1	0	1	1	0	0	1	0	0	0	40
3	1	1	1	0	0	0					30
4	1	1	0	1	1			1			50
5	1	1	1	1	1	1	1	1	1	1	100
6	1	1	1	1	0	1	0	0	1		60
7	1	1	1	0	1	1	1				60
8	0	1	1	0		1	0	0			30
9	1	1	1	1	1	1	1				70
10	1	1	0	1	1	1	1	0	1	1	80
11	1	1	1	1	1	0	0	0			50
12	1	0	1	1	1	1	0		0	0	50
13	1	1	1	1	1	1	1	0	1	0	80
14	1	1	0	1	1	0	1			1	60
15	1	1	1	1	1	1	1	1	1	1	100
16	1	1	1	0	0	0	0	0	0	0	30
17	1	1	1	0	1	1	1				60
18	1	1	0	1	0	1	0	0	0	0	40
19	1	1	1	1	1	1	1	0	0	1	80
20	1	1	1	1	1	1	1	0	1	0	80
21	1	1	1	1	1	1	1	1	0	1	90
22	1	1	0	0	0	1	1	0			40
23	1	1	1	1			0			0	40
24	1	1		1		1	1	1			60
25	1	1	1	1	1	1	1		1		80
26	1	1	1	0	1	1	0	0	0	1	60
27	1	0	1	1	0	1	1	1	1	0	70
N correct	26	24	21	20	16	20	16	7	8	7	
N try	27	27	26	27	24	25	25	19	16	16	
% correct	96	89	78	74	59	74	59	26	30	26	
% try	100	100	96	100	89	93	93	70	59	59	
N.C. levels	6	6	6	6	6	7	7	7	7	7	
N.C. ATs	3	3	3	3	3	3	3	3	3	3	

in the table above, questions are on graphs of linear equations (scales, tables of values, ordered pairs) and gradient of slopes and linear graphs.

table 5h.3: results of the test for group HH9M.

5h.4: Results of the test of group HH10T:

Stud/Quest	1a*	1b*	1c	1d**	1e	2	3**	% grade
1	1	1	1	1	1	1	1	100
2	1	1	1	1	1	1	1	100
3	1	1	1	0	1	1		71
4	1	1	1	1	1	1	1	100
5	1	1	1	0	0	1	1	71
6	1	1	1	1	1	1	1	100
7	1	1	0	0	1	1	0	57
8	1	1	1	1	0	1	1	86
9	1		1	0	1	0	0	43
10	0	1	1	0	1	1	0	57
11	1	1	1			0	1	57
12	1	1	1	1	1	1	1	100
13	1	1	1	0	1	1	1	86
14	1	1	1	1	1	0	0	71
15	1	1	1	1	1	1	1	100
16	1	1	1		1			57
17	1	1	1	1	1	1	1	100
18	1	1	1		1	1		71
19	1	1	1	1	1	1	0	86
20	1	1	1	0	1	0	1	71
21	1	1	1	1	1	1	1	100
22	1	1	1	1	1	1	0	86
23	1	1	1	0	1	1	1	86
24	1	1	1	1	1	1	1	100
25	1	1	1	1	1	1	1	100
26	1	0	1	1	1	0	1	71
N correct	25	24	25	15	23	20	17	
N try	26	25	26	23	25	25	23	
% correct	96	92	96	58	88	77	65	
% try	100	96	100	88	96	96	88	
N.C. levels	10	10	10	10	10	10	10	
N.C. ATs	4	4	4	4	4	4	4	

in the table above, all questions are on applications of sine and cosine rules in two dimensional shape problems.

table 5h.4: results of the test for group HH10T.